11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
E. Oñate, J. Oliver and A. Huerta (Eds)

SHAKEDOWN ANALYSIS OF OFFSHORE STRUCTURES UNDER IMPACT LOAD

GUO JUN, WANG JUN, YANG DI

College of Shipbuilding Engineering
Harbin Engineering University
Harbin 150001, China
e-mail: wangjun5613@163.com, www.163.com

Key Words: *Shakedown analysis, Offshore structure, impact load, Kinematic shakedown theorem.*

Abstract. Ocean engineering structures are frequently subjected to repeated dynamic loads and impact loads. The dynamic strength analysis and shakedown analysis of offshore platform structure have an important place in ensuring the safety and reliability of ocean engineering structures under impact loads. Therefore the shakedown analysis theory was introduced to the ultimate strength analysis of brace strut of semisubmersible drilling platform considering wave impact load. According to the kinematic shakedown theorem and combined with the finite element calculation and analysis, a theoretical method of upper bound shakedown analysis for offshore structures under repeated impact loads was presented and compared with existing results to verify the reasonableness. Then by applying the theoretical method to shakedown analysis of brace strut under repeated dynamic loads, influence of shell thickness, stiffener thickness and stiffener spacing on shakedown limit were studied. The results show that the theoretical calculation method is agreed with the existing results. The limit load increases with the increase of shell thickness and stiffener thickness, while decreases with the increase of stiffener spacing.

1 INTRODUCTION

The main purpose of structural design and structural analysis is to determine the safety and bearing capacity. Elastic analysis cannot take full advantage of the bearing performance when the material is in plasticity. On the other hand, the limit analysis is to find the ultimate bearing capacity of the structure with the assumption that applied loads are time-independent and proportional [1]. In most cases, the external loads on the structures are neither monotonous nor proportional. For example, the wind loads and wave loads on the ocean engineering structures. Then the structure may cause fatigue failure before the external load reaching its ultimate bearing capacity. Shakedown analysis defines the boundaries of low cycle fatigue and incremental plastic collapse [2]. When the amplitude of repeated dynamic load is less than the shakedown load, although local plastic deformation of the structure may occur in the initial loading, the plastic deformation is no longer cumulative and elastic behaviour of the structure will be presented after a certain loading cycle [3, 4]. When the amplitude of load is more than the shakedown load, each load cycle will produce plastic deformation. The structure will get into the slow process of accumulating plastic deformation and eventually cause alternating plasticity (low cycle fatigue) and incremental plastic collapse (ratcheting) [5].

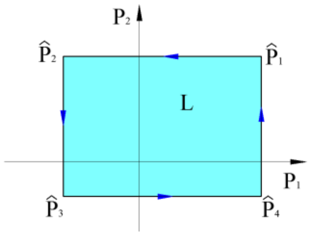
Since the kinematic shakedown theorem is proposed by Koiter, it has got great development and application [6, 7]. There are some research results of shakedown analysis considering the dynamic effects, but the calculation procedure is complex because the dynamic shakedown analysis often involves the integral of time. And it is less common in the engineering application. Meanwhile, the shakedown analysis of ocean engineering structures subjected to repeated dynamic loads is rare. The finite element method is used to evaluate the shakedown safety factor for elastic–plastic offshore structures by Fadaee et al [8]. In their work, Melan theorem of shakedown is employed, and shakedown theory is applied in the ocean engineering structures subjected to repeated wave loads.

According to the kinematic shakedown theorem, a theoretical method of shakedown analysis for typical offshore structures under repeated dynamic loads was proposed. The shakedown problem is translated into the solving of nonlinear optimization problems. Starting from upper bound theorem and based on von Mises yield criterion, shakedown bound is the minimum of the plastic dissipation function. Meanwhile it is subjected to the compatibility, incompressibility and normalized constraints. The nonlinear optimization problem is analyzed by penalty function method and generalized Lagrange multiplier method, and computed by the Newton-Raphson iteration. By applying this method, the influence of structure design parameters on shakedown limit under repeated dynamic loads were studied. The research method and results can provide a reference for the shakedown analysis of ocean engineering structures subjected to impact loads.

2 THE THEORETICAL METHOD OF SHAKEDOWN ANALYSIS

2.1 Kinematic shakedown theorem

Usually the load domain L of load amplitude applied on the structure should be determined first before shakedown analysis [9]. After load domain is known, the fictitious elastic stress $\sigma^E(t)$ need to be solved. And it is define as the response which would appear in the fictitious infinitely elastic structure if this structure was subjected to the same loads as the actual one. However, the fictitious elastic stress $\sigma^E(t)$ is time-dependent. In most practical engineering problems, the yield condition of its requirement should be satisfied in the range $t \geq 0$. This means that objective function in the subsequent nonlinear programming has infinitely many constraint equations.



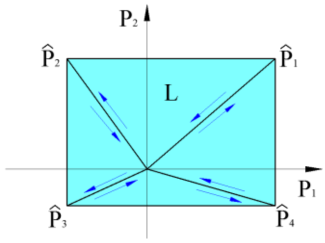


Figure 1: Convex envelope of load domain **Figure 2**: A cyclic load path containing all vertices

Therefore, in order to simplify the problem, the load space needs to be converted. König and Kleiber [10] introduced two convex-cycle theorems in 1978: "Shakedown will happen over a given load domain L if and only if it happens over the convex envelope of L", as shown in Figure 1, and "Shakedown will happen over any load path within a given load domain L if it happens over a cyclic load path containing all vertices of L", as shown in Figure 2.

