11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
E. Oñate, J. Oliver and A. Huerta(Eds)

AN ENHANCED INVERSE KINEMATIC AND DYNAMIC MODEL OF A 6-SBU STEWART PLATFORM

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Key Words: *Feedforward-feedback, Inverse Modelling, Joint Friction, Coupling Problems.*

Abstract. An inverse model for the kinematics and dynamics of a 6-SBU structured Stewart platform is generally derived for prediction of leg lengths and the force actuation to meet the desired pose within prescribed kinematic and dynamic constraints. The feedback control generated from the instantaneous difference between the measured leg lengths and predicted leg lengths at each instant of time is found to be effective along with the feedforward estimation for force actuation to the legs. In this paper an attempt has been made to accommodate intricacies involved in inverse modeling that makes the feedforward estimations precise and reduce the feedback control effort.

1 INTRODUCTION

The stability and control performance analysis of certain on-board systems under intricate inertial loading can be evaluated by using parallel manupulator. A Stewart platform [8] is the most popular parallel manipulator with six degrees-of-freedom. A Stewart platform involves six linearly extensible legs with active electric or hydraulic drive for each with integrated control for imparting a range of desired motions to a large payload within a small workspace. Coupling motions due to muliplicity of the response to a definite command are one of the main issue that makes the control more critical.

Liu et al. [6] and Marlet [7] modelled both the forward and inverse kinematics of a Stewart platform and proposed simplified solution schemes for the forward kinematics. While the inverse kinematics of the platform deals with the estimation of stroke length of the legs for a desired platform pose from its neutral position, the forward or direct kinematics are ment for control analysis to predict the platform pose from the known length of the legs. A solution scheme is necessary for the forward dynamics problem to guide the mechanism through any desired instantaneous solution among the possible multiple solutions for a intermediate pose.

A simple forward dynamic model obtained by Fichter [3] neglecting the effect of leg inertia for computing actuating forcse on a Stewart platform corresponding to input of actuation forces to the legs. Dasgupta and Mrithunjaya [1, 2] derived the inverse kinematics and dynamics and shown the effectiveness of a PD control for the force inputs to the legs at different pose dynamics where the control estimated from required and predicted leg lengths at each instant of time. Both 6-SPS and 6-UPS configurations have been widely analysed in [1-3, and 6-7] where 6 stands for the number of legs with joint assembly of same type, the first and third alphabet S or U corresponds to spherical or universal joint at bottom and top end respectively of each leg and the middle alphabet P or B refers to the prismatic joint or ball screw joint as linear actuator.

For the platform to carry heavy load at very high duty cycle smoothly the use of ball-screw joints in six linearly extensible legs of Stewart platform are now getting importance. The availability of inline linear actuator cylinder with ball bearing lead screws enables the platform to be operated at low backlash and high accuracy and also with better life expectancy. A negligible coupling error of a 6-SBU Stewart platform with the use of compensation scheme in the feedforward control has been found to occur [5] where the platform designed following an algorithm proposed in [4] considering negligible leg inertia and joint friction.

The objective of this paper is to enhance the inverse kinematics and dynamics with considerable joint friction and leg inertia that will be used in the feedforward model. This model will generate current to drive the motor of the inline actuator such a way to minimize the coupling motions as well as reduce the PD feedback controller effort.

2 PLATFORM CONFIGURATION

The schematic of the platform assembly over the top joint of all the six legs in figure 1 describing point p as the center of gravity of the circular disc at left of radius r_t and thickness

