11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
E. Oñate, J. Oliver and A. Huerta (Eds)

HARMONIC MODEL FOR NONLINEAR THERMO-MECHANICAL ANALYSIS OF HOT MILL ROLLS

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Key Words: *rolling mill, finite element, harmonic model, nonlinear analysis, multiphysics*

Abstract. Work rolls in hot rolling mills are usually subjected to cyclic thermal loadings, induced by strip heating followed by water jet cooling. This work aims to present a simplified method for assessing transient temperature and thermal stress distribution in work roll of hot rolling mill. A nonlinear elasto-plastic stress analysis is performed. In the approach, based on the finite elements (FE) method, only the work roll (without the strip) is considered with a two-dimensional model. Because the length of the roll, a plane strain state is assumed. The work roll is considered an axisymmetric structure loaded by non-axisymmetric thermal loads, which can be solved by a semi-analytical approach using a harmonic model and Fourier series expansion. The two-dimensional problem is then reduced to a one-dimensional model, with a significant decrease in overall computational time. A comparison of the results obtained by the proposed semi-analytical approach with those obtained by a two-dimensional FE model is presented.

1 INTRODUCTION

Work rolls in hot rolling mills are exposed to cyclic thermal and mechanical stresses. Thermal stresses are caused by a non-uniform temperature distribution, due to strip heating and water cooling, while mechanical stresses are produced by rolling pressures and contact actions with back-up rolls. In hot strip rolling, thermal stresses are usually comparable or even larger than mechanical stresses [1,2,3]. Thermal fatigue caused by cyclic thermal stresses, as well as other damage mechanisms (mechanical fatigue, wear, oxidation), induce a progressive surface deterioration in work rolls, which requires periodic replacing and dressing.

In order to estimate work roll surface deterioration and to conformingly schedule periodic maintenance operations, it is then required to evaluate the magnitude of thermal stresses. The analysis is generally very complex, so that analytical approaches are really impractical or even impossible. Conversely, numerical approaches based on the finite element (FE) method are very promising. Different modeling strategies, of different complexities, have been proposed in literature. Some of them model strip plastic deformation and strip/work roll mechanical interaction; computational resources and time then become extremely high and often unsuitable for current industrial needs [4,5]. An alternative simplified approach, instead, has been recently proposed, in the attempt to reduce the overall computational burden, while still providing accurate results. The approach is based on a plane (two-dimensional) model of only work roll, under rotating thermal loading [6,7]. A transient thermal and mechanical analysis was performed to estimate thermal stresses. Unexpectedly, this approach still required too high simulation times, especially for nonlinear mechanical analysis. This motivates the aim of the present work to develop a semi-analytical FE approach, which can significantly decrease the total simulation time compared to the above plane FE approach. Semi-analytical methods have been developed more than 50 years ago for FE analysis of axisymmetric structures loaded non-axisymmetrically [8]. Therefore, they can be effectively used to study a work roll in hot rolling mill, as it is an axisymmetric structure loaded by non-axisymmetric thermal loading (heating flux and convective cooling). Generally, such methods allow a three-dimensional (3D) model to be reduced to a plane (2D) model and the solution obtained as superposition of results of every component analysis, thus providing a significant reduction in computational time. For the specific application, the simplified plane model of work roll can be further analyzed by a one-dimensional (1D) FE analysis.

In the present work, a semi-analytical finite element approach is proposed in order to solve two problems: (1) transient thermal analysis of axisymmetric bodies under different types of non axis-symmetric thermal loads (temperatures, fluxes, convection) and (2) nonlinear elasto-plastic stress analysis. An 1D finite element model is developed for both thermal and structural problem. An efficient algorithm for fast and accurate time integration is also developed to solve thermal transient problems. An illustrative numerical example is finally discussed, to test the performance of the proposed semi-analytical FE approach. The example refers to an infinitely-long cylinder under rotating thermal loadings, which has been used as a simplified model of work roll in hot rolling mill. Results are compared with those given by a plane FE model. The proposed semi-analytical approach is shown to provide similar results, with a strong reduction in computational times compared to plane FE model.

2 SEMI-ANALYICAL APPROACH

A three-dimensional structure or solid is defined as "axially symmetric" or "axisymmetric" if its geometry, material properties and boundary conditions are independent of an azimuth coordinate θ of a cylindrical reference frame (r, θ, z), where z coincides with the component axis of symmetry and r is the radial distance from z-axis. Depending on the nature of external loads, two different situations can be identified. In the first, also the external loads are themselves axisymmetric with respect to same z-axis. In this case, the analysis is mathematically two-dimensional; the results are independent of θ and they are only function of r, z coordinates [9]. Examples are disks rotating at uniform speed under centrifugal forces, or cylindrical pressure vessels under internal pressure.