The above two convex-cycle theorems are adapted to convex load domains and convex yield surfaces. This allows us to consider a cyclic load path instead of all loading history. Now it only needs to calculate the stress and the strain rate field at each load corner, rather than the integration of entire loading time. In the convex load domain L, any load $P(t) \in L$ can be represented by linear combination of each corner load [11], as follows

$$P(t) = \sum_{k=1}^{m} \delta(t_k) \hat{P}_k \tag{1}$$

where $m = 2^n$ is the total number of load corners in the load domain L and n is the total number of varying loads. $\delta(t_k)$ is the Dirac function defined by

$$\sum_{k=1}^{m} \delta(t_k) = 1 \ , \ \ \delta(t_k) \geq 0 \ , \ \ \delta(t_k) = \begin{cases} 1, & t = t_k \\ 0, & t \neq t_k \end{cases} \tag{2}$$

For a given load point $P(t)$, $\delta(t_k)$ is uniquely determined. Accordingly, the strain $\varepsilon(t)$ caused by $P(t) \in L$ can also be represented by linear combination of stress ε_k corresponding to each corner load $\hat{P}_k$ such as $\varepsilon(t) = \sum_{k=1}^{m} \delta(t_k) \varepsilon_k$. At each instant or at each load vertex, the kinematical condition may not be satisfied, but the accumulated plastic strain over a load cycle must be kinematically compatible, such that $\Delta\varepsilon = \sum_{k=1}^{m} \varepsilon_k$. According to the displacement matrix, the strain can be obtained by deformation matrix *B*: $\Delta\varepsilon = Bu$. Then the kinematically compatible condition becomes $\sum_{k=1}^{m} \varepsilon_k = Bu$.

In the finite deformation theory, the elastic deformation of the material will cause the volume change, but the plastic flow here is incompressible. Since the change in volume caused by the elastic deformation is limited, and the volume of plastic deformation is incompressible, so the volume should be basically remain unchanged. In the small deformation theory, the volume change rate is:

$$\dot{V} = \left(V_1 - V_0\right)/V_0 = (1+\varepsilon_{11})(1+\varepsilon_{22})(1+\varepsilon_{33}) - 1 \approx \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} \tag{3}$$

After considering the incompressible condition, we have one more equality constraint in our system at each load corner:

$$\varepsilon_{11}^{k} + \varepsilon_{22}^{k} + \varepsilon_{33}^{k} = 0 \tag{4}$$

To facilitate the description and calculation, equation (4) can be written in matrix form $M_a \varepsilon_k = 0$ where M_a is an auxiliary matrix. According to kinematical shakedown theorem of Koiter, shakedown may happen if the following inequality is satisfied:

$$W_{ex}=\int_0^T dt\int_V \sigma^E(t)\dot{\varepsilon}^p dV \le \int_0^T dt\int_V D^p(\dot{\varepsilon}^p)dV = W_{in} \tag{5}$$

where W_{ex} is the external work, W_{in} is the internal dissipation, and D^p is the plastic dissipation function. When the yield function is taken as von Mises yield condition, D^p has the form:

$$D_p=\sigma_s\sqrt{\frac{2}{3}\dot{\varepsilon}M_d\dot{\varepsilon}} \tag{6}$$

where σ_s is the yield stress of the material, M_d is a diagonal matrix. In the three-dimensional model M_d has the size 6×6 such that $M_d = Diag\left[1 \quad 1 \quad 1 \quad 1/2 \quad 1/2 \quad 1/2\right]$.

Based on the kinematical shakedown theorem of Koiter, the upper bound analysis of shakedown limit is to compute the load multiplier α, which satisfies the kinematically compatible condition, the incompressibility condition and the following inequality:

$$\alpha\int_0^T dt\int_V \sigma^E(t)\dot{\varepsilon}^p dV \le \int_0^T dt\int_V D^p(\dot{\varepsilon}^p)dV \tag{7}$$

In all external loads which satisfy the kinematically admissible velocity fields, the minimum load corresponds to the shakedown limit load. If there is only one given load and its variation range is zero, that is to say there is only one load vertex, then shakedown analysis turns into the limit analysis. Therefore, limit analysis is a special case of shakedown analysis. So the computing of shakedown load multiplier α can be formulated as the analysis of nonlinear minimization problem:

$$\alpha = \min\left(W_{in}/W_{ex}\right) \tag{8}$$

or in normalized form:

$$\begin{aligned}&\alpha = \min\left(W_{in}\right)\\ &\text{s.t.:}\quad W_{ex}=1\end{aligned} \tag{9}$$

According to above convex-cycle theorems, the time domain integral of external work W_{ex} and internal dissipation W_{in} can convert to the superposition at all load corners. The external work W_{ex} and internal dissipation W_{in} can be calculated by Gauss-Legendre integration technique. From equation (5) and (9) one has:

$$W_{ex}=\sum_{k=1}^{m}\int_V \sigma_k^E(t)\dot{\varepsilon}_k^p dV=\sum_{k=1}^{m}\sum_{e=1}^{ne}\int_{V_e}\sigma_k^E(t)\dot{\varepsilon}_k^p dV_e=\sum_{k=1}^{m}\sum_{e=1}^{ne}\sum_{i=1}^{ng}w_i\sigma_{ik}^E(t)\dot{\varepsilon}_{ik}^p=\sum_{k=1}^{m}\sum_{i=1}^{NG}w_i\sigma_{ik}^E(t)\dot{\varepsilon}_{ik}^p \tag{10}$$

where *ne* is the total number of elements in structural body *V*, *ng* is the total number of integration points in each element V_e, and w_i is the weighting factor of the Gauss point *i*. Then the total number of Gauss points in structure *V* is $NG=ne\times ng$. Similarly, the same integration technique can be applied to external work.

