New insights into viscoelastic contact mechanics between rough solids

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Abstract. The optimized design of a large variety of rubber-made engineering applications (e.g. tires, dampers, rubber seals) needs the full solution of many mechanical problems involving viscoelasticity. Particularly interesting is the sliding contact mechanics of rough viscoelastic solids, where, in addition to the convoluted nature of the material response, the surface roughness boosts the problem complexity. In this paper, by employing the adaptive non uniform mesh developed by the authors in [1,2] and the mathematical formulation proposed in [3], we focus on the main aspects of this issue, i.e. the viscoelastic dissipative phenomena and the viscoelastic induced anisotropy.

1 INTRODUCTION

Engineering polymers are widely being assumed as smart engineering solutions thanks to the interesting combination of good mechanical and chemical properties, in terms of resilience, elasticity, durability, with an extremely low density. However, design of rubber and rubber-based components need a really careful examination of their mechanical properties. Indeed, viscoelastic dissipation is the key feature marking their complicated mechanical behavior since it is strictly related to the choice of suitable rubber-based materials for each application. Tires, V-belts, rollers, seals are only examples of components where damping effects have to be accounted for as the most important target parameters. Therefore, a large variety of scientific contributions have been dedicated to this theme [3-14].

Here, we focus our attention on the sliding steady-state contact of rigid rough surface over a viscoelastic half-space. Basically, in this case, the material is marked by two important phenomena, both induced by viscoelasticity: on one side, bulk deformation produces a viscoelastic dissipation; on the other, contact solution shows a marked anisotropy. In the paper, we explore these issues in detail.

In the past, some analytical investigations have been carried out for the case of 2D smooth contacts by Hunter [4] and, in parallel, by Goriacheva [5]; furthermore, by adopting the strong approximation of a narrow shape punch, Kalker has analyzed the 3D case [6]. However, these models are affected by significant drawbacks since, beside the approximations related to punch shape, they are only capable of handling ideal viscoelastic materials with one relaxation time. However, as widely shown experimentally (see, for example Ref. [7,8]), real viscoelastic materials shows a large spectrum of relaxation times, actually covering several order of magnitudes. Computational complexity is, furthermore, boosted when accounting for the

roughness between the contacting surfaces. Persson [9], has developed an innovative analytic approach to account for the role played by the roughness and calculate, among the other quantities, the contact area and the viscoelastic dissipation. This methodology, being exact in full contact conditions, is approximate in partial contact conditions of practical interest.

Regarding the numerical methods, in spite of the availability of many boundary element formulations able to provide, at least for the case of smooth contacts, an accurate interfacial solution, we still lack a methodology being not limited to one relaxation time [10]. Furthermore, finite element methods (FEM) [11-16] are capable of managing real materials, but, usually, they do not provide an accurate interfacial stress estimate. Indeed, this problem is particularly serious in case of rough viscoelastic contact, where roughness introduces a very large number (covering even six orders of magnitudes) of length-scales and time-scales: consequently, a very fine discretization grid could be required, thus leading to an exponential increase of the degrees of freedom and to impracticable computation times.

In this paper, our aim is to propose an effective numerical methodology capable of determining the viscoelastic rough contact solution. In details, the theory presented by the authors in [3] is shown to be able to study the contact of a rough surface sliding over a viscoelastic half space. Indeed, the computation complexity of the contact domain is solved by employing the adaptive non uniform mesh developed by the authors in [2,3]. This scheme allows us to decrease the number of elements needed to solve the problem; in such a way, the computation time results significantly reduced. By means of this approach, we calculate the viscoelastic friction and analyze the contact area anisotropy due to viscoelasticity effects .

2 MATHEMATICAL FORMULATION

In [1, 2], in order to analyze the contact between linear elastic materials, the authors developed a boundary element formulation based on the following integral equation:

$$u(\boldsymbol{x}) = \int_D G(\boldsymbol{x}-\boldsymbol{x}')\sigma(\boldsymbol{x}')dx'^2 \tag{1}$$

This relation correlates the interfacial normal stress $\sigma(\boldsymbol{x})$ to the displacement distribution $u(\boldsymbol{x})$. In [2], the domain *D* has been meshed in small squares of different size by use of a non-uniform adaptive discretization. It was, therefore, possible to reach a fully converged numerical solution with relatively low computational costs.

