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NONLINEAR VIBRATIONS OF SHELLS: EXPERIMENTS, SIMULATIONS AND APPLICATIONS

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Abstract. The present paper focuses on the theory and experiments for geometrically nonlinear vibrations of shell type structures made of traditional and advanced materials. Closed shells, curved panels and rectangular plates made of isotropic, sandwich and laminated composite materials are studied. Several original aspects of nonlinear vibrations of shells and panels including the effects of geometric imperfections, geometry and boundary conditions have been addressed and consistent reduced-order models essential to capture shell dynamics are obtained. The numerical analysis is based on multi-dimensional Lagrangian approach and pseudo arc-length continuation technique is used for bifurcation analysis. Moreover, the experimental analysis, an example of the set-up is shown in Figure 1, is carried out following a stepped-sine testing procedure and by increasing and decreasing the excitation frequency in very small steps at specific force amplitudes controlled in a closed-loop. Comparisons between experimental results and numerical simulations are performed and show good agreement for shells and panels oscillating at large-amplitude vibrations.

1 INTRODUCTION

Shell structures can be found in aircraft, spacecraft, submarines, storage tanks and the roofs of buildings. In demanding applications, shell-like structures may undergo significant deformations and stresses. Therefore, models are needed which take nonlinear effects such as large structural deflections into account.

When a circular cylindrical shell is subjected to large-amplitude vibrations, an intrinsic one-to-one internal resonance occurs between the driven mode and the orthogonal mode having the same shape and natural frequency of the driven mode but rotated by $\pi/2n$ (n is the number of nodal diameters) known as the companion mode. This modal interaction yields travelling

wave response in the circumferential direction of the shell which appears even for vibrations smaller than the thickness. Another important feature of closed circular cylindrical shells in non-linear dynamic analysis that has been addressed both theoretically and experimentally is the role of axisymmetric modes. In particular, during large-amplitude vibrations, the shell presents an inward axisymmetric dynamic contraction with twice the excitation frequency to guarantee the in-plane quasi-inextensibility of the shell. In fact, without this axisymmetric contraction, the shell increases its length along the circumference which is against the shell mechanics: A shell bends more easily than it stretches.

An extensive literature review on the nonlinear dynamics of shells in *vacuo*, filled with or surrounded by quiescent and flowing fluids has been provided by Amabili & Païdoussis (2003). The monograph of Amabili (2008) has also provided a comprehensive treatment of nonlinear vibrations and dynamic stability of shells, and has shown that many problems in this area still have unanswered questions. Systematic study of nonlinear dynamics and large-amplitude vibrations of circular cylindrical shells with and without quiescent and flowing fluid has been carried out by Amabili et al. (1999a,b) and Amabili et al. (2000a,b) by using models having a base of seven natural modes. Amabili (2003a,b) extended his previous works on nonlinear forced vibrations of imperfect simply supported empty and fluid-filled circular cylindrical shells subjected to concentrated harmonic force. In particular, in (Amabili, 2003a) Donnell's nonlinear shallow shell theory was used and the nonlinear frequency-response curves were obtained by using the Galerkin method considering several expansions that included sufficient asymmetric and axisymmetric modes. Moreover, a code based on the arc-length continuation technique was utilized to perform bifurcation analysis and several interesting nonlinear phenomena were observed which included softening type nonlinearity, different types of travelling wave response, amplitude-modulated response and modal interactions. Amabili (2003b) compared the numerical results obtained by Donnell's nonlinear theory (with and without in-plane inertia) with Sanders–Koiter, Flügge–Lur'e-Byrne and Novozhilov's theories following a Lagrangian approach.

Nonlinear vibrations of clamped fluid-filled shells have been investigated by Karagiozis et al. (2005) following two numerical models based on Donnell's shallow shell theory with and without in-plane inertia. In a series of papers Koval'chuk & Kruk (2005, 2006) and Kubenko et al. (2010) discussed the problem of nonlinear forced vibrations of completely filled simply supported circular cylindrical shells by using Donnell's nonlinear shell theory without in-plane inertia and the Krylov- Bogolyubov–Mitropol'skii averaging technique.

Nonlinear vibrations of isotropic and orthotropic shells have also been investigated by Jansen (2004) via two types of solutions with different levels of accuracy and complexity. Later, Jansen (2007, 2008) extended his work by considering the effect of geometric imperfection and static loading on the large-amplitude vibrations of composite shells, respectively. Nonlinear vibrations of symmetrically laminated composite shells have been investigated by Amabili & Reddy (2010) and Amabili (2011) based on a new higher-order shear deformation theory that takes into account nonlinear terms involving both the normal and in-plane displacements. Amabili (2012) extended his previous works and studied nonlinear vibrations of angle-ply circular cylindrical shells by taking into account the possibility of modes with nodal lines not parallel to the longitudinal axis.