The dynamic loads, such as wave loads, can be expressed as $P(t)=P_m e^{t/\tau}$. The kinematic shakedown theorem and the two convex-cycle theorems are used here. Therefore, we do not need to know the loading history. The amplitude P_m of dynamic load can be solved through the relationship between the external work W_{ex} and internal dissipation W_{in}.From the above analysis, for the structure made of elastic–perfectly plastic material, the shakedown limit load multiplier can be formulated as:

$$\alpha = \min\left(\sum_{k=1}^{m}\sum_{i=1}^{NG} w_i\sigma_s\sqrt{\frac{2}{3}\dot{\varepsilon}_{ik}M_d\dot{\varepsilon}_{ik}}\right) \quad (a)$$

$$\text{s.t.}: \begin{cases} \sum_{k=1}^{m}\varepsilon_{ik} = B_i u \quad \forall i = \overline{1,NG} & (b) \\ M_a\varepsilon_{ik} = 0 \quad \forall i = \overline{1,NG} \quad \forall k = \overline{1,m} & (c) \\ \sum_{k=1}^{m}\sum_{i=1}^{NG} w_i\sigma_{ik}^{E}(t)\dot{\varepsilon}_{ik}^{p} = 1 & (d) \end{cases} \tag{11}$$

Equation (11*a*) is the objective function. Equation (11*b*), (11*c*), and (11*d*) are the constraint conditions of nonlinear programming, among them Equation (11*b*) corresponds to the kinematically compatible condition, Equation (11*c*) represents the incompressible condition, and Equation (11*d*) is the normalized form of external work for easy computing. If the rigid or elastic perfectly plastic material model is adopted with the von Mises yield criterion, the objective function (11*a*) is only differentiable in the plastic zones while effective optimization methods require its gradient to be available everywhere. Limited analysis is a special case of shakedown analysis, and it faces the same difficulty based on kinematic formulation. Dealing with singular dissipation function in limit analysis, Andersen [12] et al. introduced a small constant η_0, where $0<\eta_0^2 \ll 1$. The dissipation function became $D_p = \sigma_s\sqrt{2\dot{\varepsilon}M_d\dot{\varepsilon}/3+\eta_0^2}$.

After the introduction of constant η_0, all elements are seen as plastic or on the plastic verge. In order to avoid the singularity of non-plastic zones, the above mentioned method is adopted here in shakedown analysis. The objective function (11*a*) of nonlinear programming for upper bound shakedown analysis can be written as:

$$\alpha = \min\left(\sum_{k=1}^{m}\sum_{i=1}^{NG} w_i\sigma_s\sqrt{\frac{2}{3}\dot{\varepsilon}_{ik}M_d\dot{\varepsilon}_{ik}+\eta_0^2}\right) \tag{12}$$

In equation (11) of nonlinear programming problem, for the sake of simplicity, the strain rate vector, stress vector, and deformation matrix are redefined as:

$$\hat{\varepsilon}_{ik} = w_i M_d^{1/2}\dot{\varepsilon}_{ik}\ ,\quad \hat{\sigma}_{ik} = M_d^{-1/2}\sigma_{ik}^{E}\ ,\quad \hat{B} = w_i M_d^{1/2} B_i \tag{13}$$

Then the calculation formulas of nonlinear programming problem for the upper bound shakedown limit through kinematical shakedown theorem become:

$$\alpha = \min\left(\sum_{k=1}^{m}\sum_{i=1}^{NG} \sigma_s\sqrt{\frac{2}{3}\hat{\varepsilon}_{ik}^{T}\hat{\varepsilon}_{ik}+\eta_0^2}\right) \quad (a)$$

$$\text{s.t.}: \begin{cases} \sum_{k=1}^{m}\hat{\varepsilon}_{ik} = \hat{B}_i\dot{u} \quad \forall i = \overline{1,NG} & (b) \\ M_a\hat{\varepsilon}_{ik} = 0 \quad \forall i = \overline{1,NG} \quad \forall k = \overline{1,m} & (c) \\ \sum_{k=1}^{m}\sum_{i=1}^{NG} \hat{\varepsilon}_{ik}^{T}\hat{\sigma}_{ik} = 1 & (d) \end{cases} \tag{14}$$

2.2 Numerical solution of shakedown analysis

In order to solve the nonlinear constrained optimization problem (14), the penalty method is used here to deal with the compatibility constraint (14*b*) and the incompressibility constraint (14*c*). The penalty function method compels iteration point to approach the feasible region through adding penalty terms composed of constraint functions to the original objective function. The penalty function of equality constraints in equation (14) can be written as:

$$F_p(\hat{\varepsilon}_{ik},\dot{u},c)=\sum_{i=1}^{NG}\left[\sum_{k=1}^{m}\sigma_s\sqrt{\frac{2}{3}\hat{\varepsilon}_{ik}^T\hat{\varepsilon}_{ik}+\eta_0^2}+\frac{c}{2}\left(\sum_{k=1}^{m}\hat{\varepsilon}_{ik}-\hat{B}_i\dot{u}\right)^T\left(\sum_{k=1}^{m}\hat{\varepsilon}_{ik}-\hat{B}_i\dot{u}\right)+\frac{c}{2}\sum_{k=1}^{m}\hat{\varepsilon}_{ik}^T M_a\hat{\varepsilon}_{ik}\right] \tag{15}$$

where c is penalty factor. When c is sufficiently large, we will approximate the optimal solution of constraint problem in equation (14). The last two items on the right side of equation (15) are addition items. If parameters $\hat{\varepsilon}_{ik}$、$\dot{u}$、c satisfy the constraint conditions, addition items are 0. And if not, additional items bring great punishment.

