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PROBABILISTIC STRENGTH ANALYSIS OF FILLED POLYMERIC COMPOSITE MATERIALS AND OF PRODUCTS BASED ON THEM

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Abstract. Work is devoted to the analysis of the reliability of polymeric composite materials. The reliability is evaluated by the calculation of the probability of failure-free operation of a fan impeller using parameters of stress-strain state.

1 INTRODUCTION

Polymeric composite materials are characterized by considerable differences in stress-strain properties. These differences depend on physical properties such as dispersion of matrix material, filling ratio, size and distribution of inclusions, etc. Irregularity of properties is defined particularly by various conditions of material molding in different parts of a product. It is practically impossible to ensure stability of these conditions by means of pressure and temperature control at various points of the product. Property variation range can be estimated by statistical characteristics such as mathematical expectation and standard deviation.

2 PROBLEM STATEMENT

One of the most important complex indexes of the quality of any product is reliability. Its quantitative characteristic is probability of failure-free operation, i.e. the probability that no failure occurs in a specified interval of time [1, 2].

The problem of reliability evaluation in quantitative statements is extremely complicated. This evaluation begins with an analysis of product designation, operating conditions and types of failures expected. Such analysis allows selecting the criterion of operational integrity defined by the functionality and performance of the product. The reliability evaluation of metallopolymeric parts is performed using strength criterion considering various linear thermal expansion coefficients for metals and composites.

Condition of operational integrity by strength criterion or strength condition considering material aging and temperature dependence of limiting and design stress can be written as follows:

$$\sigma_{eq}(t,T^{\circ}) < \sigma_{lim}(t,T^{\circ}), \tag{1}$$

where $\sigma_{eq}(t,T^{\circ})$ – equivalent stress in critical point of a product,

$\sigma_{lim}(t,T^{\circ})$ – limiting stress.

This condition of operational integrity can be applied for material aging at a fixed temperature, limiting design stress at a fixed temperature, or for both factors simultaneously [3].

3 APPROACH TO THE RELIABILITY EVALUATION

Reliability evaluation of products by strength criterion considering material aging or temperature dependence of limiting and design stress requires determining the probability that limiting stress in the material at moment in time t_i or at temperature T°_i is more than the design stress at the same moment in time or at the same temperature. One of the most used characteristics in strength calculation is yield stress. The yield stress will be assumed as a limiting stress for material of a product.

Let us introduce a random variable and define it as follows:

$$y_i = \sigma_{lim\,i}(t,T^{\circ}) - \sigma_{eq\,i}(t,T^{\circ}). \tag{2}$$

In order to ensure reliability this random variable must be positive, i.e. limiting stress (yield stress) of the polymeric material must be greater than the calculated design stress. Then the probability of failure-free operation of an element at the moment of time t_i at temperature T°_i can be equated:

$$R(t)_i = P(y_i > 0) = \sigma_{lim\,i}(t,T^{\circ}) - \sigma_{eq\,i}(t,T^{\circ}) > 0\ . \tag{3}$$

We will assume that the random variable y_i has normal distribution with mathematical expectation:

$$m_{yi} = m_{\sigma_{\lim i}} - m_{\sigma_{eqi}}. \tag{4}$$

and standard deviation:

$$S_{yi} = \sqrt{S_{\sigma_{eqi}}{}^2 + S_{\sigma_{\lim i}}{}^2}. \tag{5}$$

Expressing the probability of failure-free operation R(t) in terms of normalized distribution we have:

$$R = 1 - \Phi\left[-\frac{\bar{\sigma}_{\lim i} - \bar{\sigma}_{eqi}}{\sqrt{S_{\sigma_{\lim i}}{}^2 + S_{\sigma_{eqi}}{}^2}} \right]; \quad (6)$$

where $\bar{\sigma}_{limi}$ – mean value of limiting tensile strength;

$\bar{\sigma}_{eqi}$ – mean value of equivalent stress;

$S_{\sigma eqi}$ – standard deviation of equivalent stress;

$S_{\sigma limi}$ – standard deviation of limiting tensile strength.

3 THE PROBABILITY OF FAILURE-FREE OPERATION OF A FAN IMPELLER

Capabilities of the proposed approach are illustrated by constructing the fields of probabilities of failure-free operation [3, 4] for the central ring of the fan impeller (Fig. 1) made of a polymeric material.

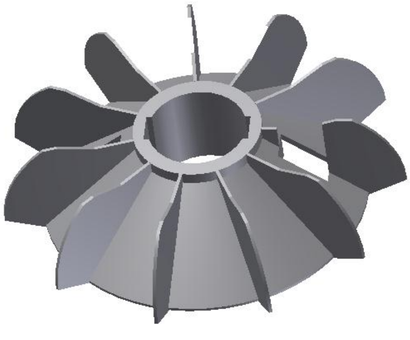

Fig.1. Fan impeller

The impeller is installed on the motor shaft and secured by a bilateral key groove. Stress state in the fan impeller occurs due to the interference when seating the impeller on a shaft and the additional interference that occurs at low temperatures due to the different thermophysical material properties of the impeller and the shaft.

