11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
E. Oñate, J. Oliver and A. Huerta (Eds)

INPUT-OUTPUT BASED MODEL REDUCTION FOR INTERCONNECTED SYSTEMS

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Key words: Elastic Multibody Systems, Model Order Reduction, Balanced Truncation, Component Mode Synthesis

Abstract. The typical application for the multibody formalism are systems, in which mechanical bodies are subject to applied forces and interactions. If deformations are to be considered, one or multiple components can be modeled as flexible bodies with the floating frame of reference approach. The flexibility information is usually obtained with a finite element discretization. For a performant simulation, a set of problem-fitted ansatz functions must be determined, leading to a reduction of the flexibility degrees of freedom. Available methods for this task are the industrially well established Craig-Bampton method, other variants from the field of component mode synthesis, sub-structuring or modal truncation. Other schemes, including Krylov subspace methods and Gramian-matrix based model reduction, focus on the input-output behavior of the related linear time-invariant systems. A definition of load cases, respectively inputs, is not straightforward in a setting where multiple bodies interact and boundary conditions may be time-dependent. In this contribution, it is demonstrated that the idea of sub-structuring and its advantages can be merged with input-output based model reduction. Thus, a new reduction method is derived which can robustly handle interacting bodies and outperforms the existing methods in terms of approximation quality respectively computational effort for the time integration.

1 INTRODUCTION

Computer aided engineering is widely used in the modern development process. The usage of numerical methods for the design and improvement of technical products can help to shorten development times. The requirements are versatile and include a desired long product lifetime and improved dynamical behavior in terms of noise or general movement-related criteria. In many applications, a modular system setup is sought for. This enables the reutilization of components and thus enforces economies of scale.

In the context of modular systems, the multibody formalism [1] is a natural choice for an adequate description. Single bodies can be exchanged without re-modeling the whole simulation environment. In order to incorporate the deformability of components, elastic multibody systems (EMBS), Figure 1, can be modeled with the floating frame of reference formulation [2]. This is especially sensible if high working speeds, large forces, or slender geometries are considered. Whenever the comfort of a tool that interacts with a human is considered, vibrations also play an important role even if deformations are not overly visible. In the given setup, a linear elastic deformation of the bodies is added to the nonlinear rigid body motion. The superposition delivers the positions and velocities of arbitrary points of the components. Interactions between the single bodies are modeled with various types of force or damping elements or joints.

The finite element method (FEM), [3], is used in many cases to describe the linear elasticity of the single bodies in terms of ordinary differential equations. Due to high requirements on accuracy, the FEM often leads to systems of equations with many, up to millions, degrees of freedom. In order to be able to keep the computational effort for the EMBS within reasonable bounds, a model order reduction must be performed. There exist a number of approaches to this problem. The methods regarded in this contribution are all based on the projection of the possible deformations of the system onto a suitably defined subspace. The sensible choice of this subspace is the key step in successful model reduction. Traditional, well-established methods are modal truncation, static condensation [4] and various forms of combinations of component modes. In the context of EMBS, the Craig-Bampton method [5] is the most widely used approach and is supported by various FE programs. While modal approximation only focuses on the homogeneous problem, other approaches specifically consider the loading and boundaries that the body is imprinted with, as well as the positions to be measured. These include moment-matching