A second type of problem, of more practical interest, is when the structure is axially symmetric but the loading is not, so that the analysis is really three-dimensional. A great simplification can be achieved by using a so-called semi-analytical approach, based on a Fourier series method. The given loading is expanded in Fourier series, as a function of azimuth angle θ, and then replaced by the sum of several component loading (harmonics). For example in the case of the thermal analysis the boundary applied temperature becomes:

$$T(r,\theta,z)=\bar{\bar{T}}_0(r,z)+\sum_{n=1}^{\infty}\bar{\bar{T}}_n(r,z)\cos n\theta+\sum_{n=1}^{\infty}\bar{T}_n(r,z)\sin n\theta \tag{1}$$

where cosine and sine terms are, respectively, the axisymmetric and anti-axisymmetric loads. All barred quantities in Eq. (1) are amplitudes, which in a three-dimensional analysis, are functions of *r*, *z* and *n* but not of *θ*. The series is truncated to a finite number N of terms. A similar series is obtained for the known applied fluxes or convection. An analysis is then performed for each load component. In linearity, theory shows that the FE analysis becomes uncoupled due to the particular topology of stiffness matrix. According to the principle of superposition, the sought solution is obtained by adding the solutions of each load component. As an example the nodal solution for the node *i* can be written in Fourier series as well:

$$u_i=\bar{u}_{i0}+\sum_{n=1}^{\infty}\bar{\bar{u}}_{in}\cos n\theta+\sum_{n=1}^{\infty}\bar{u}_{in}\sin n\theta \tag{2}$$

where here symbol *u* indicates the temperature. As above, all barred quantities are amplitudes.

In the case of structural analysis, in cylindrical coordinates, the load is:

$$\begin{Bmatrix} R(r,\theta,z) \\ T(r,\theta,z) \\ Z(r,\theta,z) \end{Bmatrix}=\begin{Bmatrix} \bar{\bar{R}}_0+\sum_{n=1}^{\infty}\left(\bar{\bar{R}}_n(r,z)\cos n\theta+\bar{R}_n(r,z)\sin n\theta\right) \\ \bar{T}_0+\sum_{n=1}^{\infty}\left(\bar{\bar{T}}_n(r,z)\sin n\theta-\bar{T}_n(r,z)\cos n\theta\right) \\ \bar{\bar{Z}}_0+\sum_{n=1}^{\infty}\left(\bar{\bar{Z}}_n(r,z)\cos n\theta+\bar{Z}_n(r,z)\sin n\theta\right) \end{Bmatrix} \tag{3}$$

to which nodal temperatures found during the previous thermal analysis, now considered as input data, has to be added. The unknown displacements in cylindrical coordinates become:

$$\begin{cases} u_i(\theta)=u_{i0}+\sum_{n=1}^{\infty}\left(\bar{\bar{u}}_{in}\cos n\theta+\bar{u}_{in}\sin n\theta\right) \\ v_i(\theta)=v_{i0}+\sum_{n=1}^{\infty}\left(\bar{\bar{v}}_{in}\sin n\theta-\bar{v}_{in}\cos n\theta\right) \\ w_i(\theta)=w_{i0}+\sum_{n=1}^{\infty}\left(\bar{\bar{w}}_{in}\cos n\theta+\bar{w}_{in}\sin n\theta\right) \end{cases} \tag{4}$$

In the FE harmonic model, only a limited number of terms of Fourier series is considered, each of them leading to an algebraic linear system. The original three-dimensional problem is then replaced by a series of two-dimensional analyses, which are solved much more quickly than the original 3D problem. Furthermore, semi-analytical approach is even more suitable to solve a plane problem (as that discussed later on in the paper), as it reduces the 2D problem to a 1D one. A problem with thousands of degrees of freedom is then replaced with one having only tens.

3 2D STATIONARY ANALYSIS

In a thermal analysis the only degree of freedom per node is temperature. In a theoretical study, different types of harmonic finite elements have been formulated for a plane thermal analysis: two-node elements (with one or two Gauss points) having linear shape functions, three-node elements (with two or four Gauss points) having quadratic shape functions [10,11]. An example is shown in Figure 1a: the mesh of a plane axial symmetric solid of revolution is one-dimensional and consists of adjacent elements located along the radius of the solid, see Figure 1b.

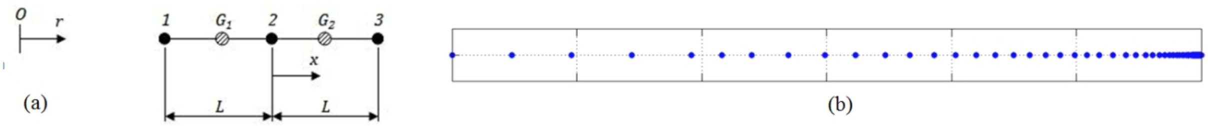

Figure 1: Three-node element with two Gauss points (a); Mesh (each dot point is a node) (b)

Due to orthogonality property of trigonometric terms of Fourier series, the element stiffness matrix is a block diagonal matrix: $[k_{el}] = diag\left(\underbrace{[\overline{k_0}],[\overline{k_1}],[\overline{k_2}]..[\overline{\overline{k_1}}],[\overline{\overline{k_2}}]..}_{2N+1}\right)$, where $[k_i]$ is the elementary stiffness matrix for i-th term of Fourier series and N is the number of harmonics. Explicit expressions for shape functions and "stiffness matrix" have been derived for each element type mentioned above [11,12]. As an example, for a two-node linear finite element it is:

$$[k_n] = \int_0^L [B_n]^T [D][B_n] r\,\mathrm{d}r, \quad [\overline{B_n}] = \begin{bmatrix} \frac{\partial N_1}{\partial r}\cos\theta & \frac{\partial N_2}{\partial r}\cos\theta \\ -\frac{n}{r}N_1 \sin\theta & -\frac{n}{r}N_2 \sin\theta \end{bmatrix}, \ [\overline{\overline{B_n}}] = \begin{bmatrix} \frac{\partial N_1}{\partial r}\cos\theta & \frac{\partial N_2}{\partial r}\cos\theta \\ \frac{n}{r}N_1 \sin\theta & \frac{n}{r}N_2 \sin\theta \end{bmatrix} \tag{5}$$

in which $[D]$ is a diagonal matrix with the material conductivity, $[B_n]$ is the "strain" - nodal temperature matrix for the n -th harmonic of Fourier series.

