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CHAOS FOR EXAMINING THE FUNDAMENTAL PERIOD OF SOILS

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Abstract. In this paper an alternative vision, based on the Chaos Theory, for the analysis of soils systems oscillating under seismic forces is described. The topological tool, Recurrence Plots RPs, enables recognition and treatment of accelerations recorded in seismological stations located on soil deposits. The chaotic attributes obtained from RPs are interpreted for determining the natural period of vibration. This dynamic non-linear characterization could help in the evolution or reinterpretation of some partially understood aspects of seismic phenomena.

1 INTRODUCTION

The most astounding damages during earthquakes are caused by amplification due to site conditions. Present seismic design practices, which incorporate information from strong motion accelerograms, very seldom reconcile the differences between accelerographic measurements and theoretical predictions. One factor involved, which is recognized as being very influential, is the effect of local conditions.

Scholars studying earthquake damage have observed the modification of earthquake motion by local soil for a long time [1]. The earliest researchers to quantify the problem were the Japanese, the most prominent ones being Sezawa ([2],[3]) and Kanai ([4], [5]). These researchers obtained algebraic expressions in the frequency domain for the surface motion to incident wave ratio from the assumption of stationary, vertically propagating, plane SH waves. Their work is limited to one and two horizontal layers of constant velocity for which they included the visco-elastic behavior and predict important amplification at the natural mode periods of the soil given by

$$T_n = \frac{4H}{V_S(2n-1)} \qquad (1)$$

where n is the mode number, H the soil depth and V_S is the soil shear wave velocity. Therefore, when Fourier spectra from earthquake accelerograms show important peaks at the natural frequencies of the soil, they are normally considered to be a consequence of the soil amplification of stationary, incoming shear waves.

However, the unexpected collapse of structures due to soil amplification effects, designed according to modern seismic codes during the Mexico 1985, Chile 2010 or Tohoku 2011 earthquakes, for mentioning a few, has taken to review soil amplification theories by deploying high-density accelerograph arrays to have a better understanding of the phenomenon. A key aspect to be elucidated in this topic is the natural periods of soils deposits. In this investigation, an alternative process to estimate the natural period of the vibration from accelerograms is presented: Recurrence Plots RPs [6].

The RPs, a topological technique from the Chaos Theory, enable the treatment of the measured accelerations for efficient interpretation of the movements. In an RPs the dynamics of soils systems vibrating under seismic forces, is reconstructed. Through the projection of the one-dimensional time series to the topological space of much higher dimensions, behaviors and trends, discovered in the reconstructed dimension, are studied. This theoretical approach captures the *true* oscillations (deterministic, chaotic or both) and permits to categorize vibrations by the fundamental period, directly from the accelerations registered. An important contribution of this study is the possibility, by means of RPs, of identifying i) materials highly sensitive to initial conditions, e.g. small differences in directivity, fault mechanism or distance, yield widely diverging outcomes (accelerations) and ii) deterministic or “predictable” stratigraphies, it means, soils systems whose behaviors can be acceptably predicted. Even more interesting is the conclusion about the changes in the conditions of soils masses that could drive the system from determinism to chaos.

2 ANALYZING COMPLEX SYSTEMS

"*The real voyage of discovery consists not in seeking new lands, but in seeing with new eyes*", M Proust (1871-1922). The strategy in any earthquake&geotechnical conception is designing based on solid criteria, intuitive imagination and emergent learning. Transformation and perpetuation, interaction and coexistence, order and conflict are words that must be integrated in the engineering knowledge because the specialist in this area have to find solutions to the problems faced by the most challenging environment: the nature.

What is needed is an alternative and reliable recording-based approach to explore, to characterize and to quantify earthquake-induced effects. Analysis from a nonlinear dynamics perspective may yield more fruitful results; still, this is not a straightforward task. Calculation of empirical global nonlinear quantities, such as Lyapunov exponents and fractal dimension, from time series data is known to often yield erroneous results ([7],[8]). The literature is replete with examples of poor or erroneous calculations and it has been shown that some popular methods may produce circumspect results ([9],[10]). Limited data set size, noise, nonstationarity and intricate dynamics are presented as additional complications. The concerns about the data are compounded by concerns about analysis. It is expected that the following consistent synthesis of the RP-analysis will enable researchers to perform studies more efficiently and more confidently. Thus it is encouraged that first at all, the user familiarizes him with the background and the methodology before attempting any analysis. To make the paper self-contained, some key concepts of Chaos Theory and Recurrence Plots are presented in the following paragraphs.

2.1 Recurrence Plots

Having established that a system contains a chaotic attractor, the process can be modeled by reconstructing the state space. Two methods are available: the method of delays and principal component analysis. We will not give a detailed description of principal component analysis because in this investigation the method of delays is used. Thus, we refer the reader interested on the former method to Broomhead and King [11].

Mutual Information. Frasier and Swinney [12] proposed *mutual information* method to obtain an estimate for delay time, τ [13]. Mutual information provides a general measure for the dependence of two variables, thus, the value of τ for which the mutual information goes to zero is preferred. Additional arguments for choosing the first zero can be found in [14].
Mutual information is a measure found in the field of Information Theory. Let S be a communication system with $s_1, s_2, \dots, s_n$ a set of possible messages with associated probabilities $P_s(s_1), P_s(s_2), \dots, P_s(s_n)$.