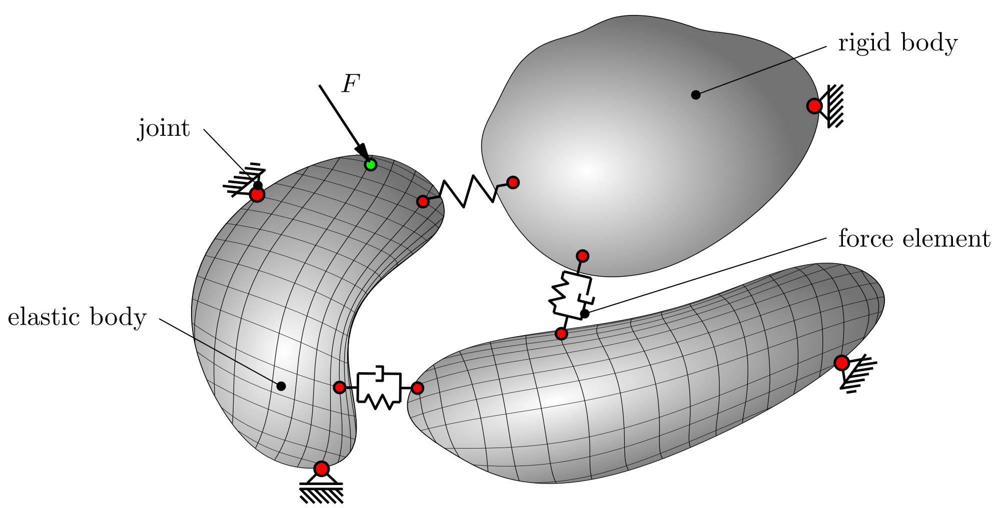


Figure 1: Elastic multibody system with interconnections and applied forces.

with Krylov subspaces and model reduction based on Gramian matrices. For several applications, the advantages of these methods have been presented [6, 7, 8]. Besides yielding an improved approximation quality, also automatized algorithms are available that deliver good results at a minimum of user input [9, 10].

In the following sections, it will be shown that ideas from the field of component mode synthesis can be combined very well with input-output based model order reduction. This results in a new method, that can outperform the established ones in various measures, as will be demonstrated for a numerical example.

2 REDUCTION OF ELASTIC DEGREES OF FREEDOM IN ELASTIC MULTIBODY SYSTEMS

The FEM is used to obtain a set of ordinary differential equations via a spatial discretization of partial differential equations, as they appear in continuum mechanics. The application of the Ritz approach in combination with d'Alembert's principle delivers the linear equation of motion

$$\boldsymbol{M}_\mathrm{e} \cdot \ddot{\boldsymbol{q}}_\mathrm{e}(t) + \boldsymbol{K}_\mathrm{e} \cdot \boldsymbol{q}_\mathrm{e}(t) = \boldsymbol{h}_\mathrm{e}(t) \tag{1}$$

with the symmetric positive definite mass matrix $\boldsymbol{M}_\mathrm{e}$ and the symmetric, at least positive semi-definite stiffness matrix $\boldsymbol{K}_\mathrm{e}$. The time-dependent coordinates $\boldsymbol{q}_\mathrm{e}(t) \in \mathbb{R}^N$ describe the positions of the nodes of the FE mesh. The force vector $\boldsymbol{h}_\mathrm{e}(t)$ describes the forces acting on the body. Due to steadily growing demands on the accuracy, the meshing is nowadays often very dense, which often leads to millions of degrees of freedom for industrially used models. In order to keep the computational burden at an acceptable level, model order reduction must be performed on Equation (1).

2.1 Projection based model order reduction for linear systems

The equation of motion describes a dynamical system. We interpret the forces acting on the system as inputs $\boldsymbol{u}(t) \in \mathbb{R}^p$, spatially distributed to the nodal coordinates by the input matrix $\boldsymbol{B}_\mathrm{e} \in \mathbb{R}^{N\times p}$. Nodal positions of interest $\boldsymbol{y}(t) \in \mathbb{R}^r$ are selected with the output matrix $\boldsymbol{C}_\mathrm{e} \in \mathbb{R}^{r\times N}$. The equation of motion (1) then is interpreted as a linear time invariant (LTI) system in the form

$$\begin{aligned} \boldsymbol{M}_\mathrm{e} \cdot \ddot{\boldsymbol{q}}_\mathrm{e}(t) + \boldsymbol{K}_\mathrm{e} \cdot \boldsymbol{q}_\mathrm{e}(t) &= \boldsymbol{B}_\mathrm{e} \cdot \boldsymbol{u}(t), \\ \boldsymbol{y}(t) &= \boldsymbol{C}_\mathrm{e} \cdot \boldsymbol{q}_\mathrm{e}(t). \end{aligned} \tag{2}$$