Then the optimization problem is solved by the penalty function $F_p(\hat{\varepsilon}_{ik},\dot{u},c)$ combined with the Lagrange multiplier method. By means of advantages of penalty function, and combined with the properties of Lagrange multiplier, we can construct a more suitable new objective function so that we will gradually achieve the optimal solution of original constraint problem when c goes to sufficiently large. That is the generalized Lagrange multiplier method. The Lagrange function is obtained by applying this method:

$$F_{pL}(\hat{\varepsilon}_{ik},\dot{u},\beta)=F_p(\hat{\varepsilon}_{ik},\dot{u},c)-\beta\left(\sum_{i=1}^{NG}\sum_{k=1}^{m}\hat{\varepsilon}_{ik}^T\hat{\sigma}_{ik}-1\right) \tag{16}$$

For finding the minimum of function $F_p(\hat{\varepsilon}_{ik},\dot{u},c)$, its first derivative of $\hat{\varepsilon}_{ik},\dot{u},\beta$ must be equal to zero [13], that is:

$$\begin{cases}\dfrac{\partial F_{PL}}{\partial\hat{\varepsilon}_{ik}}=\dfrac{2\sigma_s\hat{\varepsilon}_{ik}}{3\sqrt{2\hat{\varepsilon}_{ik}^T\hat{\varepsilon}_{ik}/3+\eta_0^2}}+c\left(\displaystyle\sum_{k=1}^{m}\hat{\varepsilon}_{ik}-\hat{B}_i\dot{u}\right)+cM_a\hat{\varepsilon}_{ik}-\beta\hat{\sigma}_{ik}=0,\ \forall i,k & (a)\\ \dfrac{\partial F_{PL}}{\partial\dot{u}}=-c\displaystyle\sum_{i=1}^{NG}\hat{B}_i^T\left(\sum_{k=1}^{m}\hat{\varepsilon}_{ik}-\hat{B}_i\dot{u}\right)=0 & (b)\\ \dfrac{\partial F_{PL}}{\partial\beta}=\displaystyle\sum_{i=1}^{NG}\sum_{k=1}^{m}\hat{\varepsilon}_{ik}^T\hat{\sigma}_{ik}-1=0 & (c)\end{cases} \tag{17}$$

To find the solutions of equations (17), the Newton-Raphson iteration with good convergence is applied here:

$$\begin{cases}\mathrm{f}_{ik}^1 d\hat{\varepsilon}_{ik}+c\sqrt{\dfrac{2}{3}\hat{\varepsilon}_{ik}^T\hat{\varepsilon}_{ik}+\eta_0^2}\left(\displaystyle\sum_{k=1}^{m}d\hat{\varepsilon}_{ik}-\hat{B}_i d\dot{u}\right)-d\beta\sqrt{\dfrac{2}{3}\hat{\varepsilon}_{ik}^T\hat{\varepsilon}_{ik}+\eta_0^2}\,\hat{\sigma}_{ik}=-\mathrm{f}_{ik}^2 & (a)\\ \displaystyle\sum_{i=1}^{NG}\hat{B}_i^T\left(\sum_{k=1}^{m}d\hat{\varepsilon}_{ik}-\hat{B}_i d\dot{u}\right)=-\sum_{i=1}^{NG}\hat{B}_i^T\left(\sum_{k=1}^{m}\hat{\varepsilon}_{ik}-\hat{B}_i\dot{u}\right) & (b)\\ \displaystyle\sum_{i=1}^{NG}\sum_{k=1}^{m}\hat{\sigma}_{ik}^T d\hat{\varepsilon}_{ik}=1-\sum_{i=1}^{NG}\sum_{k=1}^{m}\hat{\sigma}_{ik}^T\hat{\varepsilon}_{ik} & (c)\end{cases} \tag{18}$$

where:

$$\mathrm{f}_{ik}^{1}=\left[\frac{2}{3}\sigma_s I_{ik}+c\sqrt{\frac{2}{3}\hat{\varepsilon}_{ik}^{T}\hat{\varepsilon}_{ik}+\eta_0^2}M_a\right]+\frac{2}{3\sqrt{\frac{2}{3}\hat{\varepsilon}_{ik}^{T}\hat{\varepsilon}_{ik}+\eta_0^2}}\left[cM_a\hat{\varepsilon}_{ik}+c\left(\sum_{k=1}^{m}\hat{\varepsilon}_{ik}-\hat{B}_i\dot{u}\right)-\beta\hat{\sigma}_{ik}\right]\hat{\varepsilon}_{ik}^{T} \tag{19a}$$

$$\mathrm{f}_{ik}^{2}=\frac{2}{3}\sigma_s\hat{\varepsilon}_{ik}+c\sqrt{\frac{2}{3}\hat{\varepsilon}_{ik}^{T}\hat{\varepsilon}_{ik}+\eta_0^2}\left(\sum_{k=1}^{m}\hat{\varepsilon}_{ik}-\hat{B}_i\dot{u}\right)+c\sqrt{\frac{2}{3}\hat{\varepsilon}_{ik}^{T}\hat{\varepsilon}_{ik}+\eta_0^2}M_a\hat{\varepsilon}_{ik}-\beta\sqrt{\frac{2}{3}\hat{\varepsilon}_{ik}^{T}\hat{\varepsilon}_{ik}+\eta_0^2}\hat{\sigma}_{ik} \tag{19b}$$

In the systems (18) and (19), $d\hat{\varepsilon}_{ik}$ denotes the incremental vector of strain rate and $d\dot{u}$ is the incremental vector of relative displacement rate at Gaussian point *i* and load vertex *k*. $d\beta$ denotes the incremental value of Lagrange multiplier β and I_{ik} stands for the identity matrix. The singular problem arises again in (19*a*) as the denominator appears in the formulation (17*a*). Meanwhile, f_{ik}^{1} is not a symmetric matrix and it cannot be guaranteed to be a positive definite matrix. Therefore we are not sure the convergence of calculation method. To avoid the problem of calculation divergence, the second item on the right side of equation (19*a*) is eliminated and we only keep its approximate form as follows:

$$\mathrm{f}_{ik}^{1}\approx\left[\frac{2}{3}\sigma_s I_{ik}+c\sqrt{\frac{2}{3}\hat{\varepsilon}_{ik}^{T}\hat{\varepsilon}_{ik}+\eta_0^2}M_a\right] \tag{20}$$