Different types of imposed boundary conditions (temperature, flux and convection) were also studied in detail. In FE model, an imposed temperature is treated as an imposed nodal displacement in a structural analysis. Similarly, a thermal flux applied on boundary elements is converted into equivalent nodal loads. Instead, special attention has been deserved to some particular boundary conditions. For example, the application of different temperatures on different angular sectors on the boundary poses particular numerical problems, due to one-dimensional nature of harmonic model. Three methods have been proposed [11,12]: two of them (fictitious thermal flux, imposed temperature) are based on Fourier series expansion of the imposed temperature, the third is based on Lagrange multipliers. Illustrative examples showed that the performance of all three methods is actually comparable, although the third method is more flexible to represent various boundary conditions. For convection heat exchange, instead, see the final part of next Section.

For structural analysis the scheme is the same, although the matrices have more complicated form due to larger number of DOFs per node: two displacements in 2D analysis instead of only one. For the two-node linear finite element, for the n-th harmonic of Fourier series, the following expressions for $[B]$ matrices were obtained:

$$
\left[\overline{B}_n\right]=\begin{bmatrix} \frac{\partial N_1}{\partial r}\cos n\theta & 0 & \frac{\partial N_2}{\partial r}\cos n\theta & 0 \\ \frac{N_1}{r}\cos n\theta & n\frac{N_1}{r}\cos n\theta & \frac{N_2}{r}\cos n\theta & n\frac{N_2}{r}\cos n\theta \\ -\frac{nN_1}{r}\sin n\theta & \left(\frac{\partial N_1}{\partial r}-\frac{N_1}{r}\right)\sin n\theta & -\frac{nN_2}{r}\sin n\theta & \left(\frac{\partial N_2}{\partial r}-\frac{N_2}{r}\right)\sin n\theta \end{bmatrix}
$$

$$
\left[\overline{\overline{B}}_n\right]=\begin{bmatrix} \frac{\partial N_1}{\partial r}\sin \theta & 0 & \frac{\partial N_2}{\partial r}\sin \theta & 0 \\ \frac{N_1}{r}\sin \theta & n\frac{N_1}{r}\sin \theta & \frac{N_2}{r}\sin \theta & n\frac{N_2}{r}\sin \theta \\ \frac{nN_1}{r}\cos \theta & -\left(\frac{\partial N_1}{\partial r}-\frac{N_1}{r}\right)\cos \theta & \frac{nN_2}{r}\cos \theta & -\left(\frac{\partial N_2}{\partial r}-\frac{N_2}{r}\right)\cos \theta \end{bmatrix} \tag{6}
$$

starting from 2D displacement expression:

$$
\begin{cases} u_i(\theta)=u_{i0}+\sum_{n=1}^{\infty}\left(\overline{u}_{in}\cos n\theta+\overline{\overline{u}}_{in}\sin n\theta\right) \\ v_i(\theta)=v_{i0}+\sum_{n=1}^{\infty}\left(\overline{v}_{in}\sin n\theta-\overline{\overline{v}}_{in}\cos n\theta\right) \end{cases} \tag{7}
$$

Stiffness matrices become:

$$
\left[\overline{k}_n\right]=\int_{A_{el}}\left[\int_0^{2\pi} r\left[\overline{B}_n\right]^T[D]\left[\overline{B}_n\right]d\theta\right]dA \quad \text{and} \quad \left[\overline{\overline{k}}_n\right]=\int_{A_{el}}\left[\int_0^{2\pi} r\left[\overline{\overline{B}}_n\right]^T[D]\left[\overline{\overline{B}}_n\right]d\theta\right]dA \tag{8}
$$

where:

$$
[D]=\frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}
$$

The interpolation function for the two-node finite element is (also valid for thermal problem):

$$
[N(r)]=[N_1 \quad N_2]=\left[\frac{r_2}{r_2-r_1}-\frac{1}{r_2-r_1}r \qquad -\frac{r_1}{r_2-r_1}+\frac{1}{r_2-r_1}r\right]
$$

being r_1 and r_2 the nodal coordinates.

Theoretically, it is expected that increasing the node number would improve the overall numerical accuracy of results, at the expense of higher computational cost. However, systematic tests revealed how the best performance is given by the three-node element with two Gauss points (see Figure 1a), which has then been used in all subsequent simulations. Another important parameter is the number of harmonics, which should be chosen as the best balance between accuracy and simulation time. Higher harmonic number N are expected to give better precision in Fourier expansion of applied loads. It has been noted that simulation time increases roughly linearly with N harnonic number.

In the present work only the thermal stresses were analysed and therefore the boundary conditions have only to prevent the rigid body motion. This means that the boundary conditions are imposed to only two nodes.