The entropy H of the system is the average amount of information gained from measuring s and it is defined as

$$H(S) = -\sum_i P_s(s_i) \log P_s(s_i) \tag{2}$$

For a logarithmic base of two, H is measured in bits. Mutual information measures the dependency of $x(t+T)$. Let $[s,q] = [x(t), x(t+T)]$, and consider a coupled system (S,Q). Then, for sent message s and corresponding measurement s_i,

$$\begin{aligned} H(Q|s_i) &= -\sum_i P_{q|s}(q_j|s_j) \log [P_{q|s}(q_j|s_j)] \\ &= \sum_j \frac{P_{sq}(s_i, q_j)}{P_s(s_i)} \log \frac{P_{sq}(s_i, q_j)}{P_s(s_i)} \end{aligned} \tag{3}$$

where $P_{q|s}(q_j|s_i)$ is the probability that a measurement of q will result in q_j, subject to the condition that the measured value of s is s_i. Next we take the average uncertainty of $H(Q|s_i)$ over s_i,

$$\begin{aligned} H(Q|S) &= \sum_i P_s(s_i) H(Q|s_i) \\ &= -\sum_{ij} P_{sq}(s_i, q_i) \log \frac{P_{sq}(s_i, q_j)}{P_s(s_i)} \\ &= H(S,Q) - H(s) \end{aligned} \tag{4}$$

with

$$H(S,Q) = -\sum_{i,j} P_{sq}(s_i, q_j) \log P_{sq}(s_i, q_j) \tag{5}$$

The reduction of the uncertainty of q by measuring s is called the mutual information $I(S,Q)$ which can be expressed as

$$\mathrm{I(Q,S) = H(Q) - H(Q|S)} \quad (6)$$

$$\mathrm{= H(Q) + H(S) - H(S,Q) = I(S,Q)} \quad (7)$$

where $H(Q)$ is the uncertainty of q in isolation. If both S and Q are continuous, then

$$\mathrm{I(S,Q)} = \int \mathrm{P_{sq}(s,q)\log\frac{P_{sq}(s,q)}{P_s(s)P_q(q)}\,ds\,dq} \quad (8)$$

If s and q are different only as a result of noise, then $I(S,Q)$ gives the relative accuracy of the measurements. Thus, it specifies how much information the measurement of x_i provides about x_{i+1} . The mean and variance of the mutual information estimation can be calculated [15]. Although mutual information guarantees decorrelation between x_k and x_{k+t}, and between x_{k+t} and x_{k+2t}, it does not necessary follow that x_k and x_{k+2t} are also uncorrelated [16].

False Nearest Neighborhoods. Mutual information gives an estimate for τ_s, but does not determine the embedding dimension d. The Takens' Theorem [15] states that an m-dimensional attractor will be completely unfolded with no self-crossings if the embedding dimension is chosen larger than $2m$. In this work, the method of false nearest neighbors is used for finding a good value for d [17].

The method is based on the idea that two points close to each other (called neighbors) in dimension d, may in fact not be close at all in dimension $d+1$. This can happen when the lower dimensional system is simply a projection of a higher dimensional system, and it is unable to completely describe the system. Thus, the algorithm searches for "false nearest neighbors" by identifying candidate neighbors, increasing the dimension, and then inspecting the candidate neighbors for false ones. When no false neighbors can be identified, it is assumed that the attractor is completely unfolded and d, at this point, taken as the embedding dimension.

2.2 Recurrence Analysis

The set of nonlinear dynamic techniques, called Nonlinear Time Series Analysis [18], can be classified into metric, dynamical, and topological tools. The metric approach depends on the computation of distances on the system's attractor. The dynamical approach deals with computing the way nearby orbits diverge by means of estimating Lyapunov exponents. Topological methods are characterized by the study of the organization of the strange attractor, and they include close returns plots and Recurrence Plots RPs [6].

RPs are intricate and visually appealing. They are also useful for finding hidden correlations in highly complicated data. In this work the RP-analysis is extended, formalized, and systematized in a meaningful way that is based both in theory and experiments and that targets both quantitative and qualitative properties for its geotechnical and seismological application.

In this section, we briefly outline some of the basic features of RPs and describe how an RP of an experimental data set can be generated. The standard first step in this procedure is to reconstruct the dynamics by embedding the one-dimensional time series in a d_E −dimensional reconstruction space using the method of delay coordinates. Given a system whose topological dimension is d, the sampling of a single state variable is equivalent to projecting the d-dimensional phase-space dynamics down onto one axis. Loosely speaking, embedding is akin to "unfolding" those dynamics, albeit on different axes (Packard et al, 1980; [15]). Given a trajectory in the embedded space, finally, one constructs an RP by computing the distance between every pair of points (y_i, y_j) using an appropriate norm and then shading each pixel (i, j) according to that distance. The process of constructing a correct embedding is the subject of a large body of literature and numerous heuristic algorithms and arguments. Abarbanel [20] gives a good summary of this extremely active field.

Delay Coordinate Embedding. To reconstruct the dynamics, we begin with experimental data consisting of a time series:

$$\{x_1, x_2, \dots, x_N\} \tag{9}$$

Delay-coordinate reconstruction of the unobserved and possibly multi-dimensional phase space dynamics from this single observable x is governed by two parameters, embedding dimension d_E and time delay τ. The resultant trajectory in R^{d_E} is:

$$\{y_1, y_2, \dots, y_m\} \tag{10}$$

where $m = N - (d_E - 1)\tau$ and

$$y_k = (x_k, x_{k+\tau}, x_{k+2\tau}, \dots, x_{k+(d_E-1)\tau}) \tag{11}$$

for $k = 1,2, \dots, m$. Note that using d_E=1 merely returns the original time series; one dimensional embedding is equivalent to not embedding at all. Proper choice of d_E and τ is critical to this type of phase-space reconstruction and must therefore be done wisely; only "correct" values of these two parameters yield embeddings that are guaranteed by the Takens Theorem [15] to be topologically equivalent to the original (unobserved) phase-space dynamics.

Assuming that the delay-coordinate embedding has been correctly carried out, it is natural to assume that the RP of a reconstructed trajectory bears great similarity to an RP of the true dynamics. Furthermore, we expect any properties of the reconstructed trajectory inferred from this RP to be true of the underlying system as well. This is, in fact, the rationale behind the standard procedure of embedding the data before constructing a RP.