Even though theoretical studies on nonlinear vibrations of shells are quite abundant, experimental results are very scarce. A complete list of experimental works on nonlinear vibrations and fluid-structure interactions of shell structures before 2003 can be found in the

review study of Amabili & Païdoussis (2003). Comprehensive experimental analysis on large-amplitude vibrations of empty and fluid filled circular cylindrical shells subjected to radial harmonic excitation has been carried out by Amabili (2003a). Amabili et al (2005) studied the geometrically nonlinear vibrations of composite cylindrical panels following a stepped-sine testing approach. Experiments and simulations for nonlinear vibrations of imperfect cylindrical panels have been conducted by Amabili (2006). Mallon et al. (2010) performed experiments on an orthotropic circular cylindrical shell subjected to seismic like excitation. Pellicano (2011) performed experiments on cylindrical shells connected to a rigid disk and under base excitation.

The objective of this paper is to show the solutions of the open problems in the nonlinear dynamics of shell structures through experimental and numerical techniques, and to use a common approach for diverse branches of the nonlinear analysis of shells and panels. Therefore, in the present paper systems under study include shells, plates and curved panels made of traditional and composite materials and the analysis of vibrations has been performed both numerically and experimentally. The numerical investigations have been carried out by developing innovative numerical codes based on reduced-order models obtained via multi-dimensional Lagrangian approach. In order to obtain the response of the structure near resonance, a bifurcation analysis is performed by using pseudo arc-length continuation and collocation scheme and nonlinear frequency response curves are obtained. Laboratory experiments have also been conducted. In particular, a sophisticated measuring procedure has been developed, and a stepped sine testing procedure is implemented by increasing and decreasing the excitation frequency in the spectral neighbourhood of the structure's natural frequencies in order to obtain the experimental frequency response curves and validate the numerical codes.

2 THEORY

Lagrangian approach is a versatile and efficient technique for obtaining the equations of motion of shells and panels. In order to study the large-amplitude vibrations, a two-step approach should be implemented. First, the plate/shell displacements and rotations should be expanded in terms admissible functions with a large number of degrees of freedom to ensure very good accuracy and a linear analysis should be conducted by minimizing the energy functional (Rayleigh-Ritz method) to obtain the natural frequencies and mode shapes. The trial functions that are used need to satisfy only the geometric boundary conditions of the problem under investigation. Then, by selecting suitable natural modes for each displacement and rotation, the number of degrees of freedom is greatly reduced and Lagrange equations of motion are obtained as follows

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_s}\right)-\frac{\partial T}{\partial q_s}+\frac{\partial U}{\partial q_s}=Q_s\,, \quad s=1,\ldots,N\ , \tag{1}$$

where q_s are the generalized coordinates of the system under investigation and includes displacements and rotations of the middle surface of the shell or plate, N is the total number of generalized coordinates, T is the kinetic energy of the system, $\partial T/\partial q_s$ is zero in most of the applications, U is the potential energy (e.g. the elastic strain energy) and Q are the generalized

forces obtained by differentiation of the Rayleigh's dissipation function F and the virtual work W done by the external forces as follows

$$Q_s = -\frac{\partial F}{\partial \dot{q}_s} + \frac{\partial W}{\partial q_s}, \quad s = 1,\ldots,N \ . \tag{2}$$

The term derived from the potential energy of the system is complicated and gives quadratic and cubic nonlinear terms and can be written as

$$\frac{\partial U}{\partial q_s} = \sum_{i=1}^{N} q_i z_{k,i} + \sum_{j=1}^{N}\sum_{i=1}^{N} q_j q_i z_{k,i,j} + \sum_{l=1}^{N}\sum_{j=1}^{N}\sum_{i=1}^{N} q_l q_j q_i z_{k,i,j,l} \ , \tag{3}$$

where z coefficients have long expressions that also include geometric imperfections. In particular, equations (1), (2) and (3) can be conveniently written in the following matrix form:

$$\mathbf{M}\ddot{\mathbf{q}}+\mathbf{C}\dot{\mathbf{q}}+[\mathbf{K}+\mathbf{N}_2(\mathbf{q})+\mathbf{N}_3(\mathbf{q},\mathbf{q})]\mathbf{q}=\mathbf{f}\cos(\omega t) \ , \tag{4}$$

where **M** is the mass matrix, **C** is the damping matrix, **K** is the linear stiffness matrix, $\mathbf{N_2}$ gives the quadratic nonlinear stiffness terms, $\mathbf{N_3}$ denotes the cubic nonlinear terms, **f** is the vector representing external forces and w is the excitation frequency.

The set of ordinary differential equations is studied numerically using a pseudo arc-length continuation and collocation software (Doedel et al. (1998)) to perform bifurcation analysis. In particular, the shell or panel response under harmonic excitation has been studied by using an analysis in two steps: (i) first the excitation frequency has been fixed far enough from resonance and the magnitude of the excitation has been used as the bifurcation parameter; the solution has been started at zero force where the solution is the trivial undisturbed configuration of the structure and has been continued up to reach the desired force amplitude; (ii) when the desired amplitude of excitation has been reached, the solution has been continued by using the excitation frequency as the bifurcation parameter.