4 2D THERMAL TRANSIENT ANALYSIS

In numerical methods, the time domain is represented by a discrete sequence of time instants, with the time step Δt, at which the solution is calculated. The partial differential equation governing a transient thermal analysis can be solved by a finite difference method, implemented as an explicit or implicit numerical algorithm: explicit methods calculate the state of the system at a later time only considering the state of the system at current time, while implicit methods calculate the state of a system at a later time by solving an equation involving both the current state of the system and the later one. Implicit methods are usually slower (although more accurate) than explicit methods on single time step computation, as they require solving a linear system at each time step.

In the present study, time integration has been performed by two different methods. The first is Forward Finite Difference (FFD) method, which is an explicit method proven to be very quick and effective. However, it is said to be "conditionally stable", as it requires the time step Δt be smaller than a critical value to get a stable solution. This can represent a serious disadvantage, as the need of stable solution may impose relatively small time step, which results in a quite large total simulation time. The fundamental equation for FFD algorithm is:

$$\{S\}_{i+1} = [M]^{-1}\left([M]\{S\}_i - \Delta t[K]\{S\}_i + \Delta t\{F\} \right) \tag{9}$$

where $\{S\}_i$ and $\{S\}_{i+1}$ are the vectors of nodal solutions calculated at consecutive time steps i and i+1, respectively; symbol $[K]$ is the "stiffness" matrix, $[M]$ is the "mass matrix", $\{F\}$ is the vector of applied external "forces" and Δt is the time step used. Note that the mass matrix is time-independent, thus it can be inverted only once, with a considerable time saving. Explicit formulae of "mass matrix" have been derived for each element type introduced in Section 2.1 [11,12].

An alternative method has been developed for time integration. In analogy with the linear acceleration method [11], the proposed approach assumes a linear variation of first derivative, thus it has been called Linear Speed Method (LSM). Unlike FFD algorithm, this method is implicit and also "unconditionally stable". The fundamental equation of LSM method is:

$$\begin{cases} \{S\}_{i+1} = \left(\dfrac{2}{\Delta t}[M] + [K]\right)^{-1}\left(\{F\} + [M]\left(\dfrac{2}{\Delta t}\{S\}_i + \{\dot{S}\}_i\right)\right) \\ \{\dot{S}\}_{i+1} = \dfrac{2}{\Delta t}\left(\{S\}_{i+1} - \{S\}_i\right) - \{\dot{S}\}_i \end{cases} \tag{10}$$

in which $\{\dot{S}\}_i$ and $\{\dot{S}\}_{i+1}$ are the vectors of the derivatives of nodal solutions calculated at consecutive time steps i and i+1. Note that to further improve the computational speed, the system in Eq. (10) has been solved by LU factorization.

Several benchmark tests have been performed, to compare the relative performance and accuracy of FFD and LSM methods [11,12]. Both algorithms have been shown to provide almost coincident results, with a time saving for LSM of about 4% compared to FFD and even 99% with respect to a plane FE model. The use of LU factorization in LSM algorithm further reduced the computation time of about one third. The above tests then revealed that the fastest algorithm for transient thermal analysis is LSM method with LU factorization; this algorithm will be used in the following illustrative example.

The algorithms for transient analysis can also be used to solve the non-trivial problem of applied convective boundary condition. Three different methods, with various approximation levels, have been devised: one based on an average temperature, one based on a step-wise convective flux, the third based on fundamental trigonometric formulae (which has been implemented by two different algorithms: temperature calculated either at previous or at current time step) [11,12]. Without going into the details, it suffices to say that a comparative study showed how the approach based on trigonometric functions provides the highest accuracy and lowest simulation time [11,12].

5 ELASTO-PLASTIC STRUCTURAL ANALYSIS

The algorithm employed for elasto-plastic stress analysis is generally described in [14]. The plastic zone developed in the work roll has a very small thickness and occurs in the vicinity of surface roll. That is why a simplified elasto-plastic approach was utilized which allows only to slightly couple the different harmonics only for the few finite elements reaching the plastic regime. Thus the stiffness matrix becomes a little bit “less sparse” and the computer time increases insignificantly. A bilinear hardening characteristic of the material and a small tangent hardening modulus were adopted.

The well-known flow rule is:

$$d\{\varepsilon_{\mathrm{pl}}\} = d\lambda \frac{\partial F}{\partial\{\sigma\}} \tag{11}$$

were $d\{\varepsilon_{\mathrm{pl}}\}$ is the increment of plastic strain, $\{\sigma\}$ are the total stresses and F is the yield criterion, von Mises in this work:

$$F(\{\sigma\},\kappa) = (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\tau_{xy}^2 - 2\sigma_0^{*2}(\kappa) = 0 \tag{12}$$

Hardening parameter (the effective plastic strain) has the expression:

$$\kappa = \bar{\varepsilon}_{\mathrm{pl}} == \int d\bar{\varepsilon}_{\mathrm{pl}} \approx \sum \Delta\bar{\varepsilon}_{\mathrm{pl}} \tag{13}$$

where

$$d\kappa = d\bar{\varepsilon}_{\mathrm{pl}} = \sqrt{\frac{2}{3}\left((d\varepsilon_x^{\mathrm{pl}})^2 + (d\varepsilon_y^{\mathrm{pl}})^2 + (d\varepsilon_z^{\mathrm{pl}})^2 + 2(d\gamma_{xy}^{\mathrm{pl}})^2\right)} \tag{14}$$

At each iteration the current yield stress is updated according to the bilinear hardening characteristic and to the current value of the effective plastic strain in expression (13).