Our following purpose has been to formulate the viscoelastic problem in such a way that the final equation could be written in a form similar to Eq. (1). In detail, when focusing on a viscoelastic half-space that is moving at constant velocity *v* on a rigid substrate, the problem solution will require to determine the steady-state pressure distribution, the shape of the deformed interface, the contact area and the friction force.

Moving from this point of view, we observe that for a viscoelastic linear material, the relation between the displacement at the interface and the interfacial normal stress distribution can be written in the form:

$$u(\boldsymbol{x},t) = \int_{-\infty}^{t} d\tau \int d^2x' J_1(t-\tau)J_2(\boldsymbol{x}-\boldsymbol{x}')\dot{\sigma}(\boldsymbol{x}',\tau) \tag{2}$$

where, for a generic linear viscoelastic material [3], $J_1(t)$ is the creep function equal to $J_1(t) = \frac{1}{E_0} - \sum_{k=0}^{+\infty} C_k \exp(-t/\tau)$ with C_k positive coefficients, and $J_2(t)$ is the usual elastic Green's function $J_2(t) = \frac{1-\nu^2}{\pi|\boldsymbol{x}|}$.

Now, we observe that , due to the steady state assumption, $\sigma(\boldsymbol{x},t)$and $u(\boldsymbol{x},t)$ are respectively equal to $\sigma(\boldsymbol{x},t) = \sigma(\boldsymbol{x}-\boldsymbol{v}t)$ and $u(\boldsymbol{x},t) = u(\boldsymbol{x}-\boldsymbol{v}t)$. As a matter of fact, by following the mathematical procedure described in in [3], we can rewrite Eq. (2) as :

$$u(\boldsymbol{x}) = \frac{1-\nu^2}{\pi}\frac{1}{E_\infty}\int_D \frac{\sigma(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}d^2x' + \frac{1-\nu^2}{\pi}\sum_{i=1}^{n} C_i \int_0^{+\infty} \exp(-z)\int_D \frac{\sigma(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'+v\tau_i z|}d^2x' \tag{5}$$

where E_∞ is the high frequency viscoelastic modulus of the material. By adopting the iterative scheme introduced in [2] for the case of elastic materials, Eq. (5) can be quite easily inverted and the full solution of the viscoelastic problem can be achieved. As shown in details in [4], once calculated the stresses and the strains, we can determine the friction force as:

$$F_D = \int_D \sigma(x)\frac{\partial h(x,y)}{\partial x}d^2x \tag{6}$$

where $h(\boldsymbol{x}) = h(x,y)$ is the rigid surface in motion over the viscoelastic half space.

In the previous formulation, we are neglecting the Coulomb friction: basically, we are determining the normal problem solution. However, this approximation does not lead to serious limitations because the normal stress and strain distributions, which are necessary to calculate the viscoelastic dissipation, are almost negligibly affected by the presence of tangential stresses at the interface.

3 RESULTS: FRICTION & ANISOTROPY

To elucidate the main features of viscoelastic contacts, here we study the contact mechanics of a rigid rough fractal surface sliding over a viscoelastic half space ($h\to+\infty$) with one relaxation time and, specifically, the following material properties: $E_\infty=10^7$ Pa, $E_\infty/E_0=3$, and $\tau=0.01$ s. As far as the rough surface, we numerically generate self-affine fractal surfaces by means of the spectral method described in [1,17]. These surfaces have spectral components in the wavelength range $q_L<q<q_1$, where $q_L=2\pi/L$ (being L the side of the square computational cell is $L=0.01m^{-1}$), $q_1=Nq_0$ and N number of scales (or wavelengths). In particular, results shown in this section are obtained with N=64.

Our analysis starts calculating the viscoelastic friction. Indeed, in Figure 3, we analyze the viscoelastic friction as a function of the dimensionless speed $\xi=v\tau_0/L$ for the fixed normal load $F_N=0.30$ N. As expected, we have a bell-shaped curve that vanishes for very low and very high speeds, i.e. when the solid behaves as an elastic solid. Indeed, for very low speeds, the material is elastic with the soft elastic modulus E_0, whereas for very high speed, it is elastic with hard modulus E_∞ .