We now want to derive a reduced system of dimension $n \ll N$, that captures the relevant dynamics of the system. If inputs and outputs are appropriately chosen, the approximation of the input-output behavior of the system is the main goal. The reduced system lives in a subspace $\mathcal{V}$, onto which the coordinates $\boldsymbol{q}_\mathrm{e}(t)$ are projected with the projection matrix $\boldsymbol{V} \in \mathbb{R}^{N\times n}$

$$\boldsymbol{q}_\mathrm{e}(t) \approx \boldsymbol{V} \cdot \bar{\boldsymbol{q}}_\mathrm{e}(t), \tag{3}$$

which defines the generalized coordinates of the reduced system $\bar{\boldsymbol{q}}_\mathrm{e}(t) \in \mathbb{R}^n$. In general, the solution to the original problem does not live in the subspace $\mathcal{V}$ and thus a residuum $\boldsymbol{r}(t)$ is introduced

$$\boldsymbol{M}_\mathrm{e} \cdot \boldsymbol{V} \cdot \ddot{\bar{\boldsymbol{q}}}_\mathrm{e}(t) + \boldsymbol{K}_\mathrm{e} \cdot \boldsymbol{V} \cdot \bar{\boldsymbol{q}}_\mathrm{e}(t) = \boldsymbol{B}_\mathrm{e} \cdot \boldsymbol{u}(t) + \boldsymbol{r}(t), \tag{4}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}_\mathrm{e} \cdot \boldsymbol{V} \cdot \bar{\boldsymbol{q}}_\mathrm{e}(t). \tag{5}$$

By selection of a second subspace $\mathcal{W}$, the residuum can be eliminated by left-multiplying Equation (4) with $\boldsymbol{W}^T \in \mathbb{R}^{n\times N}$ due to the Petrov-Galerkin condition $\boldsymbol{W}^T \cdot \boldsymbol{r}(t) = \boldsymbol{0}$. The reduced system then reads

$$\begin{aligned} \bar{\boldsymbol{M}}_\mathrm{e} \cdot \ddot{\bar{\boldsymbol{q}}}_\mathrm{e}(t) + \bar{\boldsymbol{K}}_\mathrm{e} \cdot \bar{\boldsymbol{q}}_\mathrm{e}(t) &= \bar{\boldsymbol{B}}_\mathrm{e} \cdot \boldsymbol{u}(t), \\ \boldsymbol{y}(t) &= \bar{\boldsymbol{C}}_\mathrm{e} \cdot \bar{\boldsymbol{q}}_\mathrm{e}(t), \end{aligned} \tag{6}$$

with the reduced mass, and stiffness matrix

$$\bar{\boldsymbol{M}}_\mathrm{e} = \boldsymbol{W}^T \cdot \boldsymbol{M}_\mathrm{e} \cdot \boldsymbol{V}, \quad \bar{\boldsymbol{K}}_\mathrm{e} = \boldsymbol{W}^T \cdot \boldsymbol{K}_\mathrm{e} \cdot \boldsymbol{V}, \qquad \bar{\boldsymbol{M}}_\mathrm{e}, \bar{\boldsymbol{K}}_\mathrm{e} \in \mathbb{R}^{n\times n} \tag{7}$$

and the reduced input and output matrices

$$\bar{\boldsymbol{B}}_\mathrm{e} = \boldsymbol{W}^T \cdot \boldsymbol{B}_\mathrm{e}, \quad \bar{\boldsymbol{B}}_\mathrm{e} \in \mathbb{R}^{n\times p}, \qquad \bar{\boldsymbol{C}}_\mathrm{e} = \boldsymbol{C}_\mathrm{e} \cdot \boldsymbol{V}, \quad \bar{\boldsymbol{C}}_\mathrm{e} \in \mathbb{R}^{r\times n}. \tag{8}$$

In the following, we focus on orthogonal projection $\boldsymbol{W} = \boldsymbol{V}$, which implies symmetry of the reduced system mass and stiffness matrix. The linear system (6) can be used as superelement in FE analyses and as elastic description of one body in EMBS.