6 SIMULATION RESULTS AND DISCUSSION

6.1 Numerical example

An application which would highlight the efficiency of the semi-analytical approach introduced above is represented by the analysis of the transient temperature and stress distribution in work roll of hot rolling mill. Work roll is an axially symmetric structure, subjected to non-axisymmetric thermal loadings (thermal flux, convective cooling). In this numerical example, the performance of semi-analytical approach is compared to that of a plane FE model, which has been adopted for a simplified thermal stress analysis of work roll [10,12].

The thermal load configuration used for numerical simulations is shown in Figure 2a: an infinitely-long cylinder, rotating at constant angular speed and subjected to constant input heat flux and convective cooling. This configuration has been assumed as a simplified model of work roll under cyclic thermal loadings characteristic of hot strip rolling: the constant input flux q_0 approximates strip heating, while convective cooling is a simplified model for water cooling. Simulation parameters, assumed for simplicity as temperature invariant, are summarized in Table 1.

Table 1: Several geometrical and thermal parameters used in simulations

PARAMETER	DESCRIPTION	PARAMETER	DESCRIPTION
R=300 mm	Cylinder radius	q_0=13,7×10^6 W/m^2	Input thermal flux
ω=2,953 rad/sec	Cylinder angular speed	h=10100 W/m^{2}K	Convection coefficient
ϕ= 10°	Heating sector	T_0=20°C	Bulk temperature of cooling medium
α = 45°	Angular gap between heating and cooling	T_{roll}=20°C	Initial work roll temperature
Ψ= 90°	Cooling sector		

The numerical analysis simulates a physical time transient of 3600 seconds, which corresponds to about 1690 roll revolutions. At the initial simulation time, work roll is assumed at a constant uniform temperature T_{roll}=20°C. A thermal analysis is first performed, followed by a mechanical analysis, in which the temperature time history previously calculated is applied as input thermal load, to finally determine stress and strains. For the sake of comparison, both FE models (plane and harmonic) used the same mesh density in both thermal and mechanical analysis.

Figure 2b shows the plane FE model of work roll: an axis-symmetric mapped mesh, with 6940 elements and 6921 nodes. A mesh refinement is imposed near the surface, along the tangential and radial directions, to capture the thermal gradient here expected. Small elements are located for a depth of 10% of work roll radius, with even smaller elements placed immediately underneath the surface, for a depth of 2% of radius. Each surface element covers a 1° angular sector, with a total amount of 360 elements on the boundary.

In transient analysis, work roll rotation has been simulated by considering the roll fixed and by applying rotating thermal loadings which are moved a step forward at each next time instant (for the assigned mesh, a work roll rotation is completed after exactly 360 load steps).

In Ansys, the thermal analysis employed four-node linear thermal elements. Numerical solution used an implicit solver based on Jacoby Conjugate Gradient (JCG). The simulation required about 609'000 load steps, which were solved (as an order of magnitude) in about 3 days of simulation. It is worth mentioning how results of thermal analysis have been validated (see [10]) by an analytical solution for the stationary temperature distribution [15]. Mechanical analysis used four-node isoparametric elements under plain strain condition. An elastic-plastic material with von Mises plasticity and kinematic hardening rule has been adopted. Nonlinear numerical analysis was solved by an explicit solver code, with modified Newton-Raphson algorithm. Compared to thermal simulation, mechanical analysis required a much larger simulation time, more than an order of magnitude longer than thermal simulations. To keep the simulation time within reasonable limits, only 20 rotations were simulated, for a total running time of approximately 10 days of simulation. Further details on model and simulation parameters can be found in [6,7].

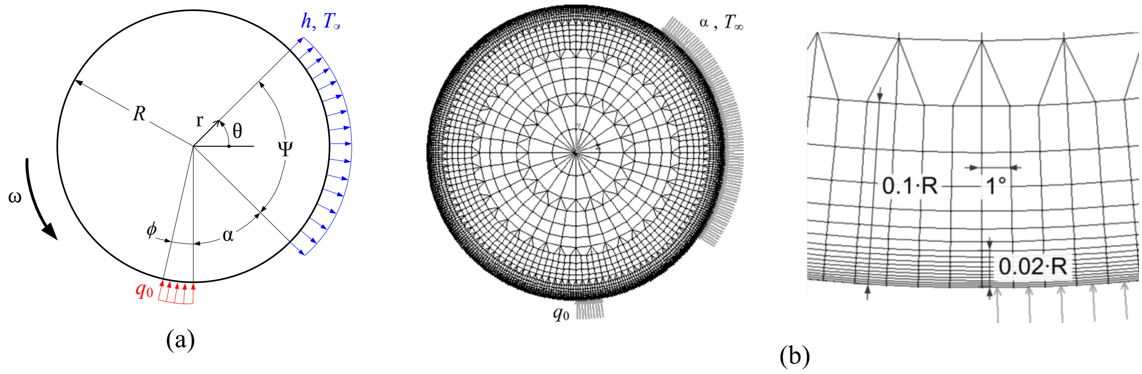

Figure 2: Thermal configuration analyzed (from [15]) (a); finite element model (global and zoom) (b)

In semi-analytical approach, the 2D geometry of work roll in Figure 2a is represented by a 1D model. A three nodes element with two Gauss points is used, see Figure 1a; the mesh, shown in Figure 1b, has a total amount of 28 elements and 57 nodes. For an effective comparison, the 1D and 2D models have an identical element distribution in radial direction, compare Figure 1b with 2b.