The following step is related to the anisotropy of the contact solution. Indeed, given the constant normal force, due to the stiffening of the material, which occurs as the sliding speed is increased, the contact area strongly decreases with the speed. However, the area decrease is not only the effect, but, due to viscoelasticity, we have a marked shrinkage of the contact area:

although the rough rigid surface in contact is isotropic, the interfacial displacement field (the deformed surfaces) of the viscoelastic solid will show a certain degree of anisotropy.

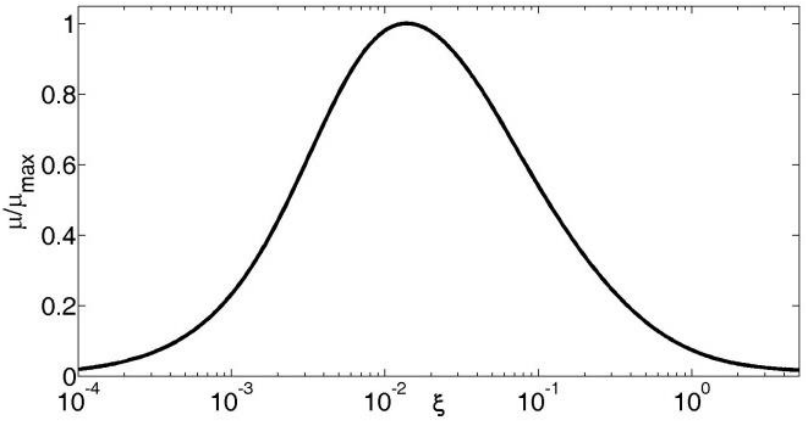

Figure 1. Friction coefficient as a function of the dimensionless sliding speed ξ for a constant normal load P=0.30 N .

Contact Anisotropy can be quantified by focusing on the quantity $m_2(\theta)$: this is the second order spectral momentum of a profile trace made at an arbitrary angle θ with respect to the x axis of the random. The following relation can be applied [18]:

$$m^2(\theta) = m_{20}cos^2(\theta) + 2m_{11}sin(\theta)cos(\theta) + m_{02}sin^2(\theta) \quad (7)$$

where m_{20} is the value of the profile second order momentum along the x axis (i.e. θ=0), m_{02} is the value of the profile second order momentum along the y axis (i.e. θ=π/2) and m_{11} is the association-variance of slope in these two directions [18] .

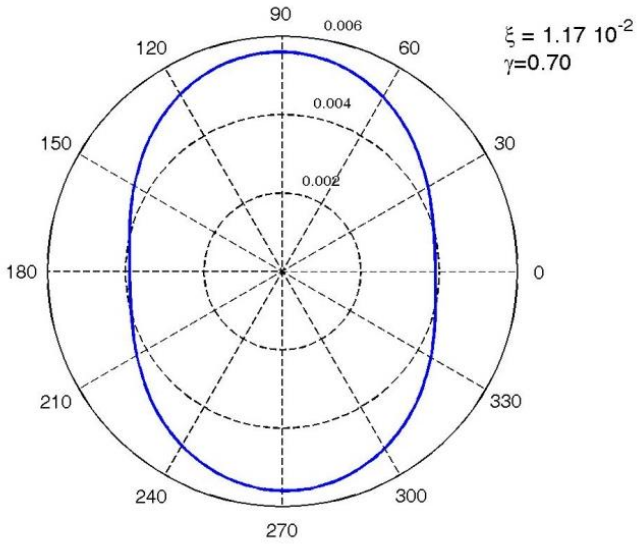

Figure 2. Polar plots of $m_2(\theta)$ for $\xi = 1.17\ 10^{-2}$.

Now, when the surface is perfectly isotropic, the momenta m_{20} and m_{20} are equals, while m_{11} is zero: therefore, when plotting $m_2(\theta)$ in a polar diagram, in this case, we have a circumference with radius $r=m_{20}=m_{20}$. On the other side, if the deformed region is anisotropic, m_{20} and m_{20} are different, and m_{11} is not equal to 0; this entails different curves for polar plot of $m_2(\theta)$. As a matter of fact, a possible way to quantify the anisotropy level is related to the introduction of the following parameters: γ that is the ratio between the maximum and minimum values for $m_2(\theta)$, and Θ being the angle of the profile with the maximum $m_2(\theta)$. In Figure 2, we observe that, for $\xi= 1.17\ 10^{-2}$, the surface shows a consistent anisotropy with $\gamma=0.70$. Furthermore, Θ is almost equal to $\pi/2$, thus being perpendicular to the sliding speed assumed parallel to x axis.