3 EXPERIMENTS AND COMPARISONS

In this section experimental results for geometrically nonlinear vibrations of shells and panels are given and comparisons are made with reduced-order models built via Lagrangian approach described in section 2. In particular, three cases have been studied that include: (i) fluid-filled circular cylindrical shells; (ii) imperfect circular cylindrical panels; (iii) free-edge sandwich panels. The experimental nonlinear analysis is carried out following a stepped sine testing procedure, increasing and decreasing the excitation frequency in very small steps in the frequency neighbourhood of the lowest natural frequencies of the structure at specific force amplitudes controlled in a closed-loop to characterize the nonlinear response in the presence of large amplitude vibrations.

3.1 Fluid-filled circular cylindrical shell

Figure 1 shows the experimental set up for a water-filled stainless steel circular cylindrical shell. The dimensions of the shell are L = 520 mm, R = 149.4 mm, h = 0.519 mm, where L is

the length of the shell in, R is the radius and h is the thickness. Two stainless-steel annular plates of external and internal radius of 149.4 and 60 mm, respectively, and thickness 0.25 mm have been welded to the shell ends to approximate the simply supported boundary condition of the shell. A rubber disk 1 mm thick has been glued to each of these annular end plates. The excitation has been provided by a shaker, model LDSV406 with power amplifier LDS PA100E, connected to the shell by a stinger glued in a position close to the middle of the shell. A piezoelectric force transducer, model B&K 8200, of mass 21 g, placed on the shaker and connected to the shell by a stinger, measured the force transmitted.

The shell response has been measured by using two accelerometers; model B&K 4393, of mass 2.4 g. For the nonlinear tests, the two accelerometers have been glued close to the middle of the shell length, at different angular positions corresponding to an antinode and a node of the excited driven mode to measure the nonlinear response. The time responses have been measured by using the Difa Scadas II front-end connected to a workstation with the software CADA-X of LMS for signal processing and data analysis; the same front-end has been used to generate the excitation signal. The CADA-X closed-loop control has been used to keep the value of the excitation force constant for any excitation frequency, during the measurement of the nonlinear response. The geometric imperfections of the tested shell are measured by using a dial gauge on a grid of 100 points, i.e., five equidistant circumferences and 20 positions around each circumference. Moreover, a fine grid of additional 68 points has been measured around the longitudinal weld where small deformations of the shell are present.

Figure 2(a, b) shows the accelerations measured by the two accelerometers versus the excitation frequency for five different force levels: 0.1, 1.5, 3, 4.5 and 6 N. The measured accelerations have been filtered in order to eliminate any other frequency, except the excitation frequency. Experiments have been performed both increasing and decreasing the excitation frequency and hysteresis is clearly visible. The frequency resolution used is 0.025 Hz. Sudden increments (jumps) of the vibration amplitude are observed changing the excitation frequency; these are characteristic of softening type nonlinearity. When the vibration amplitude is equal to the shell thickness, the peak of the response appears for a frequency lower of more than 1 % with respect to the linear one (i.e. the one measured with force 0.1 N). The travelling wave response around the shell has been detected in a relatively large frequency range around the resonance, especially for the excitation force of 6 N.

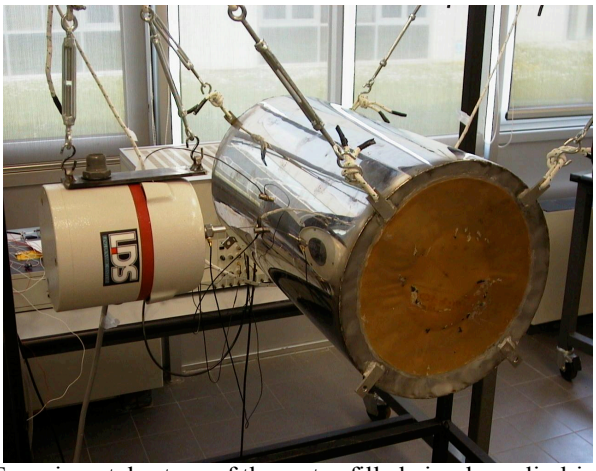
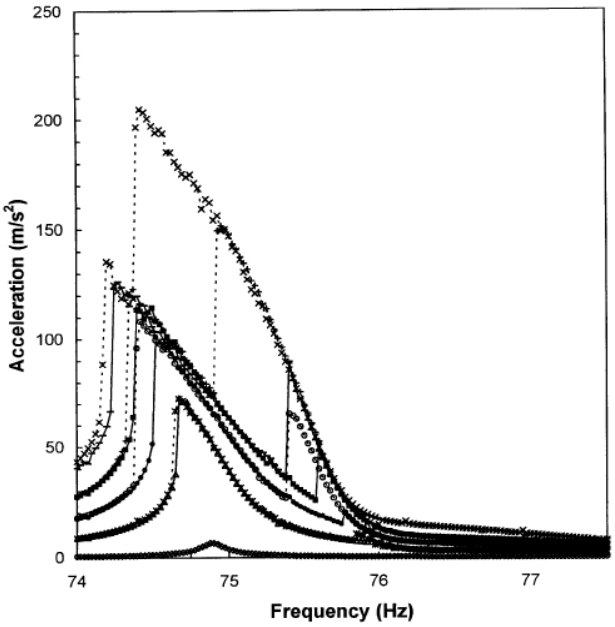
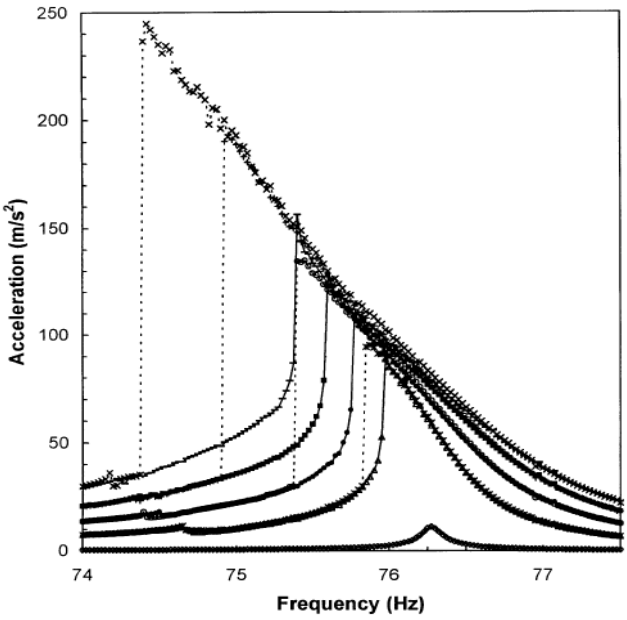