2.2 Force application on the elastic part in EMBS

The floating frame of reference formulation splits overall movement of a single body into rigid body motion and elastic parts. The equation of motion for one elastic body reads according to [11]

$$\begin{bmatrix} m\boldsymbol{I} & m\tilde{\boldsymbol{c}}^T(\boldsymbol{q}_\mathrm{e}) & \boldsymbol{C}_\mathrm{t}^T \\ m\tilde{\boldsymbol{c}}(\boldsymbol{q}_\mathrm{e}) & \boldsymbol{J}(\boldsymbol{q}_\mathrm{e}) & \boldsymbol{C}_\mathrm{r}^T(\boldsymbol{q}_\mathrm{e}) \\ \boldsymbol{C}_\mathrm{t} & \boldsymbol{C}_\mathrm{r}(\boldsymbol{q}_\mathrm{e}) & \boldsymbol{M}_\mathrm{e} \end{bmatrix} \cdot \begin{bmatrix} \dot{\boldsymbol{v}}_\mathrm{IR} \\ \dot{\boldsymbol{\omega}}_\mathrm{IR} \\ \ddot{\boldsymbol{q}}_\mathrm{e} \end{bmatrix} = - \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{K}_\mathrm{e} \cdot \boldsymbol{q}_\mathrm{e} \end{bmatrix} + \boldsymbol{h}_\mathrm{gen}, \tag{9}$$

where $\dot{\boldsymbol{v}}_\mathrm{IR}, \dot{\boldsymbol{\omega}}_\mathrm{IR}$ are the accelerations of the reference frame, describing the rigid body motion and $\boldsymbol{h}_\mathrm{gen}$ are generalized forces. Time dependencies are not explicitly in the following. For a detailed explanation, the reader may refer to [1, 2]. It is clearly visible that the rigid body movement induces inertia forces, that act on the elastic part via the coupling terms $\boldsymbol{C}_\mathrm{t}, \boldsymbol{C}_\mathrm{r}$. The representation of the elastic body depends on the choice of the reference frame, see [12] for a review. Obviously, the directly applied forces on the elastic part are not the only ones to be considered during model reduction. A similar effect appears if boundary conditions are applied on elastic degrees of freedom. The resulting forces must be taken into account in the reduction algorithm if input-output based model reduction is performed in order to guarantee good performance for the final EMBS model.

3 CHOICE OF A PROBLEM-FITTED SUBSPACE

The best known reduction method is modal truncation. Eigenmodes (and eigenfrequencies) of the system are defined as the solution of the eigenproblem of the unconstrained body

$$(\boldsymbol{K}_\mathrm{e} - \boldsymbol{\Lambda} \cdot \boldsymbol{M}_\mathrm{e}) \cdot \boldsymbol{V}_\mathrm{modal} = \boldsymbol{0}, \qquad \boldsymbol{\Lambda} = \mathrm{diag}(\omega_j^2), \quad j = 1, 2, \ldots, N. \tag{10}$$

In order to obtain a reduction, only the n most important modes are kept as basis for the subspace $\mathcal{V}$. Since eigenmodes are the solution of the homogeneous problem, no force excitation or loading scenario is considered for the reduction. For non-free-floating bodies, the straight forward modal truncation of the free body thus often delivers unsatisfactory results. Another drawback of modal truncation is the selection of important modes, which usually is judged solely on the corresponding eigenfrequencies.