Constructing the Recurrence Plot. RPs are based upon the mutual distances between points on a trajectory, so the first step in their construction is to choose a norm D. In this work the maximum norm is used, although in one dimension the maximum norm is, of course, equivalent to the Euclidean p-norm. We chose the maximum norm for two reasons: for ease of implementation and because the maximum distance arising in the recurrence calculations (the difference between the largest and smallest measurements in the time series) is independent of embedding dimension d_E for this particular norm. This means that we can make direct comparisons between RPs generated using different values of d_E without first having to re-scale the plots. Next, we define the recurrence matrix A as follows:

$$A(i,j) = D(y_i, y_j), 1 \leq i,j \leq m \tag{12}$$

$$D(y_i, y_j) = \max_{1 \leq k \leq d_E} \left| x_{i+(k-1)_\tau} - x_{j+(k-1)_\tau} \right| \tag{13}$$

The time series spans both ordinate and abscissa and each point (i, j) on the plane is shaded according to the distance between the two corresponding trajectory points y_i and y_j (Figure 1). The pixel lying at (i, j) is color-coded according to the distance. For instance, if the 117th point on the trajectory is 14 distance units away from the 9435th point, the pixel lying at (117, 9435) on the RP will be shaded with the color that corresponds to a spacing of 14.

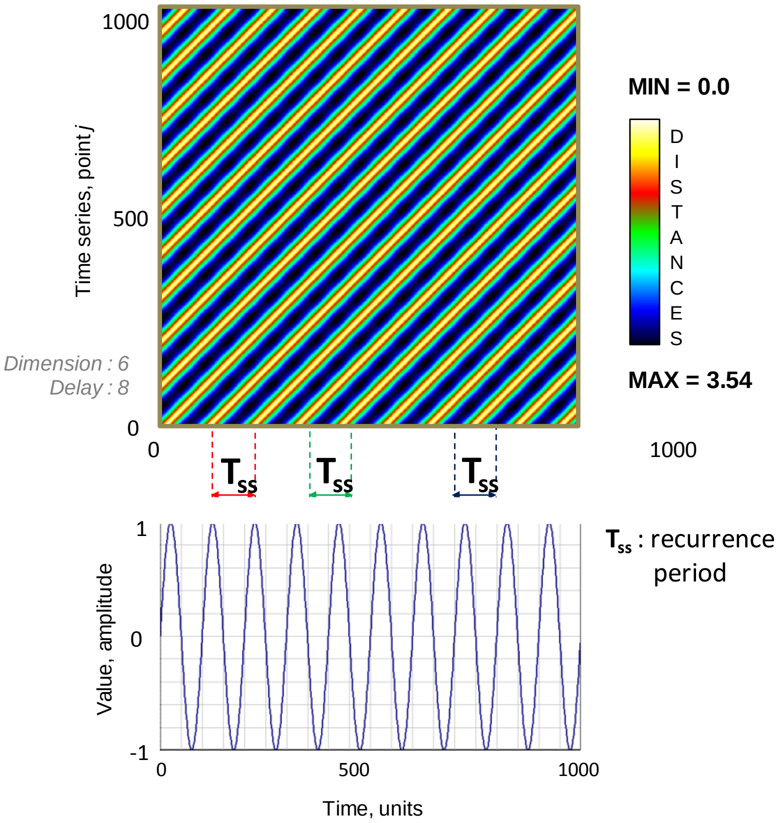

Figure 1: RP graphique description

Figure 2 shows RPs generated from very different data sets: from a time series derived by sampling the function sine t until a noise series. The colors on these plots range from white-yellow for very small spacing to dark blue for large inter-point distances (see calibration bar in Figure 1). With this in mind, the sine-wave RP is relatively easy to understand; each of the “blocks” of color simply represents half a period of the signal. The lower RPs in the Figure 2, generated from chaotic data sets, are far more complicated, although they too have block-like structures resembling what might be expected from a periodic signal. These signals, though, are not periodic, so the repeated structural elements in the plot request an explanation. Alternatively, recurrence points for the white noise (at the bottom of Figure 2) are simply distributed in a homogeneous random pattern, signifying that the variable lacks of deterministic structures.