Figure 1: Experimental set-up of the water-filled circular cylindrical shell.

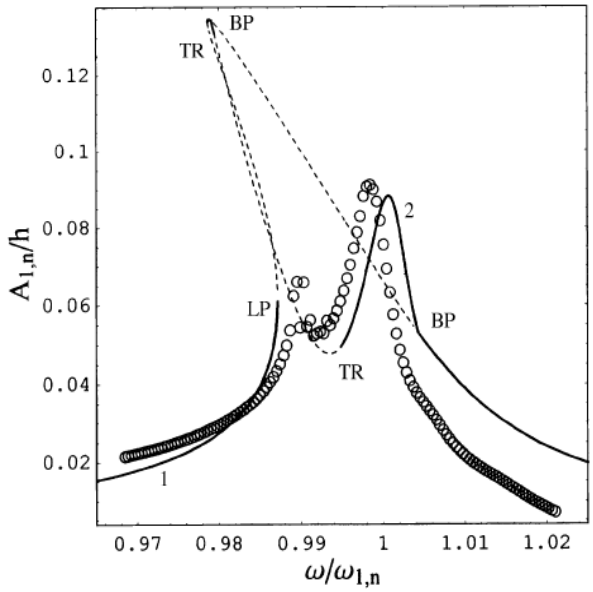


(a)

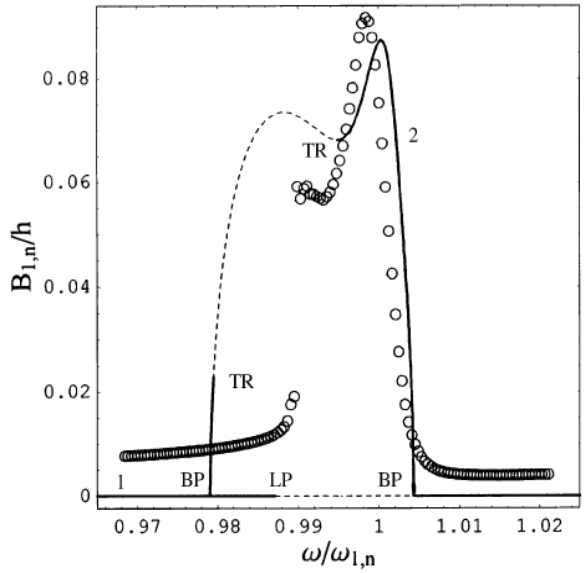


(b)

Figure 2: Experimentally measured acceleration versus the excitation frequency for the fundamental mode of the water-filled shell.—◇—,force 0.1N, —△—, force 1.5N up, --∗-- force 1.5N down, —●— force 3N up, --○-- force 3N down, —■— force 4.5N up, --+-- force 4.5N down, --▪-- force 6N up, --×-- force 6N down.



(a)



(b)

Figure 3: Amplitude-Frequency response for mode (n = 10, m = 1) of water-filled shell; force 3N.○ experimental data; ——— stable theoretical solutions; – – – unstable theoretical solutions. (a) Displacement/h from the first accelerometer; (b) displacement/h from the second accelerometer. BP, TR and LP denote, pitchfork bifurcation, Neimarck-Sacker bifurcation and limit point, respectively.