As discussed above, an important modeling parameter which has to be defined in the harmonic model is the number of harmonics N. The number of harmonics is a critical parameter, since it controls both total simulation time and accuracy of results. The thermal configuration in Figure 2a is characterized by a constant flux applied on roll boundary over a relatively small angular sector (ϕ = 10°). When expanding in Fourier series a periodic step function (as the thermal flux applied on work roll boundary), the approximation error tends to increase as the step width decreases, because of Gibbs' phenomenon at jump discontinuity. In the analyzed geometry, the thermal flux extends only over a 10° angular sector, which would then require an appropriate high number of harmonics to minimize the approximation error. Some benchmark tests were performed [11,12] to identify the optimal number of harmonics for the configuration here analyzed. A comparison of the maximum transient temperature, calculated by the plane model and various harmonic models with different number of harmonics revealed that N=50...100 would be an optimal compromise between accuracy and computing time.

For both, the plane FE and harmonic approaches, the work roll was considered fixed while thermal loading rotates.

6.2 Thermal analysis

A qualitative comparison of results between plane and harmonic FE model is given in Figure 3, which shows, as an example, the temperature field in work roll after 1800 seconds. The map of temperature at other time instants (not included here) would show a progressive heating of the entire work roll, although the largest temperature gradients develop very close to the surface (this justifies the use there of very small elements in the mesh). Within the roll there is a progressive temperature increase up to 180°C, while at surface the maximum temperature reaches 388°C within the bite region. The temperature field calculated by semi-analytical FE model is very similar to that given by plane FE model, although they do not seem exactly coincident, the difference being ascribed to the slightly different color map adopted.

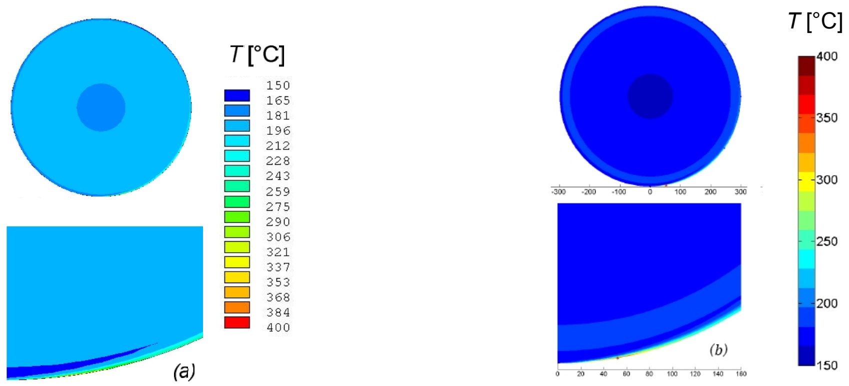

Figure 3: Temperature distribution in work roll after 1800 sec. (a) plane and (b) harmonic FE model

Figure 4, instead, compares the temperature time history within a 60-sec time interval, for a point located on work roll surface. Each peak temperature occurs when the monitored point enters the heating zone, thus the series of equally-spaced peaks identifies the sequence of work roll rotations. The transient nature of thermal phenomenon here investigated is clearly confirmed by the continuous increase of peak temperature during consecutive rotations. The diagrams show a very similar trend, both for the overall temperature history and the maximum temperature reached during every rotation. Only small differences may be captured, which may be attributed to the different rate of results saving on computer hard disk.

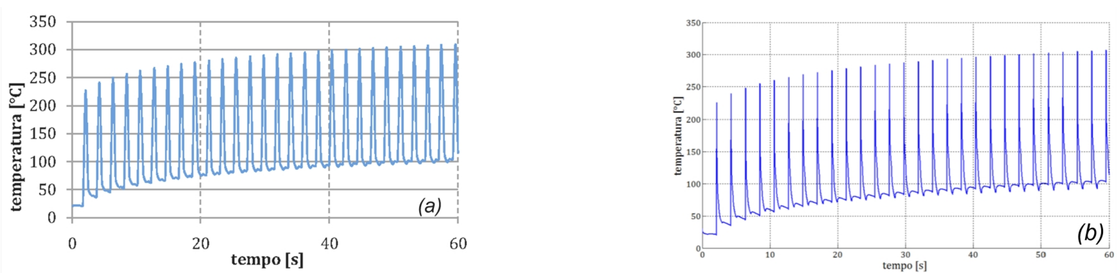

Figure 4: Temperature history for a point on work roll surface: (a) plane and (b) harmonic FE model

Figure 5 shows the variation of the temperature along the radial direction, in different instants. The results of plane model are compared to those obtained in [15] under steady-state condition. A very high temperature gradient in the vicinity of the roll surface is noticed.