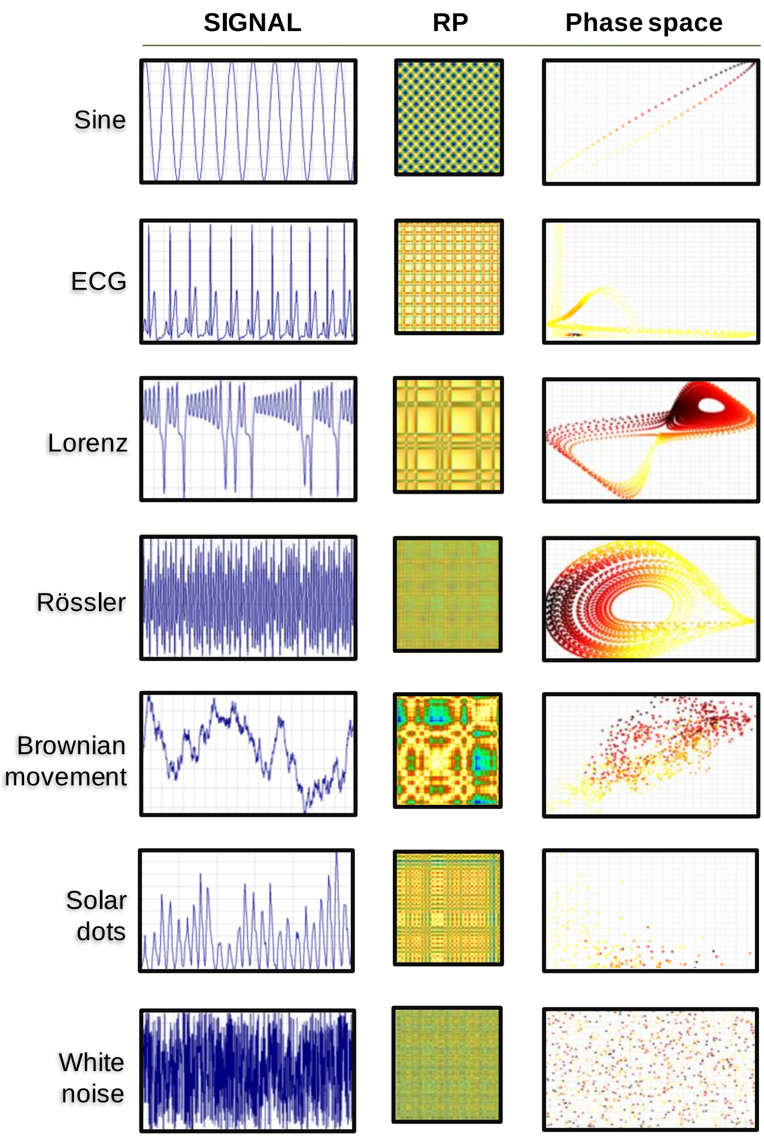

Figure 2: Examples of Recurrence Plots and Phase Space Plots of different data sets.

Structures in RPs. As already mentioned, the initial purpose of RPs was to visualize trajectories in phase space, which is especially advantageous in the case of high dimensional systems. RPs yield important insights into the time evolution of these trajectories, because typical patterns in RPs are linked to a specific behavior of the system. Following the phase space characteristics, the path in the correlation dimension curve and the large scale patterns in RPs, designated as *typology*, the RPs structures can be classified as *homogeneous*, *periodic*, *drift* and *disrupted* ones [21]:

- Homogeneous RPs are typical of systems in which the relaxation times are short in comparison with the time spanned by the RP. An example of such an RP is that of a stationary random time series. See the uniformly distributed white noise example shown in Figure 2.
- Periodic and quasi-periodic systems have RPs with diagonal oriented, periodic or quasi-periodic recurrent structures (diagonal lines, checkerboard structures). Figure 2 shows the RP of the sine signal, example of a periodic system. Irrational frequency ratios cause more complex quasi-periodic recurrent structures (see the ECG, Lorenz, Rössler and sun spots examples); however, even for oscillating systems whose oscillations are not easily recognizable, RPs can be very useful.
- A drift is caused by systems with slowly varying parameters, i.e. non-stationary systems. The RP pales away from the line of identity, LOI (the main diagonal line in a RP, $R_{i,j} = 1$).
- Abrupt changes in the dynamics as well as extreme events cause white areas or bands in the RP, for example the Brownian motion. RPs allow finding and assessing extreme and rare events easily by using the frequency of their recurrences.

A closer inspection of the RPs reveals also the texture or small-scale structures [6], which can be typically classified in single dots, diagonal lines as well as vertical and horizontal lines (the combination of vertical and horizontal lines obviously forms rectangular clusters of recurrence points); in addition, even bowed lines may occur [21,6]:

- Single, isolated recurrence points can occur if states are rare, if they persist only for a very short time, or fluctuate strongly.
- A diagonal line $R_{i+k,j+k} \equiv 1|_{k=0}^{l-1}$ (where l is the length of the diagonal line) occurs when a segment of the trajectory runs almost in parallel to another segment for l time units:

$$\overrightarrow{x_i} \approx \overrightarrow{x_j}, \overrightarrow{x_{i+1}} \approx \overrightarrow{x_{j+1}}, \dots, \overrightarrow{x_{i+l-1}} \approx \overrightarrow{x_{j+l-1}}$$

- A *vertical* (*horizontal*) *line* $R_{i,j+k} \equiv 1|_{k=0}^{v-1}$ (with *v* the length of the vertical line) marks a time interval in which a state does not change or changes very slowly:

$$\overrightarrow{x_i} \approx \overrightarrow{x_j}, \overrightarrow{x_i} \approx \overrightarrow{x_{j+1}}, \dots, \overrightarrow{x_i} \approx \overrightarrow{x_{j+v-1}}$$

  The state is trapped for some time. This is a typical behavior of laminar states (intermittency) [21].

- *Bowed lines* are lines with a non-constant slope. The shape of a bowed line depends on the local time relationship between the corresponding close trajectory segments.

RPs of paradigmatic systems provide an instructive introduction into characteristic typology and texture but the visual interpretation of RPs requires some experience. A deeper explanation about typology and texture is presented in [22].

2 RECURRENCE IN ACCELEROGRAMS

A set of acceleration time series recorded in the soft soils of the Mexican metropolis are used to study its chaotic nature. The soil systems and the recorded responses studied here are on the surface of 16 soft soils (clays) and 4 stiff deposits within the urban area of Mexico City. The soft soils are located on the lacustrine basin where soils were deposited by air or water transportation (very soft clay formations with large amounts of microorganisms interbedded by thin seams of silty sand), some others are product of volcanic effusions that took place within the last one million years (fly ash and volcanic glass) and there are stations on a third type of soils that are considered firm or materials rock-like. The elastic natural periods T_n (key parameter in ground motions categorization) of the sites included in the database vary from $T_n = 2\ s$ to $T_n = 4.2\ s$. Fifteen earthquakes were selected having at least 100 s and high signal-to-noise ratios. These events are representative of the tectonic regions (different source mechanism) that affects the valley. The set is denser in events from the subduction of the Cocos Plate into the Continental Plate because they are associated to the most damaging shocks.

Information accumulated over the last four decades has firmly established that the seismic movements within the Basin of Mexico can differ considerably from one site to the other [22]. The statutory regulations have tried to take into account these facts but still there are dangerous doubts about the outcome patterns. The purpose of the following analysis is to illustrate an alternative way in which the oscillations can be described and to provide a qualitative understanding of the complex system responses.