Figure 3(a,b) shows the comparison between numerical and experimental nonlinear frequency response curves for the same water-filled shell and for mode (n = 10, m = 1) when the force level is 3 N. The numerical reduced-order model is obtained by using Donnell's shallow shell theory and considering 16 degrees of freedom which has shown good

convergent solutions. It is particularly interesting to note that amplitudes of the driven ($A_{1,n}$(t)) and companion ($B_{1,n}$(t)) modes are coincident in this case in a frequency range of almost 2 Hz around the resonance. The companion mode participation grows with the vibration amplitude (and the magnitude of the excitation) from zero to the driven mode curve; after this point, driven and companion mode amplitudes increase simultaneously, giving rise to a pure travelling wave response. The modal damping used to obtain the theoretical curves is 0.003 for both driven and companion modes. The agreement between the theoretical and experimental results is good. In the figure, the main branch "1" corresponds to zero vibration amplitude of the companion mode; this branch has pitchfork bifurcations at $\omega/\omega_{1,n}$ = 0.9791 and 1.0043 where branch "2" appears. Branch "2" loses stability through two Neimark-Sacker (torus) bifurcations at $\omega/\omega_{1,n}$ = 0.9795 and 0.9944. No stable response is found in Figure 3(a,b) for 0.9871 < $\omega/\omega_{1,n}$ < 0.9944. Moreover, the calculated response of the shell for that region is modulated in amplitude as it is also evident from the experimental results. In fact, the modulated response becomes apparent with increasing the force excitation resulting in quasi-periodic oscillation.

3.2 Imperfect circular cylindrical panels

Tests have been conducted on a stainless steel panel, with the following dimensions; L = 0.199 m, R = 2 m, α = 0.066 rad, h = 0.0003 m, where L is the length of the panel in x direction, R is the radius, h is the thickness and α is the opening angle. The panel was inserted into a heavy rectangular steel frame made of several thick parts, see Figure 4, having V-grooves designed to hold the panel and to avoid transverse (radial) displacements at the edges; silicon was placed into the grooves to fill any gap between the panel and the grooves. Practically all the in-plane displacements normal to the edges were allowed because the constraint given by silicon on these displacements was very small; in-plane displacements parallel to the edges were elastically constrained by the silicon. The excitation has been provided by an electrodynamical exciter, model B&K 4810.

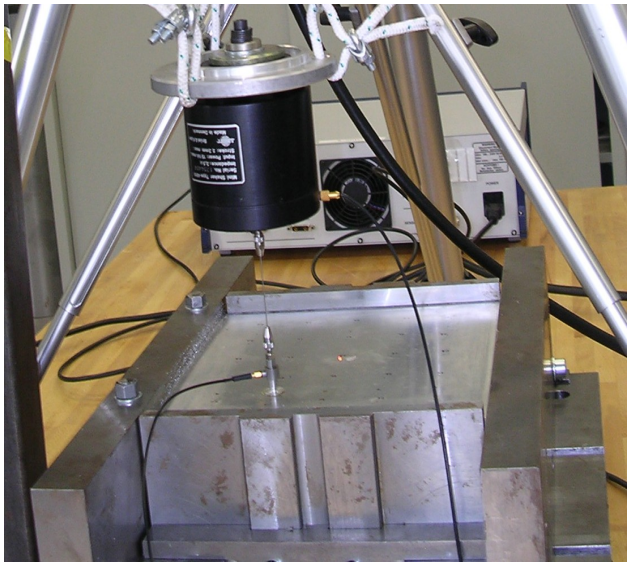


Figure 4: Experimental set-up for the imperfect circular cylindrical panel.

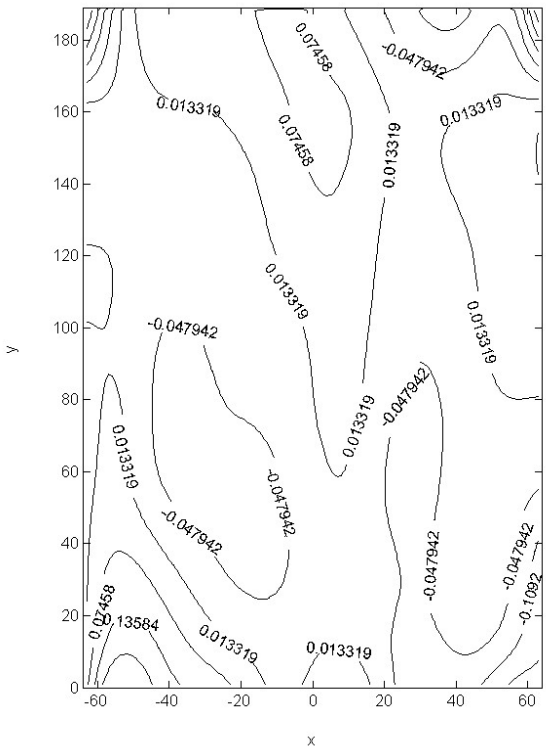


Figure 5: Experimental set-up for the imperfect circular cylindrical panel.

A piezoelectric miniature force transducer B&K 8203 of the weight of 3.2 grams, has been glued to the panel and connected to the shaker with a stinger, to measure the force transmitted. Unlike previous section, in this case the panel response has been measured by using a very accurate laser Doppler vibrometer Polytec (sensor head OFV-505 and controller OFV-5000) in order to have non-contact measurement without introduction of inertia. The time responses have been measured by using the Difa Scadas II front-end, connected to a workstation, and the software CADA-X 3.5b of LMS for signal processing, data analysis, experimental modal analysis and excitation control. The same front-end has been used to generate the excitation signal. The CADA-X closed-loop control has been used to keep constant the value of the excitation force for any excitation frequency, during the measurement of the nonlinear response.