An improved set of ansatzfunctions can be obtained with the various methods from the field of component mode synthesis. The system is partitioned into $N - b$ internal and b boundary degrees of freedom and the partitioned system reads

$$\begin{aligned} \begin{bmatrix} \boldsymbol{M}_\mathrm{bb} & \boldsymbol{M}_\mathrm{bi} \\ \boldsymbol{M}_\mathrm{ib} & \boldsymbol{M}_\mathrm{ii} \end{bmatrix} \cdot \begin{bmatrix} \ddot{\boldsymbol{q}}_\mathrm{b} \\ \ddot{\boldsymbol{q}}_\mathrm{i} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_\mathrm{bb} & \boldsymbol{K}_\mathrm{bi} \\ \boldsymbol{K}_\mathrm{ib} & \boldsymbol{K}_\mathrm{ii} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{q}_\mathrm{b} \\ \boldsymbol{q}_\mathrm{i} \end{bmatrix} &= \begin{bmatrix} \boldsymbol{B}_\mathrm{b} \\ \boldsymbol{B}_\mathrm{i} \end{bmatrix} \cdot \boldsymbol{u}, \\ \boldsymbol{y} &= \begin{bmatrix} \boldsymbol{C}_\mathrm{b} & \boldsymbol{C}_\mathrm{i} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{q}_\mathrm{b} \\ \boldsymbol{q}_\mathrm{i} \end{bmatrix}. \end{aligned} \tag{11}$$

We assume for the sake of simpler presentation that the system is ordered and then partitioned in that way, otherwise it can be permuted. The Craig-Bampton procedure for example combines eigenmodes of the fixed body with static deformations (Guyan condensation). The Galerkin approximation of the system with

$$\boldsymbol{V}_\mathrm{Guyan} = \begin{bmatrix} \boldsymbol{I} \\ -\boldsymbol{K}_\mathrm{ii}^{-1} \cdot \boldsymbol{K}_\mathrm{ib} \end{bmatrix}, \quad \boldsymbol{V}_\mathrm{Guyan} \in \mathbb{R}^{N \times b}, \tag{12}$$

delivers a statically exact approximation of the system, if forces or boundary conditions are applied on the set of boundary degrees of freedom. This is a property that is shared with (static) moment matching with Krylov subspaces [13]. Fixed interface normal modes are defined as the solution of the eigenproblem

$$(\boldsymbol{K}_\mathrm{ii} - \boldsymbol{\Lambda}_\mathrm{i} \cdot \boldsymbol{M}_\mathrm{ii}) \cdot \boldsymbol{V}_\mathrm{modal,i} = \boldsymbol{0}, \qquad \boldsymbol{\Lambda}_\mathrm{i} = \mathrm{diag}(\omega_{\mathrm{i},j}^2), \quad j = 1, 2, \ldots, N - p. \tag{13}$$

Again, only the most important $n - p$ modes are kept for the reduction. The idea is now to combine static condensation, which ensures inter-component compatibility, with the fixed interface normal modes into one projection matrix

$$\boldsymbol{V}_\mathrm{CB} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ -\boldsymbol{K}_\mathrm{ii}^{-1} \cdot \boldsymbol{K}_\mathrm{ib} & \boldsymbol{V}_\mathrm{modal,i} \end{bmatrix}, \quad \boldsymbol{V}_\mathrm{CB} \in \mathbb{R}^{N \times n}. \tag{14}$$

The Craig-Bampton scheme is well-established and frequently used in many applications. In contrast to standard modal truncation, load cases are accounted for by the use of Guyan condensation. But still, for the internal dynamics, an approximation that is only based on the homogeneous problem is used and a sensible selection of fixed interface normal modes can be tricky.

3.1 Frequency weighted balanced truncation

The basic idea of balanced truncation is to keep only degrees of freedom that significantly contribute to the input-output behavior. Energy-related criteria are derived with the use of Gramian controllability and observability matrices, [14]. Two Gramian matrices for non-conservative second order system are the position controllability Gramian matrix $\boldsymbol{P}_\mathrm{p}$ and the position velocity observability Gramian matrix $\boldsymbol{Q}_\mathrm{pv}$

$$\boldsymbol{P}_\mathrm{p} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \boldsymbol{L}^{-1}(\omega)\cdot\boldsymbol{B}_\mathrm{e}\cdot\boldsymbol{B}_\mathrm{e}^T\cdot\boldsymbol{L}^{-H}(\omega)\mathrm{d}\omega, \tag{15}$$