2.1 RPs- large scale

Examples of RPs obtained from accelerograms recorded during the earthquakes are shown in Figure 3. One intriguing and puzzling characteristic of the RPs is the structural similitude that they exhibit with different seismic and site conditions. Evaluating RPs constructed from accelerograms recorded during the same event on different site conditions, one question is obligated: do soft soils and rocks, when are excited by the same seismic force, move similarly? On the other hand, keeping constant the soil properties (dividing the database in a separated sets: soft-soils/rock -stiff- masses) and varying the seismic inputs (earthquakes), the RPs structures are exceptionally comparable and the doubt is evident: do earthquake mechanism, distance (from epicenter to the site) and directivity have a tangible, real impact on materials vibrations? The answers, based on large scale, do not correspond with engineering criteria and experience: soils and rocks do move differently and seismic inputs do have a deep effect on the response recorded.

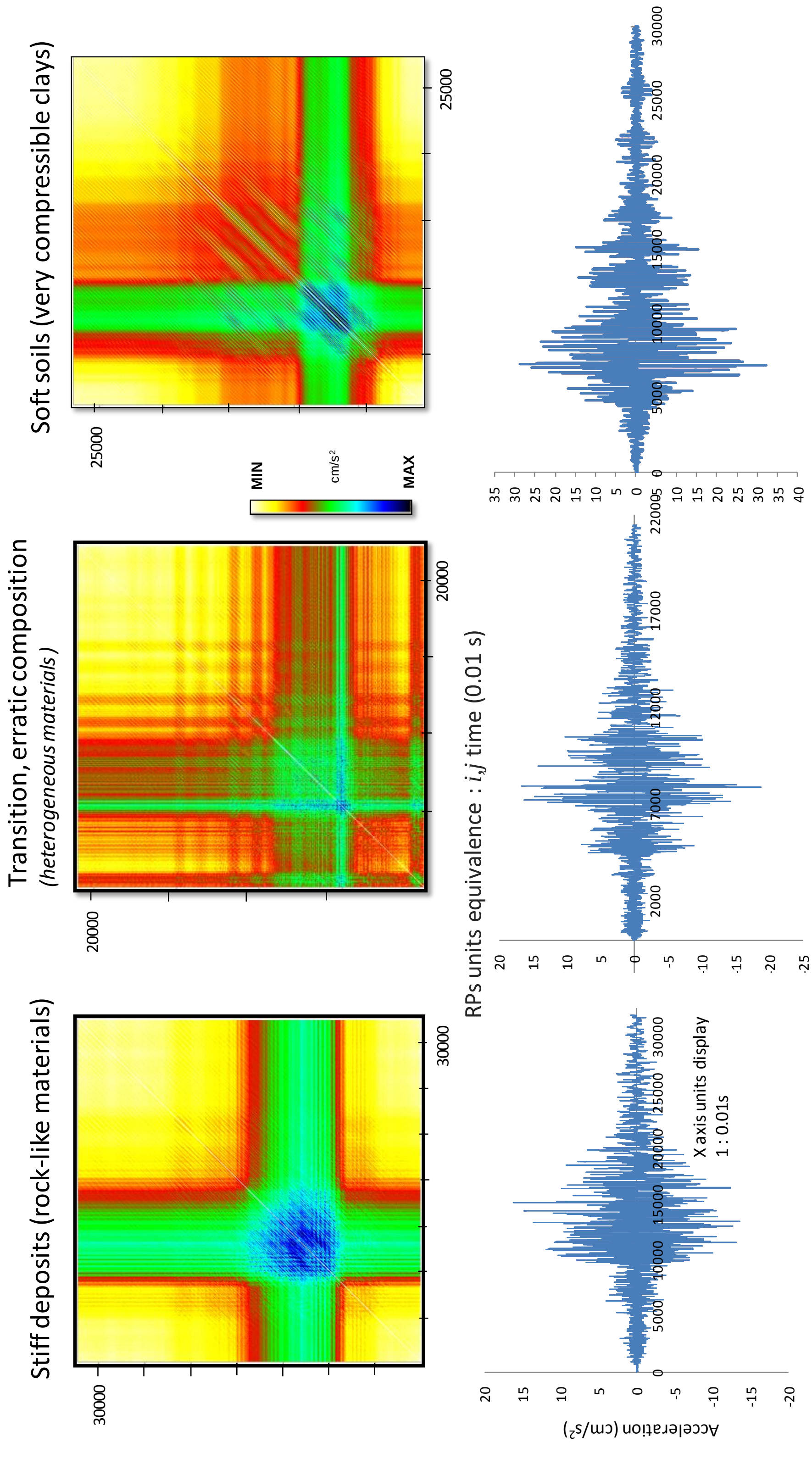

Figure 3: Recurrence Plots from soft and stiff deposits, different input seismic loads.

Through a deep inspection of the RPs and using nonlinear concepts we found that the ground structures (soft and stiff, homogeneous and heterogeneous deposits) when are affected by seismic forces, in a macro scale, evolve in a similar way. The time evolution of the ground accelerations exposes well defined white areas and cold bands (green/blue strips), hallmark in nonstationary systems. The combination of vertical and horizontal strips forms rectangular clusters where the maximum accelerations are located. Because of the abrupt changes between the beginning, the intense and the final part of the movements, a ground motion can be defined as an event that contain *extreme* sub-events (maximum accelerations) where the ground conditions are anomalous for some seconds to then vanish until the movement finishes. The number of clusters that appears in the RPs is the recurrence of the sub-events. The study of vertical RP structures makes the identification of *trapping time* (seconds that soil and rocks are being truly perturbed) possible. The ground response is very far from the deterministic behavior of the very well-known pendulum's oscillation (under damped vibration - low drive frequencies or if the drive frequency is raised and the attractor undergoes a series of bifurcations). None of the RPs of accelerograms can be related with the patterns of the pendulum's attractors. The underlying process in seismic ground motions seems much more complex, large-scale observed, and it cannot be directly labeled as "deterministic", "chaotic" or "random".