From (18*a*), the incremental vector of strain rate has the form:

$$d\hat{\varepsilon}_{ik}=-c\sqrt{\frac{2}{3}\hat{\varepsilon}_{ik}^{T}\hat{\varepsilon}_{ik}+\eta_0^2}(\mathrm{f}_{ik}^{1})^{-1}\left(\sum_{k=1}^{m}d\hat{\varepsilon}_{ik}-\hat{B}_i d\dot{u}\right)+d\beta\sqrt{\frac{2}{3}\hat{\varepsilon}_{ik}^{T}\hat{\varepsilon}_{ik}+\eta_0^2}(\mathrm{f}_{ik}^{1})^{-1}\hat{\sigma}_{ik}-(\mathrm{f}_{ik}^{1})^{-1}\mathrm{f}_{ik}^{2} \tag{21}$$

The above equation can be expanded by load vertexes from *k*=1 to *m*, and then sum these *m* equations up. We obtain:

$$\sum_{k=1}^{m}d\hat{\varepsilon}_{ik}=c(\mathrm{f}_i^3)^{-1}\sum_{k=1}^{m}\sqrt{\frac{2}{3}\hat{\varepsilon}_{ik}^{T}\hat{\varepsilon}_{ik}+\eta_0^2}(\mathrm{f}_{ik}^{1})^{-1}\hat{B}_i d\dot{u}+d\beta(\mathrm{f}_i^3)^{-1}\sum_{k=1}^{m}\sqrt{\frac{2}{3}\hat{\varepsilon}_{ik}^{T}\hat{\varepsilon}_{ik}+\eta_0^2}(\mathrm{f}_{ik}^{1})^{-1}\hat{\sigma}_{ik}-(\mathrm{f}_i^3)^{-1}\sum_{k=1}^{m}(\mathrm{f}_{ik}^{1})^{-1}\mathrm{f}_{ik}^{2} \tag{22}$$

where:

$$\mathrm{f}_i^3=I_i+c\sum_{k=1}^{m}\sqrt{\frac{2}{3}\hat{\varepsilon}_{ik}^{T}\hat{\varepsilon}_{ik}+\eta_0^2}(\mathrm{f}_{ik}^{1})^{-1} \tag{23}$$

and I_i is the identity matrix in (23). Substitute (22) into (18*b*) and we can get:

$$\sum_{i=1}^{NG}\hat{B}_i^T\mathrm{f}_i^4\hat{B}_i d\dot{u}=\sum_{i=1}^{NG}\hat{B}_i^T\left[d\beta(\mathrm{f}_i^3)^{-1}\sum_{k=1}^{m}\sqrt{\frac{2}{3}\hat{\varepsilon}_{ik}^{T}\hat{\varepsilon}_{ik}+\eta_0^2}(\mathrm{f}_{ik}^{1})^{-1}\hat{\sigma}_{ik}-(\mathrm{f}_i^3)^{-1}\sum_{k=1}^{m}(\mathrm{f}_{ik}^{1})^{-1}\mathrm{f}_{ik}^{2}\right]+\sum_{i=1}^{NG}\hat{B}_i^T\left(\sum_{k=1}^{m}\hat{\varepsilon}_{ik}-\hat{B}_i\dot{u}\right) \tag{24}$$

where:

$$\mathrm{f}_i^4=I_i-c(\mathrm{f}_i^3)^{-1}\left[\sum_{k=1}^{m}\sqrt{\frac{2}{3}\hat{\varepsilon}_{ik}^{T}\hat{\varepsilon}_{ik}+\eta_0^2}(\mathrm{f}_{ik}^{1})^{-1}\right] \tag{25}$$

By means of reduction, substituting (23) into (25) leads to:

$$\mathrm{f}_i^4=I_i-c(\mathrm{f}_i^3)^{-1}\left[\sum_{k=1}^{m}\sqrt{\frac{2}{3}\hat{\varepsilon}_{ik}^{T}\hat{\varepsilon}_{ik}+\eta_0^2}(\mathrm{f}_{ik}^{1})^{-1}\right]=I_i-(\mathrm{f}_i^3)^{-1}\left(\mathrm{f}_i^3-I_i\right)=(\mathrm{f}_i^3)^{-1} \tag{26}$$

From (26), the system (24) can be simplified as the following form:

$$\mathrm{f}^5 d\dot{u} = d\beta \mathrm{f}^6 + \mathrm{f}^7 \tag{27}$$

Functions f^5, f^6, and f^7 in the above equation have the following forms respectively:

$$\mathrm{f}^5 = \sum_{i=1}^{NG} \hat{B}_i^T \mathrm{f}_i^4 \hat{B}_i \tag{28a}$$

$$\mathrm{f}^6 = \sum_{i=1}^{NG} \hat{B}_i^T \mathrm{f}_i^4 \sum_{k=1}^{m} \sqrt{\frac{2}{3}\hat{\varepsilon}_{ik}^T \hat{\varepsilon}_{ik} + \eta_0^2} (\mathrm{f}_{ik}^1)^{-1} \hat{\sigma}_{ik} \tag{28b}$$

$$\mathrm{f}^7 = -\sum_{i=1}^{NG} \hat{B}_i^T \mathrm{f}_i^4 \sum_{k=1}^{m} (\mathrm{f}_{ik}^1)^{-1} \mathrm{f}_{ik}^2 + \sum_{i=1}^{NG} \hat{B}_i^T \left(\sum_{k=1}^{m} \hat{\varepsilon}_{ik} - \hat{B}_i \dot{u} \right) \tag{28c}$$