Geometric imperfections of the panel have been detected by using a 3D laser scanning system VI-910 Minolta to measure the actual panel surface. The contour plot indicating the deviation from the ideal panel surface is reported in Figure 5. Geometric imperfections are always present in actual panels. Actually in the tested panel these imperfections are associated to initial stresses, which have been minimized with accurate positioning in the frame.

Figure 6 shows the measured oscillation (displacement, directly measured by using the Polytec laser Doppler vibrometer with displacement decoder DD-200 in the OFV-5000 controller; measurement position at the centre of the panel) around the fundamental frequency, i.e. mode (1, 1), versus the excitation frequency for different force levels: 0.01, 0.05, 0.1, 0.15 and 0.2 N. The excitation point was at $L/4$ and $R\alpha/3$. The level of 0.01 N gives a good evaluation of the natural (linear) frequency, identified at 96.2 Hz. The measured oscillation reported in Figure 5 has been filtered in order to eliminate any frequency except

the excitation frequency (1st harmonic of the response). Experiments have been performed increasing and decreasing the excitation frequency (up and down); the frequency step used in this case is 0.025 Hz, 16 periods have been measured with 128 points per period and 200 periods have been waited before data acquisition every time that the frequency is changed. The hysteresis between the two curves (up = increasing frequency; down = decreasing frequency) is clearly visible for the three larger excitation levels (0.1, 0.15 and 0.2 N). Sudden increments (jumps) of the vibration amplitude are observed when increasing and decreasing the excitation frequency and indicate softening-type nonlinearity.

It must be observed that the force input around resonance was distorted with respect to the imposed pure sinusoidal excitation; this is probably the reason for not perfect superposition of part of "up" and "down" responses. Figure 7(a) shows the harmonic components in the excitation signal for 0.15 N in the frequency range investigated. In particular, the 2nd harmonic of the excitation signal reaches amplitudes much larger than the 1st harmonic itself, which is the only one controlled. Fortunately in this case higher order harmonics do not have a significant effect on the panel dynamics, as shown by the harmonic components in the response signal shown in Figure 7(b) for excitation of 0.15 N "up" and "down".

A comparison of theoretical model with 36 degrees of freedom (dof) and experimental results for excitation 0.15 N at (L/4, $R\alpha$/3) is shown in Figure 8 (modal damping of 0.012 is assumed for all the generalized coordinates). The numerical model has been obtained by using Donnell's nonlinear shallow shell theory including in-lane inertia and consists of 30 in-plane modes and 6 transverse modes. Comparison of numerical and experimental results is excellent for the 1st harmonic value. Calculations have been obtained after introducing the geometric imperfection having the form of mode (1, 1) and magnitude of 0.35 h in the numerical model as reported in Figure 5. Moreover, the value of damping has been identified by using the nonlinear experimental response. The 1st harmonic of the response is the most significant one because it is directly excited and additional harmonics of the response are much smaller, as shown in Figure 7.

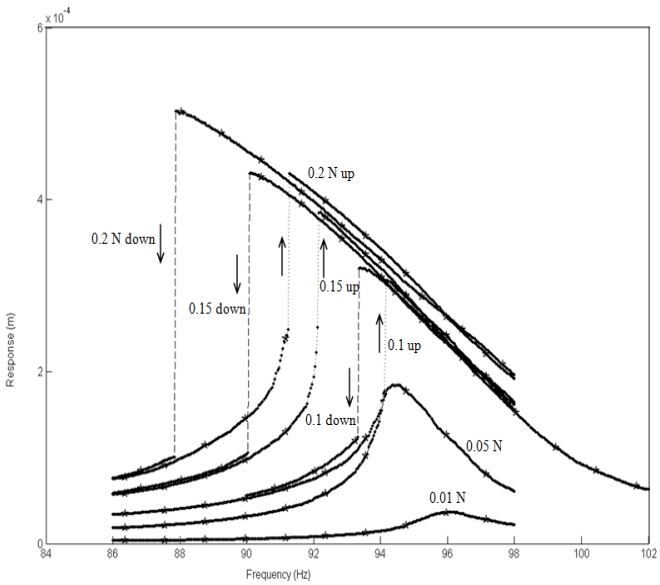
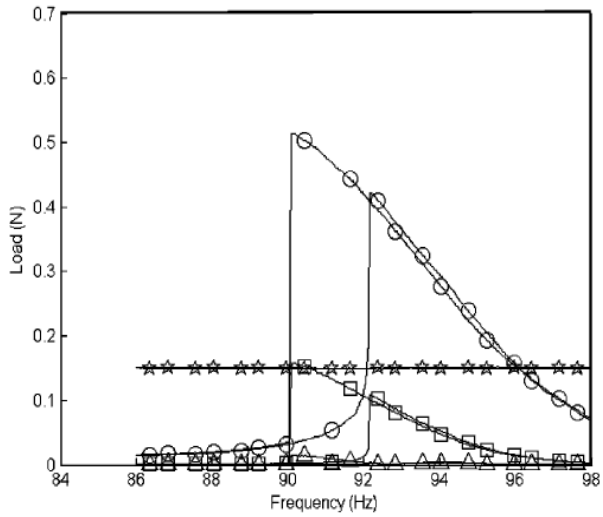
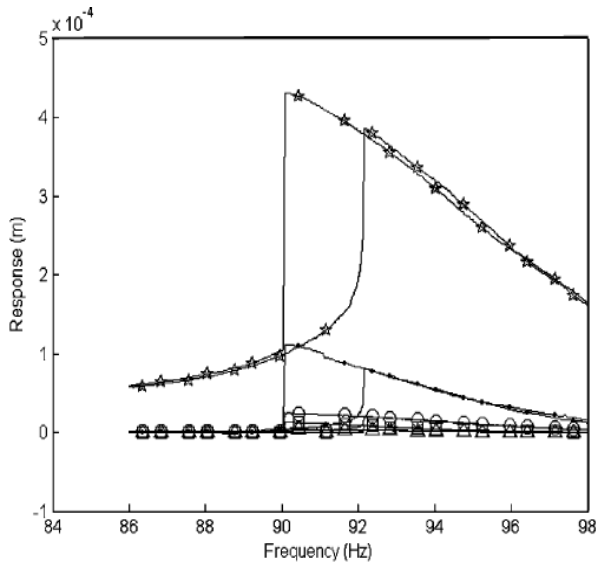