As a conclusion we can say that soils and rocks are nonlinear devices because they become activated when their reaction potential crosses a certain threshold. The activity of large formations of geological materials (soils and rocks) is macroscopically measurable in the accelerogram which results from a spatial integration of many reaction potentials (the environment interacting). The RPs-vertical structures displayed by the layered natural materials should be related with *intermittence* (alteration of phases of seemingly periodic systems). The apparent periodic phases of the ground behavior are not quite, but only nearly periodic. Thus, rather than a truly-periodic series of values, the data are apparently periodic but where the chaotic nature of the system becomes apparent after certain ground acceleration is reached. It is very important to point out that *intermittence* is more patent during large earthquakes, and a probably reason, which looks a paradox, is that the energy released from the source is not permanently continuous on time, there are relax intervals in between without important seismic wave arrivals from the source. The *intermittence* in accelerograms could be related with the intense part of the earthquake.

2.2 RPs- small scale

Delay-coordinate embedding produces clean, easily analyzable pictures of the ground dynamics and the results suggest that the dynamical behavior of the soils/rocks is very high-dimensional. This implies that the system is probably influenced by variables that can be hardly identified or that are beyond the limits of our current understanding [23]. However, a proper definition of initial conditions permits characterizing the system evolutions and extracting meaningful (for engineering purposes) conclusions about the behavior of this complex natural structure. Zooming into the RPs, above the intense part of the response (blue/green clusters) the soil response can be studied from the clear and suggestive signatures in the sub-plots. The width of the vertical band indicates the time in which the *intense* state does not change or changes very slowly. The strong ground accelerations are trapped for some seconds (the cluster base) and because this extreme situation is not an isolated point (rare) the possibility that this alteration had been produced by noise is eliminated. In this *intense* time, periodic, chaotic or random patterns can be recognized and the parameters range over which the system is stable and where the trajectories are divergent could be identified.

Harmonics in soft soils oscillations. Observe Figure 4. These clusters were obtained from accelerograms recorded in soft soils deposits. These examples show diagonal oriented recurrent structures that can be related with the vibration of one degree of freedom 1D oscillator. Due to space restrictions only some examples are showed, but they are representative of the structures displayed for the whole soft-soils set.

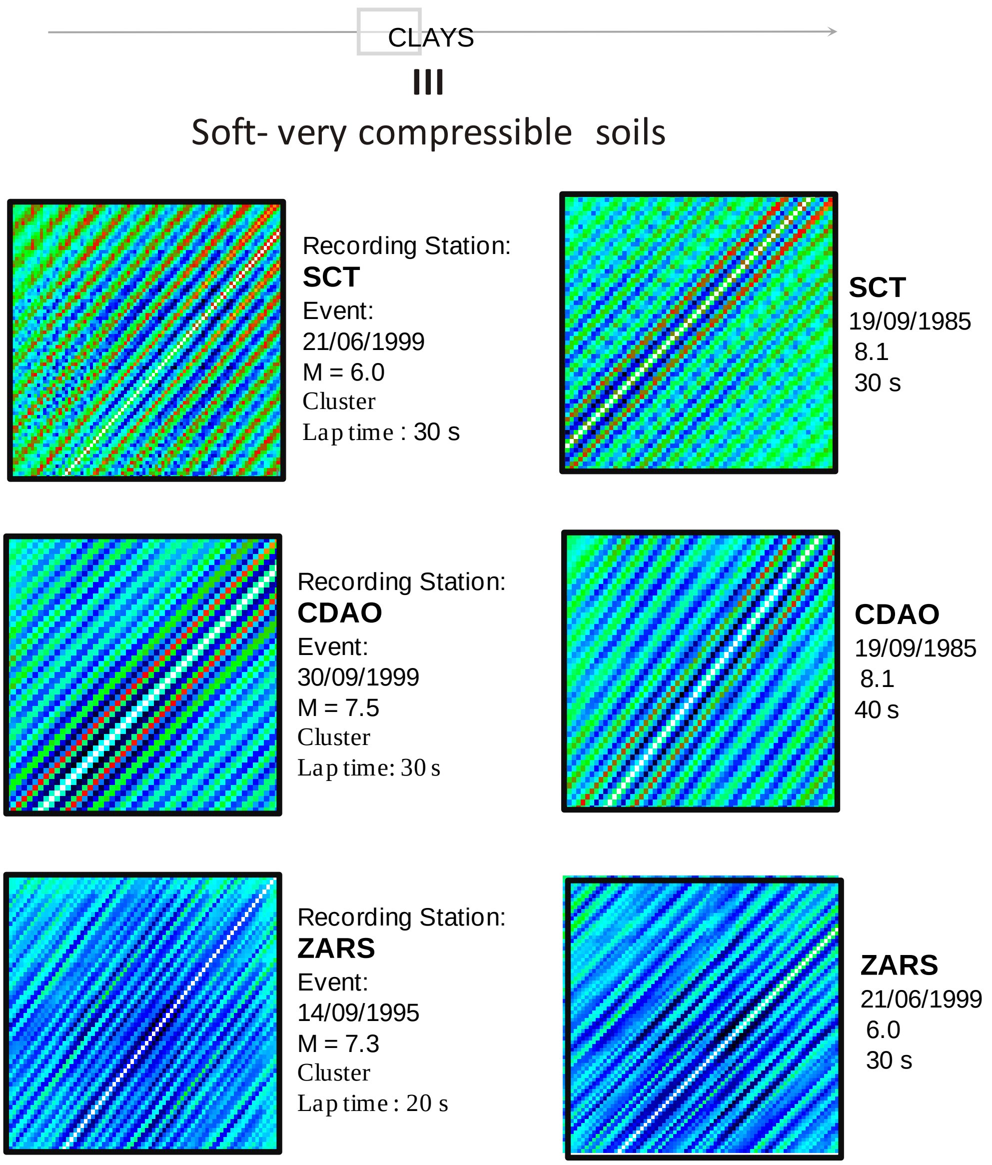

Figure 4: Diagonal structures in soft soils.