Let us consider the examination of the probability of failure-free operation of a fan impeller in a given temperature operating range. The goal of the calculation is to obtain the appropriate material selection which would guarantee the target level of fan impeller failure-free operation.

The investigated task has two mutually perpendicular axes of symmetry, thus a computational domain representing one quarter of the fan impeller cross section is examined. The elastic problem is solved in (Fig. 2). Tension Δt defines deformation on a part of the computational domain.

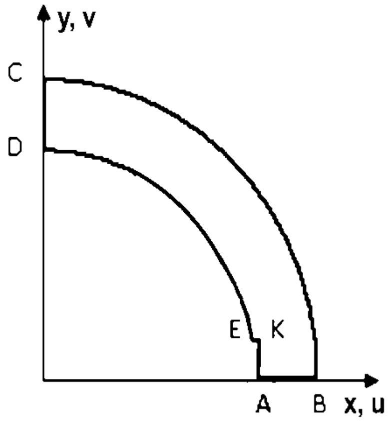

Fig. 2. Computational domain

Boundary conditions:

At boundary AB: V = 0, $\sigma_\tau = 0$ – sliding conditions along a rigid wall which are typical for tasks with presence of symmetry.

At boundary BC: $\sigma_t = 0$, $\sigma_n = 0$ – free surface.

At boundary CD: U = 0, $\sigma_\tau = 0$ – sliding conditions along a rigid wall.

At boundary DE: $U = \Delta t \cdot \cos\alpha$, $V = \Delta t \cdot \sin\alpha$ – condition of continuous contact and tension Δt.

At boundaries EK and KA: $\sigma_t = 0$, $\sigma_n = 0$ – free surface.

Program testing is carried out by the analytical solution of Lame's theory applied to an elastic pipe under internal pressure. This solution becomes applicable for testing if we exclude the groove ЕКА from the scheme. The task is carried out by the Finite Element Method.

Initial data is represented by mean value and standard deviation of Young's modulus and tensile strength of investigated materials at various temperatures.

Tables 1, 2, 3 represent material characteristics (E - Young's modulus, μ – Poisson ratio) at various temperatures. Nominator contains mean value, denominator – standard deviation of relevant variables.

Table 1: Material characteristics at 20 ºC

Material	E, MPa	μ	α, 10^6 1/°C
Ethylene-propylene block copolymer 22007-16	1170/95	0,37/0,020	98/12
Frost-resistant polypropylene 15-04	1110/77	0,36/0,018	104/14

Table 2: Material characteristics at – 40 ºC

Material	E, MPa	μ	α, 10^6 1/°C
Ethylene-propylene block copolymer 22007-16	2100/145	0,22/0,017	98/12
Frost-resistant polypropylene 15-04	2040/127	0,21/0,016	104/14

Table 3: Material characteristics at – 60 ºC

Material	E, MPa	μ	α, 10^6 1/°C
Ethylene-propylene block copolymer 22007-16	2310/165	0,17/0,014	98/12
Frost-resistant polypropylene 15-04	2240/147	0,16/0,013	104/14

Processing an array of parameters of stress-strain state of construction obtained as a result of numerical and natural experiments was carried out by means of the theory of probability and mathematical statistics.

The possibility of product failure has probabilistic nature and it must be estimated by relevant quantitative characteristics. In assessment of probability of failure-free operation it is necessary to compare computational and experimental stress-strain state data in all points of construction with strength criterions.

Obtained stress is compared with strength by mathematical expectation and standard deviation presented in table 4.

Table 4: Limiting stress

Temperature	20°C	- 40°C	- 60°C
Ethylene-propylene block copolymer 22007-16	25/0,94 MPa	66/2,7 MPa	77/2,8 MPa
Frost-resistant polypropylene 15-04	23/0,94 MPa	66/2,7 MPa	78/2,8 MPa

Tests were carried out [3] using heat chamber. Yield strength of materials based on polypropylene at normal conditions is equal to tensile strength. At low temperatures failure strain is small and failure is brittle. Temperature increase leads to greater material yield and failure strain increases significantly. Thus, tensile strength is used as a uniform strength index in this investigation.

In case of examination by critical strain criterion (2nd criterion) an ultimate strain is set for the impeller. This limiting strain is experimentally obtained and corresponds to the yield stress at 20 °C and ultimate strength at lower temperatures (table 5).

Table 5: Ultimate strain

Temperature	20°C	- 40°C	- 60°C
Ethylene-propylene block copolymer 22007-16 Frost-resistant polypropylene 15-04	7%	3%	2%

Calculations show that the probability of failure-free operation in the ring zone of the impeller produced of discussed materials equals to a value not less than 0.99. Figure 3 and Figure 4 illustrate the results of the calculations in the form of distribution surface of strain intensity values and the probability of failure-free operation over the impeller cross section and corresponding isolines.