A comparison between plane and harmonic models is presented in Figure 6, which refers to the radial temperature distribution at different time instants at the neighboring of the roll surface. A very similar trend is observed for both FE models, although temperatures tend to be slightly lower for harmonic model. The continuous temperature increases with time, especially inside the work roll, is indicated by the different curves. Both diagrams also confirm that work roll remains at a rough uniform temperature, except for a very small portion very close to the surface (wide about 1% of roll radius), where a steep temperature gradient is observed (note that diagrams only plot radial coordinates close to roll surface). The very localized nature of temperature variation within this narrow region (usually called "thermal boundary layer" in technical literature) requests the use of an appropriate mesh refinement close to work roll surface that is very useful also in the structural analysis.

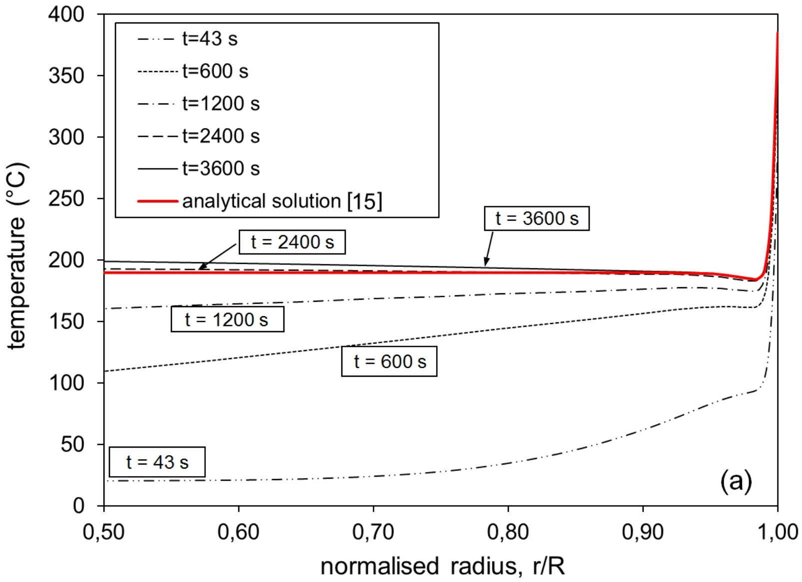

Figure 5: Radial temperature at different time instants: plane model

Figure 7, instead, describes the time-evolution of temperature for points at different radial depths, along a fixed angular position. The figure further emphasizes that the thermal gradient is confined in a small region close to the boundary. In fact, on work roll surface the temperature ranges of about 200°C, while at 1 mm depth from surface the variation is only 100 °C, and even negligible at 6 mm below surface. In addition, the detail in the same figure highlights a sort of "thermal inversion" phenomenon induced by forced convection cooling, in which the work roll material on the surface is at a lower temperature than material inside.

For what concerns the comparison of computing time, the simulations confirmed how the performance of harmonic model gives a strong reduction of running time, compared to plane FE simulations. As a rough estimate, the harmonic model requires about 60 minutes to complete the 3600 seconds of work roll simulation, while the plane model needs about 10 hours. Obviously, such a comparison is only indicative, since computation times greatly depend on the specific computer layout and performance.

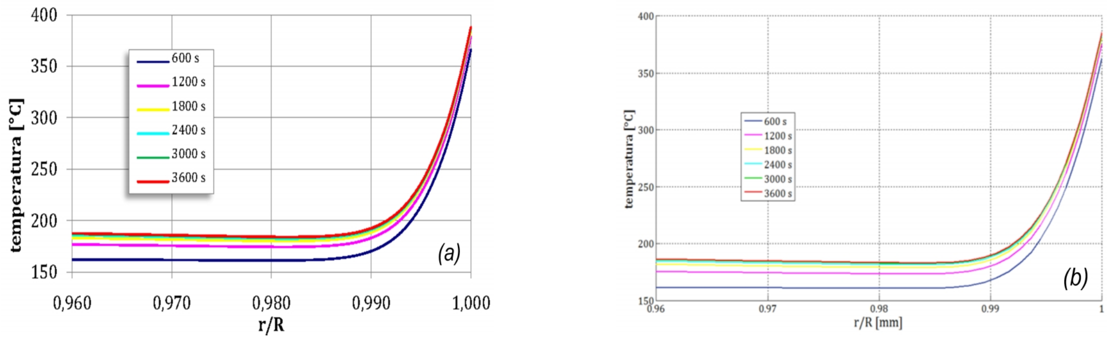

Figure 6: Radial temperature at different time instants: (a) plane and (b) harmonic FE model

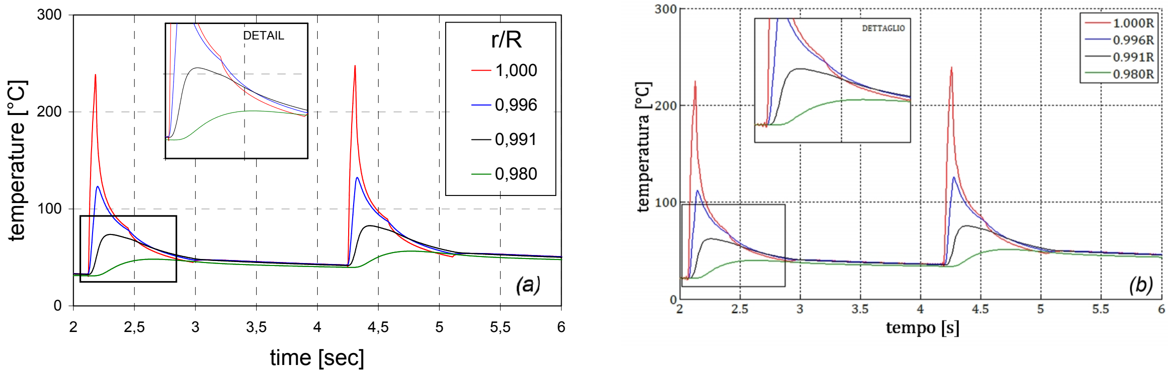

Figure 7: Temperature time history for points at different radial depths, along the same angular position. (a) plane and (b) harmonic FE model