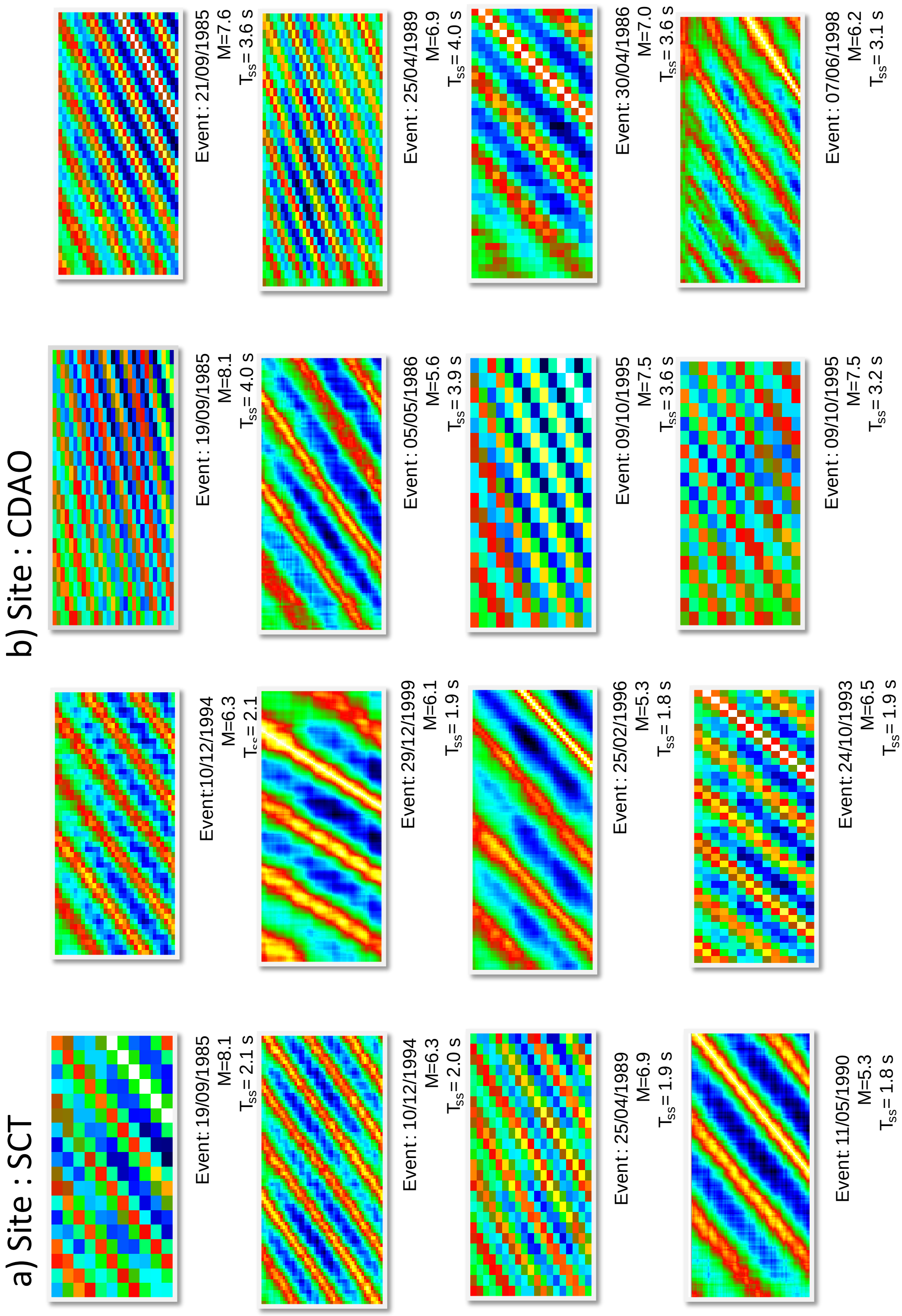

Figure 5: Recurrence period for soft soils.

Assuming that during *intermittence* soft soils behave as a 1D oscillator, the period of soil vibration during the semi-sinusoidal oscillation (in this investigation called T_{ss}) can be obtained from the distance between diagonals, as shown for sine signal in Figure 1. For a same site, no important degradation is observed in T_{ss}, even when the records came from different intensity, frequency and duration input conditions. Beyond the scope of this work is the discussion about the impact of the differences between T_{ss} and T_n in the aseismic design, but no doubt exists that the T_{ss} values are more authentic than those obtained from spectral analyses and many important conclusions about nonlinearity and site effects must be re-evaluated using these findings. In Figure 5 some sub-plots are presented to make clear this assumption. The sites are SCT and CDAO, two emblematic deposits because of their astounding amplification behavior. During the catastrophic earthquake suffered in Mexico City (Sept 19th 1985 Michoacán event) these two stations recorded more than 10 times the accelerations measured in stiff deposits (rock like materials) and resonance phenomena is added to the scientific explanations about the enormous movements in this zone. In Mexican construction codes the recommended fundamental period for SCT is $T_n = 2.1\ s$ and for CDAO $T_n = 3.5\ s$.

Chaotic vibrations in stiff soils structures. The clusters in Figure 6 are far more complicated. The checkboard structures and the upward diagonal lines result from strings of vector patterns repeating themselves multiple times down the dynamics. This type of recurrent structure indicates that the dynamics is visiting the same region of an attractor at different times; therefore, the presence of diagonal lines indicates that deterministic rules are present in the dynamics. The set of lines parallel to the main diagonal is the signature of determinism, however, it is not so clear as in soft-soils (e.g., the size of the lines being relatively short among a field of scattered recurrent points), i.e., the RPs contain subtle patterns not easily ascertained by visual inspection.