The relative displacement rate $d\dot{u}$ can be calculated by (27). We substitute (19*b*) into (28*c*), and from (14*b*) it can be simplified as:

$$d\dot{u} = (\beta + d\beta)(\mathrm{f}^5)^{-1} \mathrm{f}^6 - \dot{u} \tag{29}$$

Substituting (19*b*) and (22) into (21), then from (19a), (25), (26) and (29), simplifying the equation and the incremental vector of strain rate can be obtained:

$$d\hat{\varepsilon}_{ik} = -\hat{\varepsilon}_{ik} + (\beta + d\beta) \mathrm{f}_{ik}^8 \tag{30}$$

where:

$$\mathrm{f}_{ik}^8 = c\sqrt{\frac{2}{3}\hat{\varepsilon}_{ik}^T \hat{\varepsilon}_{ik} + \eta_0^2} (\mathrm{f}_{ik}^1)^{-1} (\mathrm{f}_i^3)^{-1} \hat{B}_i (\mathrm{f}^5)^{-1} \mathrm{f}^6 + \sqrt{\frac{2}{3}\hat{\varepsilon}_{ik}^T \hat{\varepsilon}_{ik} + \eta_0^2} (\mathrm{f}_{ik}^1)^{-1} (\mathrm{f}_i^3)^{-1} \hat{\sigma}_{ik} \tag{31}$$

In order to calculate $(\beta + d\beta)$, from (18*c*) we have:

$$\sum_{i=1}^{NG} \sum_{k=1}^{m} \hat{\sigma}_{ik}^T (\hat{\varepsilon}_{ik} + d\hat{\varepsilon}_{ik}) = 1 \tag{32}$$

Substituting (30) into (32) and solving for unknown $(\beta + d\beta)$, we get:

$$\beta + d\beta = \left(\sum_{i=1}^{NG} \sum_{k=1}^{m} \hat{\sigma}_{ik}^T \mathrm{f}_{ik}^8 \right)^{-1} \tag{33}$$

Refer to (9), $\hat{\sigma}_{ik}^T \hat{\varepsilon}_{ik}$ is normalized from (17c) and it satisfies the following equation:

$$\sum_{i=1}^{NG} \sum_{k=1}^{m} \hat{\sigma}_{ik}^T \hat{\varepsilon}_{ik} = 1 \tag{34}$$

Through (33), the normalized expression of $(\beta + d\beta)$ is

$$\beta + d\beta = \sum_{i=1}^{NG} \sum_{k=1}^{m} \hat{\sigma}_{ik}^T \hat{\varepsilon}_{ik} \cdot \left(\sum_{i=1}^{NG} \sum_{k=1}^{m} \hat{\sigma}_{ik}^T \mathrm{f}_{ik}^8 \right)^{-1} \tag{35}$$

According to (29) and (30), the displacement rate vector $\dot{u}$ and the global strain rate vector $\hat{\varepsilon}$ can be calculated iteratively. Thereinto, $\hat{\varepsilon}$ may be expressed as $\hat{\varepsilon} = \left[\hat{\varepsilon}_{11} \quad \cdots \quad \hat{\varepsilon}_{ik} \quad \cdots \quad \hat{\varepsilon}_{NG,m} \right]^T$.

Thus, the new displacement rate vector $\dot{u}$ and the global strain rate vector $\hat{\varepsilon}$ obtained by an iterative method simultaneously satisfy (17*b*) and (32). During the initial calculation, the normalization of $\hat{\sigma}_{ik}^T \hat{\varepsilon}_{ik}$ is processed, so as to satisfy the equation (17*c*). We can get Lagrange multiplier β updated as $(\beta + d\beta)$ from (35). Iterating those computation steps may get the solutions of $\dot{u}$, $\hat{\varepsilon}$, β which satisfy all the constraint conditions in (17).

Based on the above analysis, by the kinematical shakedown theorem of Koiter, the calculation steps to solve the upper bound shakedown load multiplier α of structure are:

1) Initialize the displacement rate vector $\dot{u}^0$ and the global strain rate vector $\hat{\varepsilon}^0$ so that the normalized condition is satisfied:

$$\sum_{i=1}^{NG}\sum_{k=1}^{m}\hat{\sigma}_{ik}^{T}\hat{\varepsilon}_{ik}^{0}=1 \tag{36}$$

Usually the fictitious elastic stress σ_{ik}^{E} need to be calculate first, so that the load domain L can be determined on the basis of σ_{ik}^{E} . And therefore $\dot{u}^0$ and $\hat{\varepsilon}^0$ can take the normalized fictitious values for their initial values corresponding to the fictitious elastic stress σ_{ik}^{E} . Then set the initial value of penalty factor *c* and small constant η_0 . And determine the convergence criterion of iterative algorithm. According to the computation requirement, the maximum number of iterations allowed to proceed also can be set.

2) Calculate f^5, f^6 and f^7 from (28) at the current value of $\dot{u}$ and $\hat{\varepsilon}$.

3) Compute $(\beta + d\beta)$ from (35), then calculate $d\dot{u}$ and $d\hat{\varepsilon}_{ik}$ from (29) and (30) respectively.

4) According to the searching method with decrease in dimension to find ξ_0 , so as to satisfy:

$$\xi_0 = \min F_p\left(\dot{u} + \xi d\dot{u}, \hat{\varepsilon} + \xi d\hat{\varepsilon}_{ik}\right) \tag{37}$$

Update the displacement rate vector $\dot{u}$, the global strain rate vector $\hat{\varepsilon}_{ik}$ and the Lagrange multiplier β by the following formulas:

$$\dot{u} = \dot{u} + \xi_0 d\dot{u} \ ,\ \hat{\varepsilon}_{ik} = \hat{\varepsilon}_{ik} + \xi_0 d\hat{\varepsilon}_{ik} \ ,\ \beta = \beta + d\beta \tag{38}$$

5) Check the convergence criterion of iterative calculation. If they meet all requirements then stop calculation, the upper bound shakedown load multiplier α is taken as the current value of ξ_0 . Otherwise repeat steps 2 to 4.