$$\boldsymbol{Q}_\mathrm{pv} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \boldsymbol{L}^{-H}(\omega)\cdot\boldsymbol{C}_\mathrm{e}^T\cdot\boldsymbol{C}_\mathrm{e}\cdot\boldsymbol{L}^{-1}(\omega)\mathrm{d}\omega, \tag{16}$$

$$\text{with}\quad \boldsymbol{L}(\omega) = -\omega^2\boldsymbol{M}_\mathrm{e} + \mathrm{i}\omega\boldsymbol{D}_\mathrm{e} + \boldsymbol{K}_\mathrm{e}. \tag{17}$$

For frequency weighted balanced truncation, a weighting can be introduced. In the case of a bandwidth filter from ω_1 to ω_2, the position controllability Gramian matrix is defined

$$\boldsymbol{P}_\mathrm{p}^\mathrm{F}(\omega_1,\omega_2) = \frac{1}{\pi}\int_{\omega_1}^{\omega_2} \boldsymbol{L}^{-1}(\omega)\cdot\boldsymbol{B}_\mathrm{e}\cdot\boldsymbol{B}_\mathrm{e}^T\cdot\boldsymbol{L}^{-H}(\omega)\mathrm{d}\omega. \tag{18}$$

With the Gramian matrices, the system can be transformed into a balanced realization, in which states can be sorted by significance on the input-output behavior. Balanced states relate to the Hankel singular values, that directly express the impact on the transfer behavior. States that relate to small Hankel singular values are then neglected for the reduced model.

There exist a-priori error bounds on the reduced system and automated procedures are available [6]. Besides, the approximation quality significantly outperforms modal truncation for the same size of the reduced system, compare [15]. However, if the exact input is not know during the reduction, the approximation quality can drastically worsen. The error bounds do not hold any more if subsystems are connected [16].

4 COMBINATION OF SUBSTRUCTURING IDEAS AND INPUT-OUTPUT BASED REDUCTION

We want to eliminate the drawbacks of unknown inputs by applying Gramian-matrix based model reduction only on the internal dynamics. In order to keep the compatibility to correctly connect the elastic body, Guyan condensation, with $\boldsymbol{\Psi}_\mathrm{c} = -\boldsymbol{K}_\mathrm{ii}^{-1}\cdot\boldsymbol{K}_\mathrm{ib}$, is used. In contrast to Craig-Bampton, the fixed interface normal modes are replaced with a projection matrix $\boldsymbol{V}_{\text{in}}$

$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{\Psi}_{\text{c}} & \boldsymbol{V}_{\text{in}} \end{bmatrix}, \quad \boldsymbol{V} \in \mathbb{R}^{N \times n}. \tag{19}$$

The inputs to the internal dynamics that are considered are directly applied forces which do not act on the set of boundary degrees of freedom and inertia related forces, that are induced by the movement of the boundary nodes. The acceleration of the Guyan modes leads to a distributed force excitation on the internal dynamics with

$$\boldsymbol{f}_{\text{inertia}} = -(\boldsymbol{M}_{\text{ib}} + \boldsymbol{M}_{\text{ii}} \cdot \boldsymbol{\Psi}_{\text{c}}) \cdot \ddot{\boldsymbol{q}}_{\text{b}} = \tilde{\boldsymbol{B}}_{\text{i}} \cdot \ddot{\boldsymbol{q}}_{\text{b}}. \tag{20}$$

The input matrix for the internal dynamics is thus defined by the mass distribution and the choice of boundary degrees of freedom. With this definition, input-output motivated model order reduction, such as frequency weighted balanced truncation, is possible for the internal dynamics. The approach is explained in more detail and results for an industrially used FE model are shown in [17].

5 NUMERICAL EXAMPLE

In order to verify the benefits of the presented method, a numerical example is provided. The setup consists of two elastic bodies, Figure 2, that are clamped in two positions. Besides, the upper body is locked to the inertial system in two points. A force is applied on the left side and measurements are taken on the right side. All interface nodes are master nodes of respective `RBE2`-elements.