Figure 6: Experimental frequency-response curves of the imperfect circular cylindrical panel at different force levels.



(a)



(b)

Figure 7: Harmonic components of the excitation force and of the experimentally measured response of the panel. ● = mean value, ☆ = first harmonic, ○ = second harmonic, □ = third harmonic, △ = fourth harmonic. (a) Excitation force 0.15N; (b) measured response for excitation 0.15N.

3.3 Free-edge sandwich panel

Experimental tests have been performed on an all edge free sandwich panel with in-plane dimensions a = 460 mm (along x direction), b = 900 mm (along y direction). The skin of the panel is fabricated from Carbon/Epoxy composite with (0/90°) lay-up (where the 0 angle is the exterior layer, with fibers parallel to y direction). The thickness of each Carbon/Epoxy layer is 0.17 mm. Moreover, the panel has a DIAB® Divinycell foam core with and an averaged core thickness of 2.62 mm. In order to simulate completely free boundary conditions, the sandwich panel was suspended by using very soft elastic cords, to let the edges move freely in both in-plane and transverse directions.

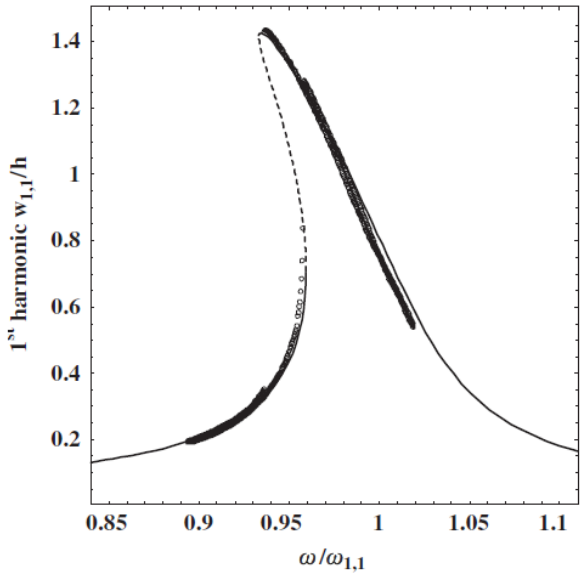


Figure 8: Comparison of the first harmonic of numerical and experimental frequency-response curves; force 0.15N. ○ experimental data; —— stable theoretical solutions; - - - unstable theoretical solutions.

In this case, the excitation has been provided by an electro-dynamic shaker type B&K 4824 driven by B&K power amplifier type 2732 via a stinger located between the shaker and the load cell (B&K type 8203) attached to the sandwich panel at $x = 0.4a$ and $y = 0.56b$. A very accurate Polytec single point laser Doppler vibrometer (sensor head OFV-505 and controller OFV-5000) has been used for measuring the velocity at several different excitation levels. In this case, the time responses have been measured by using a LMS SCADAS III front-end connected to a work station and the MIMO Sweep & Stepped Sine Testing application of the software LMS Test.Lab 11B has been used for signal processing and excitation control. The same front-end has been used to generate the excitation signal, and its closed loop control system has been used to keep the value of the excitation force constant once the excitation frequency is spanned.