3 RATIONALITY VERIFICATION OF THEORETICAL MODEL

In order to verify the reasonableness of the theoretical model for upper bound shakedown analysis proposed in this paper, a classical example of shakedown analysis is adopted here and the result of theoretical method is compared with exiting results in related literatures. As shown in Figure 3, the square plate with a central circular hole is subjected to the biaxial uniform loads $P_1$and P_2. The aperture and the plate width have the ration of D/L=0.2.

The computation applied the present method is shown in Figure 4. The present result is compared with the shakedown lower bound obtained by Groβ-Weege [14] and the shakedown upper bound obtained by Carvelli [15]. As can be seen from the figure, the computing method in this paper is based on the upper bound shakedown analysis, and therefore the present result is much closer to the Carvelli’s upper bound. In general, the computation is ideal, which consequently validates the effectiveness of computing method for upper bound shakedown analysis presented in this article.

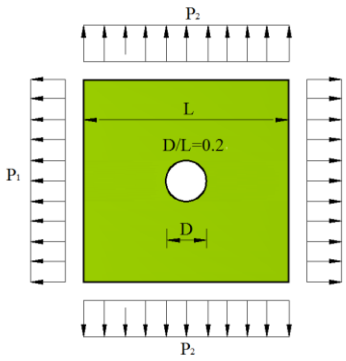


Figure 3: Diagram of plate with a circular hole

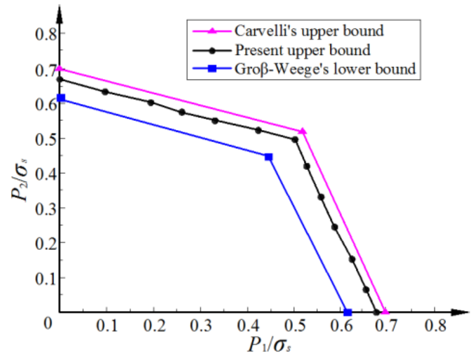


Figure 4: The comparison of shakedown domains

4 SHAKEDOWN ANALYSIS FOR LOCAL STRUCTURES OF SEMISUBMERSIBLE PLATFORM

The strut is at the flexure of structure and stress concentration is more likely to occur. Its destruction would endanger the safety of the entire platform. The strut is in long-term immersion of seawater and subjected to cyclic loads of variable amplitude frequently. It is often difficult to guarantee that the strut completely work at elastic state. Therefore, using the method presented in this paper, the shakedown characteristics of strut is analyzed here, and to determine its ultimate strength subjected to repeated dynamic loads.

The whole model of semisubmersible platform shown in Figure 5 provides boundary conditions for further analysis of the local strut structure. The nodal displacement of corresponding position from whole model is taken as the boundary conditions of the refined strut. The transverse section is shown in Figure 6. The strut is mainly composed of shell, stiffening plate and stiffening rib. And its two terminals are connected with columns. The design parameters of shell thickness, stiffener thickness and stiffener spacing are considered here and their influence on shakedown limit of strut structure were studied. According to actual force condition of strut, the external loads on strut are simplified to axial force P_1 which is applied on the two terminals of strut along the *x* axis and lateral concentrated load P_2 which is perpendicular to strut along the *y* axis. During calculations, P_1 and P_2 are along the negative direction of *x* and *y* axis respectively. Parameters t_1 and t_2 represent the shell thickness and stiffener thickness respectively. And parameter *l* is the stiffener spacing.

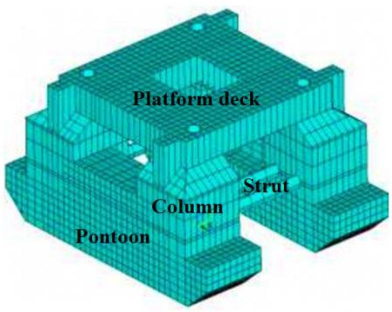


Figure 5: The finite element model of semisubmersible

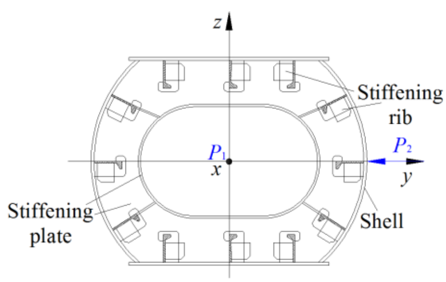


Figure 6: cross section of strut

While the rest structure design parameters remained unchanged, changing the shell thickness, make its value is $0.75t_1$, t_1 and $1.5t_1$ respectively. Following the method of upper bound shakedown analysis in this article, the shakedown limit curves of different shell thickness are shown in Figure 7. The shakedown domain is different with three kinds of shell thickness. P_1 and P_2 are interacted on each other for shakedown domain. Simultaneously with the increase of shell thickness, the shakedown limit also will increase.