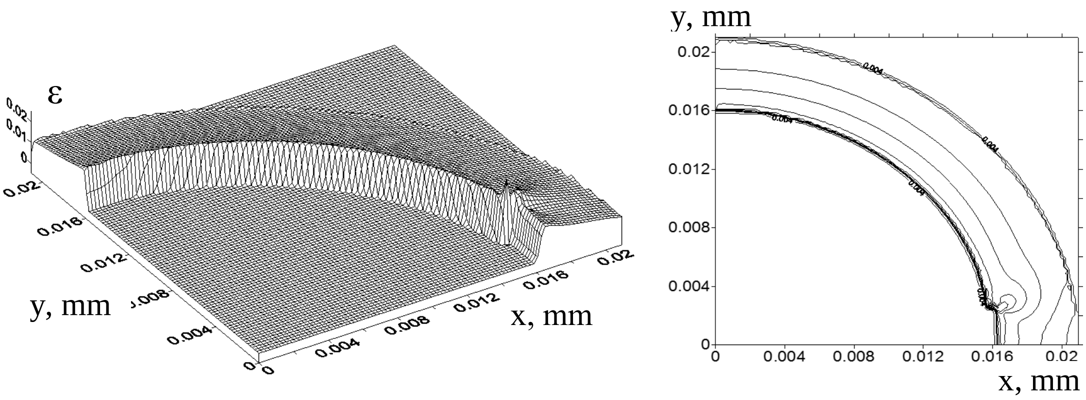

Fig. 3. Strain intensity for ethylene-propylene block copolymer 22007-16 at – 60 °C

The reduction of the probability of failure-free operation is observed in the angular part of the key groove (Fig. 4) as a result of stress and strain concentration in this area (Fig. 3). Values of the probability of failure-free operation dependent on temperature are obtained in the most critical area of impeller which is the area of the key groove shown in table 6.

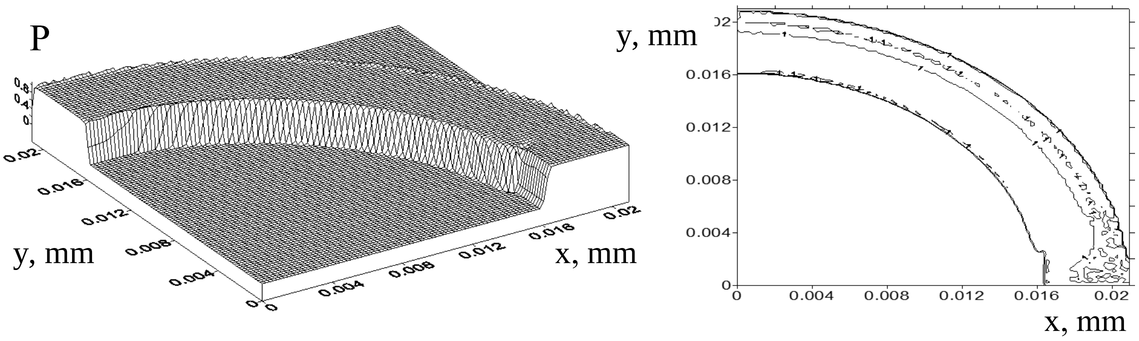

Fig. 4. Probability of failure-free operation for ethylene-propylene block copolymer 22007-16 at – 60 °C

Table 6: Probability of failure-free operation (Pmin)

Material	Frost-resistant polypropylene 15-04			Ethylene-propylene block copolymer 22007-16		
Temperature	20°C	- 40°C	- 60°C	20°C	- 40°C	- 60°C
1st criterion	0.94	0.98	0.98	0.95	0.98	0.97
2nd criterion	1	1	0.87	1	1	0.89

Calculation of the probability of failure-free operation demonstrated that the impeller made of Ethylene-propylene block copolymer 22007-16 has greater reliability at given temperatures.

The probability of failure-free operation at temperature – 60 °C calculated according to ultimate strain criterion is relatively small in the area of the key groove for both materials. This can lead to the development of local failure and to the failure of the whole construction as a result.

The probability of failure-free operation can be increased by the variation of the following parameters: changing the geometry of construction, such as size and shape of key groove or size of impeller utilizing a different material.

The reliability evaluation due to the considered approach is defined in terms of the probability of failure-free operation of material in all points of a product. The results can be presented by the fields of probabilities of failure-free operation throughout the volume of the construction. The transition from local strength assessments to strength of construction can be done using the criterions that depend on the volume fraction of the material in a product, where a failure condition was satisfied.

4 CONCLUSION

1. An approach presented in this paper accounts a random nature of various parameters which define stress-strain state of construction as well as the random nature of strength criteria. This allows one to avoid additional experimentation and to obtain values of numerical characteristics by means of numerical modeling.

2. The provided technique allows one to determine the dependence of the probability of failure-free operation on the values of one or more control parameters. This implies the identification of the most crucial parameters and zones where failure is the most expectable. These factors need particular attention in the production design process.

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