Figure 9 compares the experimental results and numerical nonlinear frequency-response curve of the panel when the amplitude of excitation is 4.5N. The numerical simulations were carried out by using 67 and 70 dof models. Both models include 66 in-plane modes. In addition to that, the 70 dof model has the first 4 transverse modes while the 67 dof model has only the fundamental mode. The reduced-order models were built by using von Kármán nonlinear theory (since the panel is very thin, there is no advantage in accuracy in using shear deformation theories). It can be seen that, although for both numerical models the trend of nonlinearity is very well simulated and indicate a hardening nonlinearity, the 67 dof model gives inaccurate results after the limit point at $\Omega = 1.05\omega_{A,A\text{-}1}$ (A,A-1 stands for the first anti-symmetric/anti-symmetric mode of the panel with respect to panel axes of symmetry). In fact, since the point of excitation is close to the node of the fundamental mode, after the resonance the second mode which does not have a node close to the excitation point, is activated and significantly contributes to the nonlinear response of the panel.

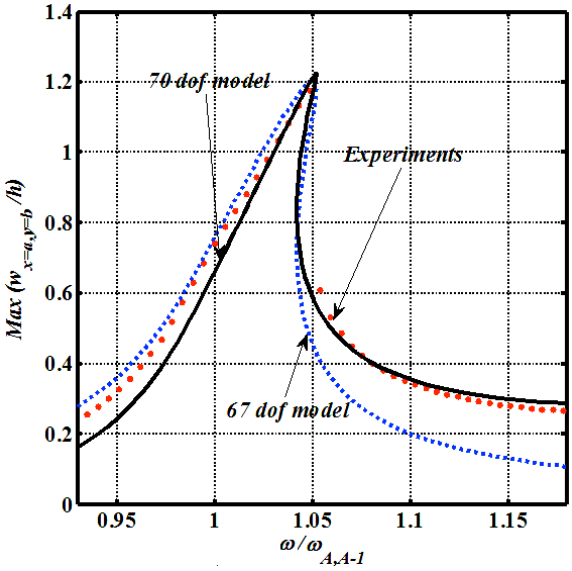


Figure 9: Comparison between experimental and numerical frequency response curves of the foam core sandwich panel; 67 and 70 dof models. Modal damping is 0.0164 for all generalized coordinates and force magnitude is 4.5N.

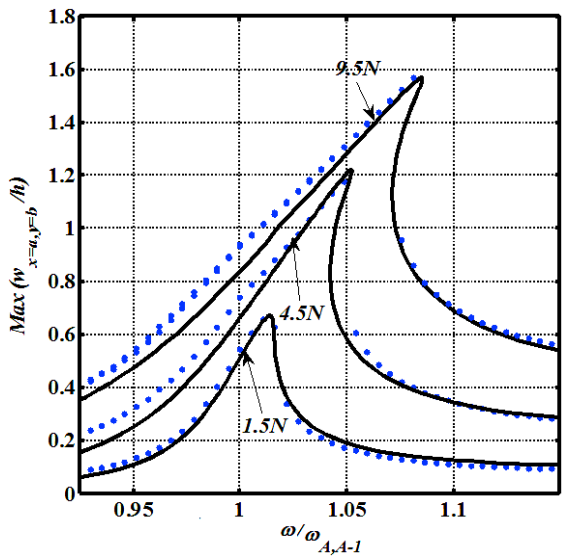


Figure 10: Comparison between experimental and numerical frequency response curves of the foam core sandwich panel for different force levels using the 70 dof model.

Further comparisons between the numerical models with 70 dof and the experimental frequency-response curves of the sandwich panel for different excitation levels are shown in Figure 10. The damping ratios used in the numerical simulations to reproduce the experimental results are given in Figure 11. It is interesting to see that for amplitudes larger than the thickness order, damping ratio varies nonlinearly and increases significantly for large amplitude vibrations. In particular, the linear damping is more than doubled for vibration amplitude around 1.5 times the panel thickness.

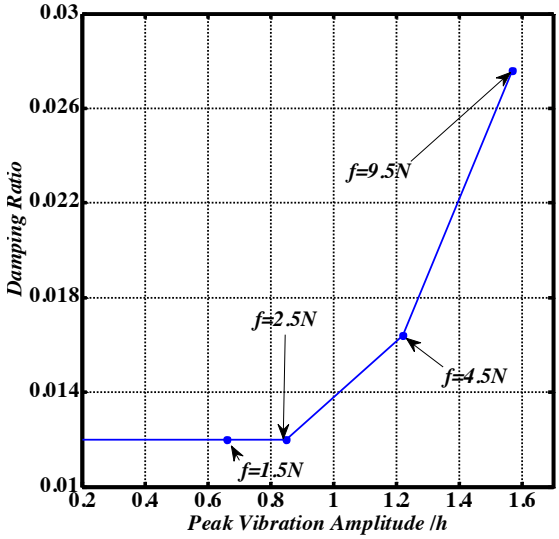


Figure 11: Damping ratio used in the numerical simulations.

4 CONCLUSIONS

The aim of this paper was to show the solutions of the open problems in the nonlinear dynamics of shell structures through novel experimental and numerical techniques, and to use a unified approach for diverse branches of the nonlinear analysis of shells and panels that have proceeded separately in the past. Several original problems including the effects of geometric imperfections, geometry, lamination, fluid-structure interaction and boundary conditions have been addressed and problems were tackled both numerically and experimentally. The numerical models were built via a convenient Lagrangian approach and the experimental tests were carried out by using a stepped sine testing approach to characterize the nonlinear behaviour of the structure.

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