6.3 Mechanical analysis

The temperature distribution calculated in thermal analysis is used as input load in the next mechanical analysis to compute thermal stresses. In present study an elastic-ideal plastic steel was considered (the yield stress was σ_y=500 MPa). As an example, Figure 8 shows the distribution of von Mises stress in work roll calculated by both plane and harmonic FE model. Results refer to the stress calculated by a nonlinear transient analysis, resulting from the temperature distribution after 20 rotations (time instant 43 sec). The maximum von Mises stress is limited to approximately 542 MPa, slightly higher than σ_y=500 MPa due to the tangent hardening modulus.

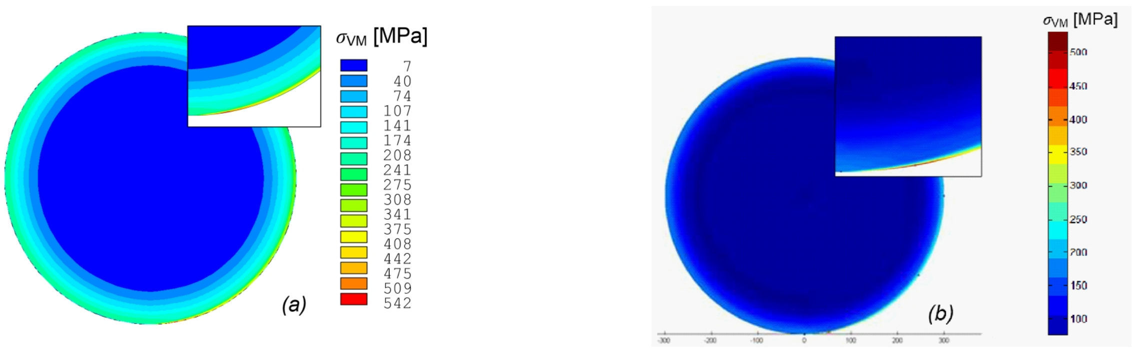

Figure 8: Von Mises stress (global and zoom view on heating zone): (a) plane and (b) harmonic FE model

Note that, because of plasticity, the mechanical analysis would become nonlinear and an iterative convergence algorithm should be performed at each time step. However tacking into account the highly localized plastic zone, a simplified approach was used and in this case the different linear systems, corresponding to different harmonics, become slightly coupled and therefore the computer time would not increase significantly.

7 CONCLUSIONS

The present work proposes a semi-analytical approach for the transient thermal analysis and stress analysis of axisymmetric structures under non axis-symmetric loads. A typical example is represented by work rolls in hot rolling mills, which are exposed to hot strip heating and water cooling on different parts of their. The work roll revolution induces cyclic heating and cooling phases on the surface, responsible of cyclic tensile and compressive stresses. Such cyclic stress may cause surface cracks to appear and propagate, hence demanding for periodic work roll replacement and dressing.

The FE theory on semi-analytical approach for stationary and transient thermal analysis and for stress analysis of axi-symmetric bodies under non axis-symmetric thermal loads is briefly reviewed and discussed. Different aspects of FE numerical simulations are discussed, as element type, modeling of boundary condition, numerical algorithms for transient analysis. The present paper only illustrates the main advantages or disadvantages of different methods, while all the theoretical details are discussed in detail elsewhere (the interested reader can refer to [11,12]). A numerical example is finally presented to test the performance of the proposed semi-analytical approach and compared to a plane FE model. The example refers to a rotating cylinder under imposed input flux and convective cooling, which has been used as a simplified model to study temperature and stress distribution in the work hot rolling mill [6,10,11,12].

The transient thermal history in work roll is first simulated, which allows identifying the transient temperature distribution and the so-called "thermal boundary layer" close to wok roll surface, where the largest thermal gradients usually occur. The calculated temperature distribution is applied as input load in subsequent mechanical analysis to calculate thermal stresses in elasto-plastic range. Semi-analytical approach is shown to provide results in very good agreement with plane FE simulations.

The example confirms the accuracy of semi-analytical approach, which however gives a strong reduction of overall simulation time compared to plane FE simulations, as it reduces a plane model to a 1D one, therefore a problem having thousands of degrees of freedom is replaced with one having only tens. Plastic zones are very localized in the vicinity of the roll surface and for this reason the time computer is very short also for stress analysis problem even if few iterations are performed at each time step.

The obtained numerical results are quite promising and confirm the validity of the proposed semi-analytical approach. However, an experimental validation of obtained numerical results would be required, as well as an experimental evaluation of both thermo-mechanical materials properties and thermal coefficient used in numerical simulations.

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