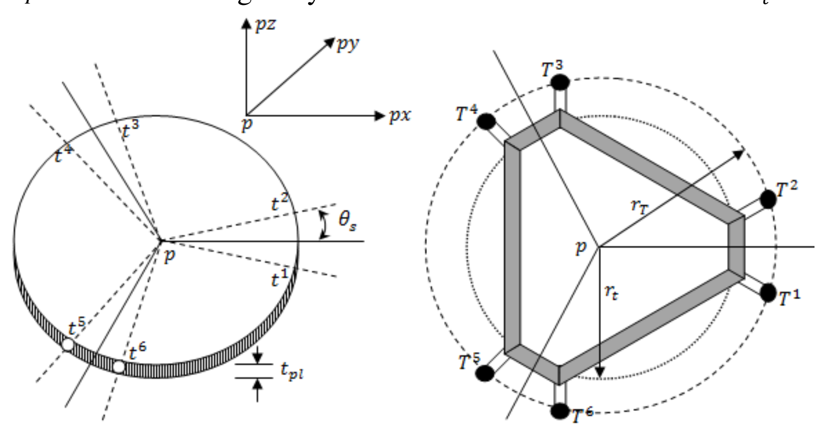

Figure 1: Schematic of the top assembly of the Stewart platform

t_{pl} which is mounted on the semi-regular hexagonal frame at right side with thickness a and width c and used as the base for the payload. The hexagonal frame consists of three smaller and three larger solid trapezoids each have two right triangular prism of length b at their ends. The trapezoids have mean length and mass pair of L_l, M_l and L_s, M_s respectively.

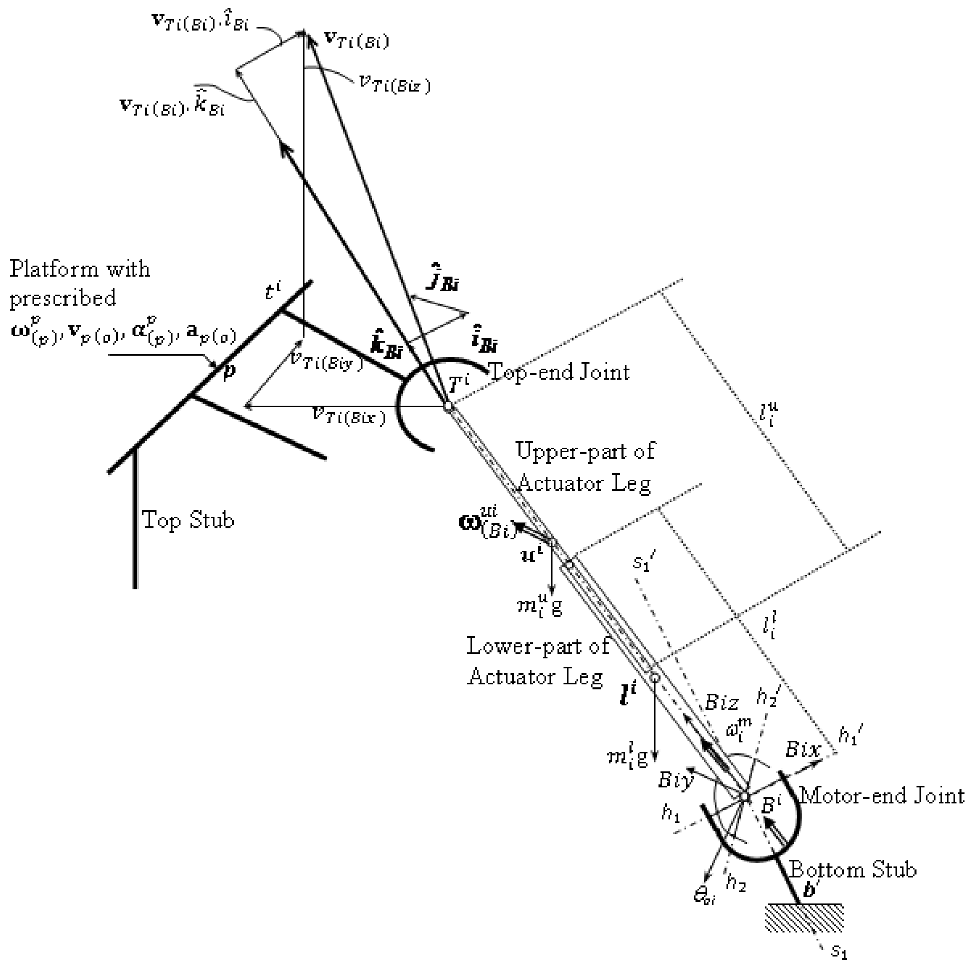

Figure 2: Description of velocity and force of a actuated leg and joints assembly of the Stewart platform