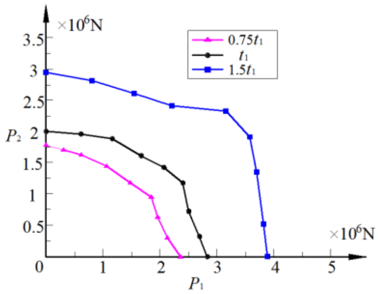


Figure 7: Shakedown domains of different shell thickness

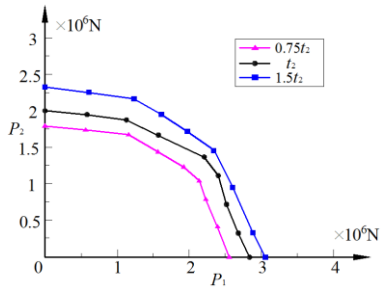


Figure 8: Shakedown domains of different stiffener thickness

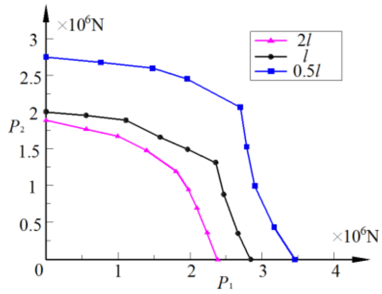


Figure 9: Shakedown domains of different stiffener spacing

And similar to the above analysis of shell thickness, keeping the rest structure design parameters all the same, change the stiffener thickness and make its value is $0.75t_2$, t_2 and 1.5 t_2 respectively. The shakedown domain of strut with different stiffener thickness is shown in Figure 8. The influence of stiffener thickness on shakedown domain of strut is not greater than the shell thickness. But the shakedown limit also increases with the increase of stiffener thickness. Then Figure 9 shows the shakedown domain with different stiffener spacing. The stiffener spacing calculated here is $0.5l$, l and $2l$ respectively. Thus it can be seen that without changing the rest structure design parameters, the stiffener spacing has significant effect on the shakedown domain. With decrease of the stiffener spacing, the global stiffness of strut is enhanced and the shakedown limit increases too.

5 CONCLUSIONS

According to the kinematical shakedown theorem of Koiter and combined with finite element analysis, the shakedown analysis method of ocean engineering structures subjected to repeated dynamic loads is proposed and the rationality of the calculation method is verified by an example. By taking the semisubmersible platform as the research object, the local refining model of strut is established. Influence of shell thickness, stiffener thickness and stiffener spacing on shakedown limit of strut is studied under repeated dynamic loads. It can be seen from the results that:

1) With the increase of shell thickness and stiffener thickness, the shakedown limit also will increase;

2) The influence of stiffener thickness on shakedown domain of strut is not greater than the shell thickness. Meanwhile, the stiffener spacing has significant effect on the shakedown domain;

3) With decrease of the stiffener spacing, the global stiffness of strut is enhanced and the shakedown limit increases too.

REFERENCES

[1] J.-W. Simon, D. Weichert. Shakedown analysis of engineering structures with limited kinematical hardening. *International Journal of Solids and Structures*, Vol. 49, pp. 2177-2186, 2012.

[2] Jaan-Willem Simon, Dieter Weichert. Numerical lower bound shakedown analysis of engineering structures. *Computer Methods in Applied Mechanics and Engineering*, Vol. 200, pp. 2828-2839, 2011.

[3] Shenshen Chen, Yinghua Liu, Zhangzhi Cen. Lower bound shakedown analysis by using the element free Galerkin method and non-linear programming. *Computer Methods in Applied Mechanics and Engineering*, Vol. 197, pp. 3911-3921, 2008.

[4] Chen Shenshen, Liu Yinghua, Cen Zhangzhi. A meshless local Petrov-Galerkin method for static shakedown analysis of elasto-plastic structures. *Chinese Journal of Theoretical and Applied Mechanics*, Vol. 41, pp. 713-721, 2009.

[5] Yang Sun, Shui-Long Shen, Xiao-He Xia, Zheng-Liang Xu. A numerical approach for predicting shakedown limit in ratcheting behavior of materials. *Materials* & *Design*, Vol. 47, pp. 106-114, 2013.

[6] H.X. Li. Kinematic shakedown analysis under a general yield condition with non-associated plastic flow. *International Journal of Mechanical Sciences*, Vol. 52, pp. 1-12, 2010.

[7] H.X. Li, H.S. Yu. A nonlinear programming approach to kinematic shakedown analysis of frictional materials. *International Journal of Solids and Structures*, Vol. 43, pp. 6594-6614, 2006.

[8] M.J. Fadaee, H. Saffari, R. Tabatabaei. Shakedown limit of elastic–plastic offshore structures under cyclic wave loading. *Ocean Engineering*, Vol. 35, pp. 1854-1861, 2008.

[9] Hai-Sui Yu, Juan Wang. Three-dimensional shakedown solutions for cohesive-frictional materials under moving surface loads. *International Journal of Solids and Structures*, Vol. 49, pp. 3797-3807, 2012.

[10] König J.A., Kleiber M. On a new method of shakedown analysis. *Bulletin del'Academie Polonaise des Sciences, Serie des sciences techniques*, Vol. 26, pp. 165-171, 1978.

[11] Thanh Ngọc Tran, R. Kreißig, Duc Khôi Vu, Manfred Staat. Upper bound limit and shakedown analysis of shells using the exact Ilyushin yield surface. *Computers* & *Structures*, Vol. 86, pp. 1683-1695, 2008.

[12] Knud D. Andersen, Edmund Christiansen, Andrew R. Conn, and Michael L. Overton. An Efficient Primal-Dual Interior-Point Method for Minimizing a Sum of Euclidean Norms. *SIAM Journal on Scientific Computing*, Vol. 22, pp. 243–262, 2000.

[13] Zhang guangcheng. Computational Methods for Nonlinear Optimization[M]. *Beijing: Higher Education Press*, 2005.

[14] Johannes Groβ-Weege. On the numerical assessment of the safety factor of elastic-plastic structures under variable loading. *International Journal of Mechanical Sciences*, Vol. 39, pp. 417-433, 1997.

[15] V. Carvelli, Z. Z. Cen, Y. Liu, G. Maier. Shakedown analysis of defective pressure vessels by a kinematic approach. *Archive of Applied Mechanics*, Vol. 69, pp. 751-764, 1999.