The methodology just introduced relies on the analysis of the deformed surface; therefore, the approach could be not fully effective when considering very small contact area, since, in this case, the non-contact regions can have a prominent contributions blurring contact anisotropy. An alternative procedure, introduced in , deals with the contact area. This approach moves from defining the characteristic function $\chi(x)$, which is equal to 1 if x is in the contact region or, otherwise, to 0. We can introduce the power spectral density $C_\chi(\mathbf{q})$ of $\chi(x)$: clearly, if the contact area is anisotropic, the function $\chi(x)$ and the power spectrum $C_\chi(\mathbf{q})$ will be themselves anisotropic. Indeed, in Figure 3, given a dimensionless speed ξ again equal to $\xi= 1.17\ 10^{-2}$, we observe that the contour curve $C_\chi(q)/C_\chi(0)=0.476$ is an ellipse stretched in the speed direction. This is perfectly coherent with the previous analysis, since the spatial frequencies are clearly related to the inverse of the spatial vectors.

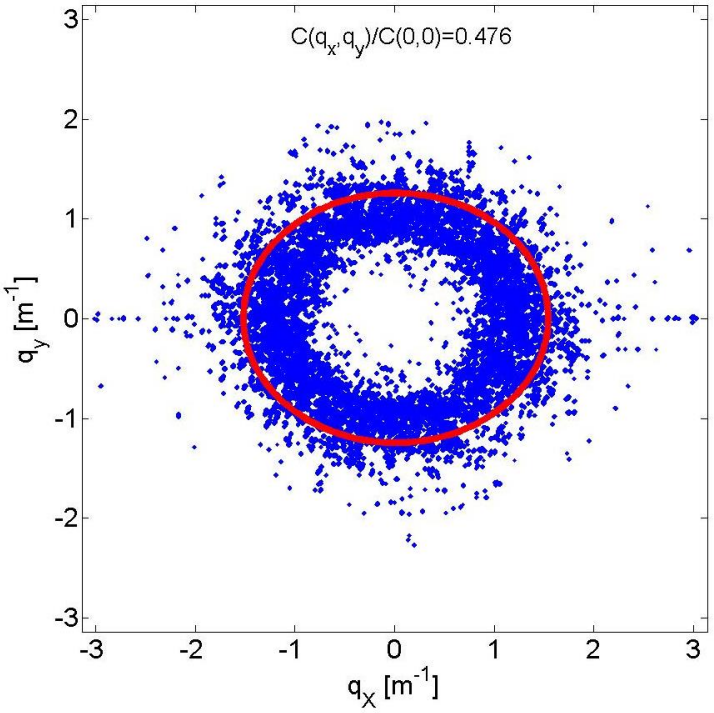

Figure 3. Contour curve $C_\chi(q)/C_\chi(0)=0.476$ for $\xi= 1.17\ 10^{-2}$.

6 CONCLUSIONS

In this paper, the numerical theory already proposed by the authors in [3] is shown to be capable of exhaustively solving the sliding contact mechanics between a viscoelastic solid and a rigid randomly rough surface. In detail, by use of this numerical approach, we have evaluated the viscoelastic friction: as expected, this quantity shows a strong dependence on the sliding speed and, more specifically, tends to vanish for very high and very low speeds, i.e. when the solid shows an elastic behavior. Furthermore, we have shown how the viscoelasticity induces a marked anisotropy of the contact solution. Indeed, the contact area and, consequently, the deformed region are stretched along the speed direction. Anisotropy has been then quantified by following two different procedures, and, interestingly, the maximum degree of anisotropy has been found for the same speed of the maximum friction. We observe that both quantities, namely the friction and the anisotropy degree, have an outstanding importance in applications since influences phenomena, like energy dissipation and leakage. In this context, the presented numerical methodology contributes to simulate these phenomena, thus aiding an optimized design.

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