In the figure 2 a i^{th} leg system the points b^i, B^i, T^i and t^i mark the extent of bottom stub, the actuator cylinder assembly and the top stub with B^i and T^i represents the spherical and the universal joint respectively. The six bottom stubs rigidly connected with stationary base with circumscribing circle of radius r_b. The height of point p at neutral pose is $z_{p0(o)}$ with reference to stationary system (x, y, z) and origin at o.The vectors $\mathbf{x}_{bi(o)}$, $\mathbf{x}_{Bi(o)}$ and $\mathbf{x}_{Ti(o)}$ represents the coordinate of points b^i, B^i, T^i with respect to point o. the translation of point p along x, y and z axes respectively known as surge, sway and heave i.e. $\mathbf{x}_{p(o)} = (x \quad y \quad z_{p0(o)} + z)^T$and that of rotational motion known as roll, pitch and yaw i.e. $\boldsymbol{\theta}_{p(o)} = (\alpha \quad \beta \quad \gamma)^T$.

3 INVERSE MODEL OF THE PLATFORM

The expressions involved in translation and orientation of payload are meant for the point p only. The resultant moment of inertia of the top rigid assembly including the payload is determined about the centroid of the top plate. During the calculation of lower leg inertia only the static load is considered. The ball-screw friction coefficient is assumed to be constant during operation. The superscripts (d), a and u, is meant for demanded value, value corresponding to a actuator and upper leg variable.

3.1 Equations of platform configuration

The variables l_i^t and l_i^b are the length of the top and bottom stub. The angles θ_s is found from the geometry in figure 1.Constants $\theta_{Ti(p)}$ and $\theta_{Bi(o)}$ are the angular positions of the universal and spherical joints with respect to body fixed and stationary coordinate systems respectively.

$$\theta_{bi(o)} = -\left(\frac{\pi}{3} - \theta_s\right) + (i-1)\frac{\pi}{3} \;\;\&\;\; \theta_{ti(p)} = -\theta_s + (i-1)\frac{\pi}{3};\;\; \forall\, i = 1, 3\,\&\,5 \tag{1}$$

$$\theta_{bi(o)} = \frac{\pi}{3} - \theta_s + (i-2)\frac{\pi}{3} \;\;\&\;\; \theta_{ti(p)} = \theta_s + (i-2)\frac{\pi}{3}\;;\; \forall\, i = 2, 4\,\&\,6$$

$$L_i = l_i^t + l_i^b + l_i^a \tag{2}$$

$$L_{i0} = \left|\mathbf{x}_{bi(o)}|_0 - \mathbf{x}_{ti(p)}|_0\right| \tag{3}$$

$$\theta_{Ti(p)} = \theta_{ti(p)} + \left(\theta_{bi(o)} - \theta_{ti(p)}\right)(L_{i0} - l_i^t)/L_{i0} \;\; \forall\, i = 1{:}\,6 \tag{4}$$

$$\theta_{Bi(o)} = \theta_{bi(o)} - \left(\theta_{bi(o)} - \theta_{ti(p)}\right)l_i^b/L_{i0} \;\; \forall\, i = 1{:}\,6 \tag{5}$$

$$\mathbf{x}_{bi(o)} = (r_b\cos\theta_{bi(o)} \quad r_b\sin\theta_{bi(o)} \quad 0)^T \tag{6}$$

$$\mathbf{x}_{Bi(o)} = \left(r_B\cos\theta_{Bi(o)} \quad r_B\sin\theta_{Bi(o)} \quad z_{p(o)} \times l_i^b/L_{io}\right)^T \tag{7}$$

$$\mathbf{x}_{Ti(p)} = \left(r_T\cos\theta_{Ti(p)} \quad r_T\sin\theta_{Ti(p)} \quad -z_{p0(o)} \times l_i^t/L_{io}\right)^T \tag{8}$$

$$\mathbf{x}_{ti(p)} = (r_t\cos\theta_{ti(p)} \quad r_t\sin\theta_{ti(p)} \quad 0)^T \tag{9}$$