For the analysis of the approximation quality of each reduction method, both single bodies are reduced without incorporating the other bodies properties. After that, both reduced bodies are coupled in an EMBS. The full finite element solution serves as reference result.

The first criterion we want to inspect is the approximation of the transfer function. We want to approximate the system well in the frequency range $f = [0, 1000]$ Hz, which can be used for frequency weighted balanced truncation. The single bodies are each

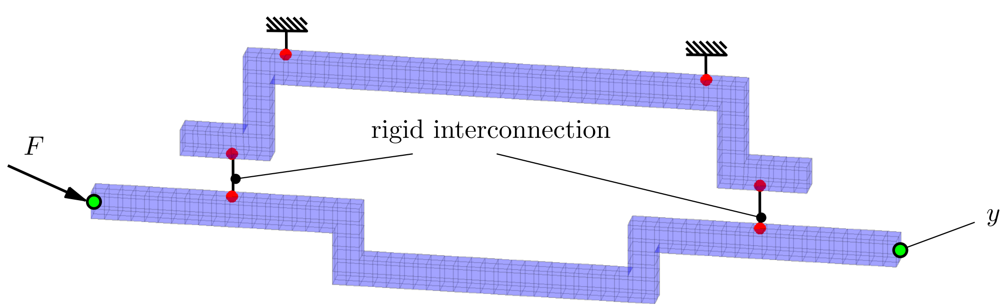


Figure 2: Model setup with force application F and measurement y.

reduced to $n = 25$ degrees of freedom with the Craig-Bampton method and the presented combination of CMS ideas and Gramian matrix based model reduction.

The Frobenius norm of relative error of the transfer function

$$\varepsilon(f) = \frac{||\boldsymbol{H}(f) - \bar{\boldsymbol{H}}(f)||_F}{||\boldsymbol{H}(f)||_F}, \tag{21}$$

where $\boldsymbol{H}(f)$ denotes the transfer matrix of the full system and $\bar{\boldsymbol{H}}(f)$ the transfer matrix of the reduced system, is depicted in Figure 3..

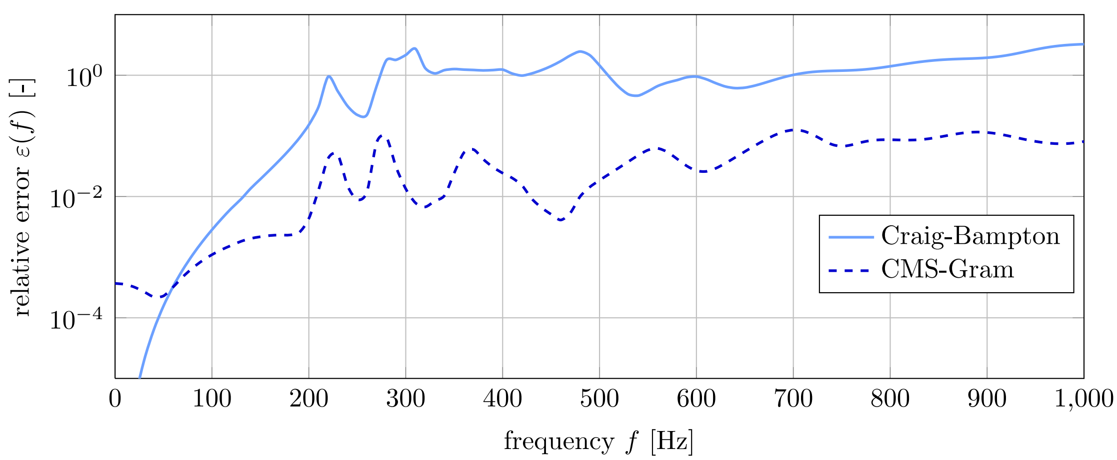
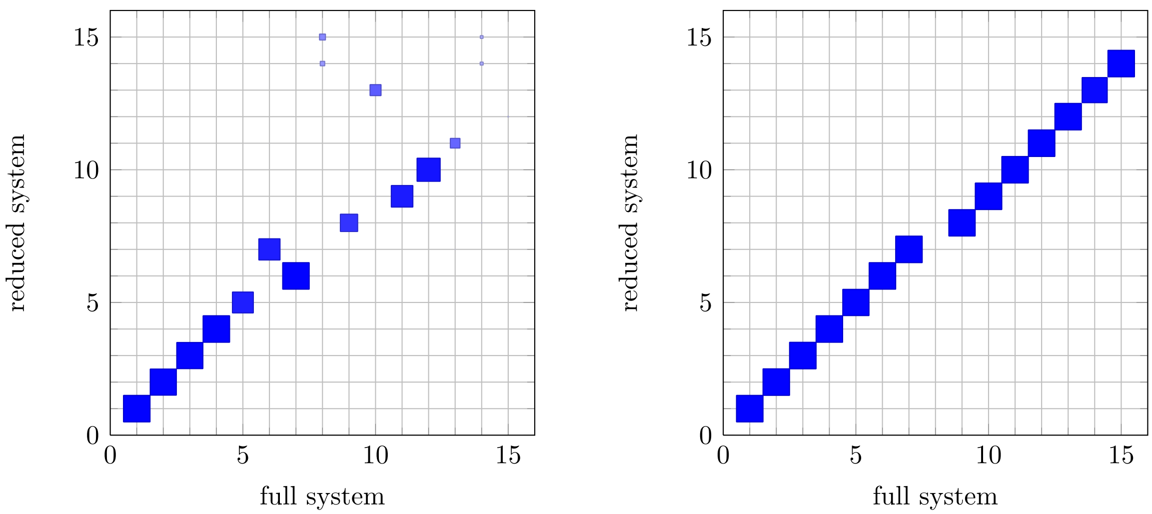