Although the blocklike structures resembling to what might be expected from a periodic signal, the rock-like materials exhibit a complex recurrent behavior with irregular cyclicities that qualifies them as dynamical systems and their behavior as typical for nonlinear or chaotic systems. This means that the deposits in Hill zone are highly sensitive to initial conditions, e.g. small differences in directivity, fault mechanism or distance, yield widely diverging outcomes.

As in many natural systems, the geological materials constitute systems that can be called deterministic, meaning that their future behavior is fully determined by their initial conditions, with no random elements involved. The deterministic nature of rock (stiff materials) systems does not make them predictable. The rock-like deposits behavior can be described as deterministic chaos, or simply chaos.

3 CONCLUSIONS

Based on the findings of this study, recorded accelerograms on soils and rocks should be considered as a sequence of episodes of seismic wave arrivals alternated with free soil vibrations episodes, behavior related with intermittence.

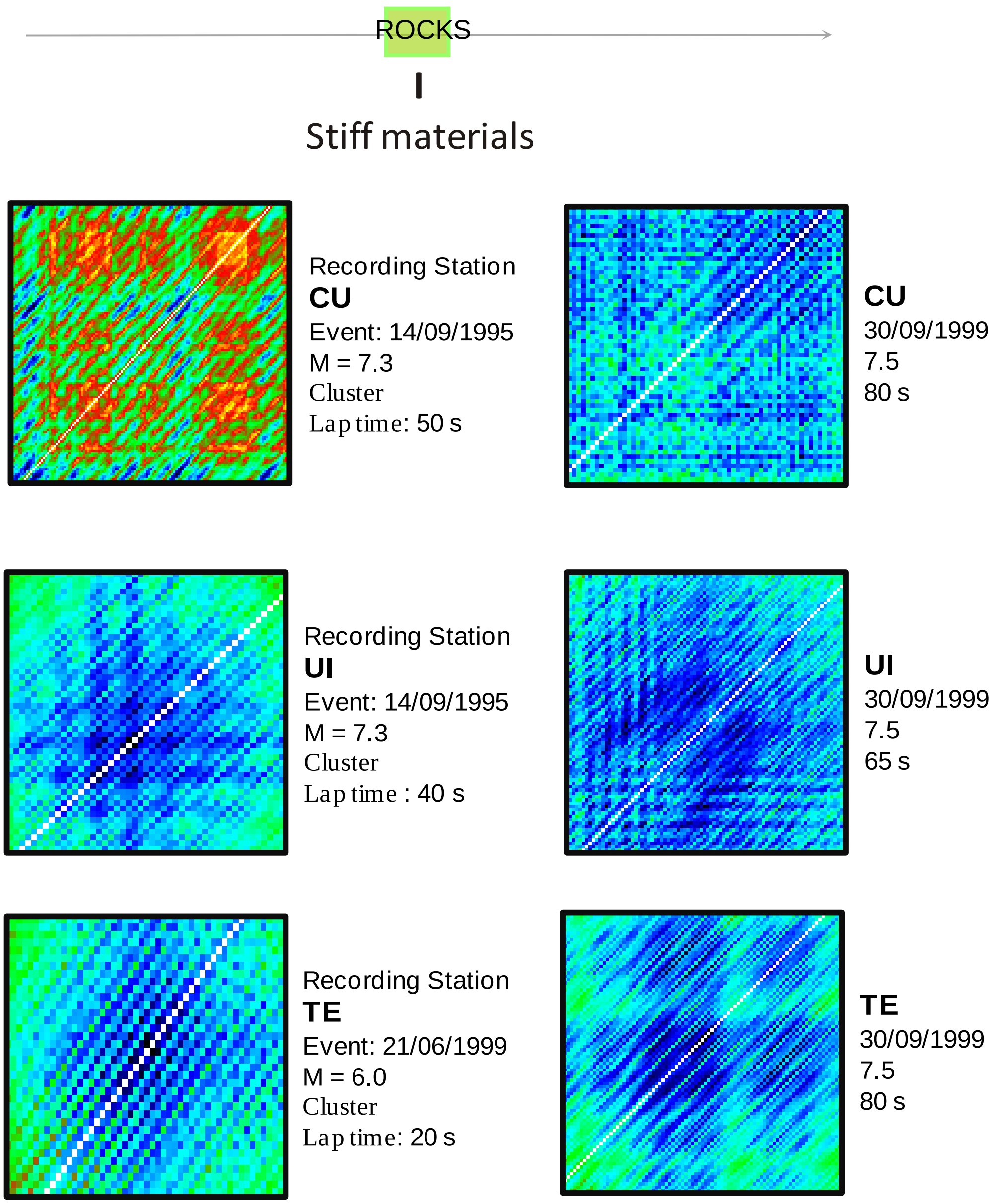

Figure 6: Quasi-periodic structures in stiff deposits.

It has been noticed, from the analyzed cases (different fault mechanisms, epicentral distances and magnitudes), that there are no significant differences between soil and rock time evolutions (macro-scale). The study of the alteration of phases in RPs drives to the conclusion that soils and rocks deposits responses can be characterized only in the intense part of the time series (clusters blue/green bands, maximum accelerations zone).

RPs of stiff materials, in general, display more complicated structures but they resemble chaotic movements for the universe of initial conditions analyzed. Despite being chaotic, the trajectories are actually quite organized, but a vibration period cannot be determined.

Soft soils deposits progress from quasi-periodic to periodic oscillations as the amplitude of the seismic responses exceeds certain acceleration thresholds. The deposits studied can be linked to certain determinism, the diagonal bars in the intense part of the response are directly related with the natural period of each stratigraphy.

The inconsistency between soil amplification theories and accelerographic measurements for large earthquakes could be re-interpreted through Chaos theory: Geological materials are systems that evolve in a similar way, in the macro-scale, but is in the micro-scale that the materials display particular trajectories.

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