Figure 3: Relative error of the coupled reduced bodies.

While the Craig-Bampton method (——) provides the exact static solution, the approximation gets very poor for frequencies larger than 200Hz. The CMS-Gram approach (- - -) can in contrast approximate the transfer behavior in acceptable quality over the whole frequency range of interest.

One point that is often criticized about the idea of input-output based model reduction is that the global approximation quality worses in comparison to the use of eigenmode-related schemes. A measure that is suited to analyze the global approximation, and is frequently used also in modal analysis, is the modal assurance criterion (MAC). The MAC is basically a correlation test for single vectors [18], which can be modified to judge on the correlation of mass-orthogonal eigenmodes. The MAC-values in this case identify how much the eigenvectors of the reduced system correlate with eigenvectors of the full coupled system. The results for both reduced systems are shown in Figure 4. Even though no eigenvalue problem is solved in the presented method, we can approximate the eigenmodes of the coupled system much better than with the Craig-Bampton approach. The Craig-Bampton method fails to approximate more than seven eigenmodes in acceptable quality with the chosen dimension of the reduced system n. With the CMS-Gram approach on the other hand, only the eighth eigenmode of the full system is not represented in the



(a) Craig-Bampton method.

(b) CMS-Gram (presented approach).

Figure 4: Modal assurance criterion for the eigenmodes of the coupled system.

reduced system. The reason is that this eigenmode does not contribute significantly to the input-output behavior of the system and is therefore not important for the approximation we seek.

6 CONCLUSIONS AND OUTLOOK

In this contribution it is shown how input-output based model order reduction can be used in the simulation of coupled systems. With the help of a newly defined input matrix, it is possible to replace the modes of vibration in the Craig-Bampton method with Gramian matrix based ansatzfunctions. For the same dimension of the reduced coupled system, the approximation quality significantly improves, which is show in terms of the transfer matrix approximation and the modal assurance criterion.

A drawback, which is shared with the Craig-Bampton apprach, is that the reduced systems get large if a large number of inputs or measurement points must be considered. In order to tackle this problem, the set of Guyan ansatzfunctions could be replaced with a more sophisticated approach, especially if some or all of the boundary conditions are known in detail during the reduction.

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