The top and bottom stub length are given by respectively

$$l_i^t = \left|\mathbf{x}_{ti(o)}^{(d)} - \mathbf{x}_{Ti(o)}^{(d)}\right| \tag{10}$$

$$l_i^b = \left|\mathbf{x}_{Bi(o)} - \mathbf{x}_{bi(o)}\right| \tag{11}$$

Moment of inertia of the frame with respect to its centroid

$$I_{xcf} = M_l\left[\frac{{L_l}^2}{4} + \frac{c^2}{4} - \frac{1}{12}\left(\frac{bc}{L_l + b}\right)^2\right] + M_s\left[\frac{{L_s}^2}{4} + \frac{c^2}{4} - \frac{1}{12}\left(\frac{bc}{L_s + b}\right)^2\right] + \frac{1}{2}abc\rho[(L_l + b)^2 + (L_s + \\ + 3M_l\left[\frac{c}{6} + r_t\cos(60^0 - \theta_s)\right]^2 + 3M_s\left[\frac{c}{6} + r_t\cos\theta_s\right]^2 \tag{12}$$

$$I_{ycf} = M_l\left[\frac{5}{24}a^2 + \frac{{L_l}^2}{12} + \frac{c^2}{8} - \frac{1}{24}\left(\frac{bc}{L_l + b}\right)^2\right] + M_s\left[\frac{5}{24}a^2 + \frac{{L_s}^2}{12} + \frac{c^2}{8} - \frac{1}{24}\left(\frac{bc}{L_s + b}\right)^2\right] + \frac{1}{6}abc\rho[(L_l + \\ + 3M_l\left[\left\{\frac{c}{6} + r_t\cos(60^0 - \theta_s)\right\}\sin 60^0\right]^2 + 3M_s\left[\left\{\frac{c}{6} + r_t\cos\theta_s\right\}\cos 30^0\right]^2 \tag{13}$$

$$I_{zcf} = M_l\left[\frac{5}{24}a^2 + \frac{{L_l}^2}{8} + \frac{c^2}{12} - \frac{1}{36}\left(\frac{bc}{L_l + b}\right)^2\right] + M_s\left[\frac{5}{24}a^2 + \frac{{L_s}^2}{12} + \frac{c^2}{8} - \frac{1}{36}\left(\frac{bc}{L_s + b}\right)^2\right] + \frac{1}{4}abc\rho[(L_l + \\ + 3M_l\left[\left\{\frac{c}{6} + r_t\cos(60^0 - \theta_s)\right\}\cos 60^0\right]^2 + 3M_s\left[\left\{\frac{c}{6} + r_t\cos\theta_s\right\}\sin 30^0\right]^2 \tag{14}$$

3.2 Equations of inverse kinematic model

The unit vectors $\hat{\mathbf{e}}_p^{(d)}$, $\hat{\mathbf{e}}_o^{(d)}$ and $\hat{\mathbf{e}}_{Bi}^{(d)}$represents the desired body fixed, stationary and leg coordinate systems respectively with $\mathrm{R}_{p,o}^{\mathrm{T}(d)}$ and $\mathrm{R}_{Bi,o}^{\mathrm{T}(d)}$ are the rotation matrix used to convert body fixed coordinate into stationary coordinate.

$$\hat{\mathbf{e}}_p^{(d)} = \left(\hat{\imath}_p^{(d)} \quad \hat{\jmath}_p^{(d)} \quad \hat{k}_p^{(d)}\right)^T \tag{15}$$

$$\hat{\mathbf{e}}_o^{(d)} = (\hat{\imath} \quad \hat{\jmath} \quad \hat{k})^T \tag{16}$$

$$\hat{\mathbf{e}}_p^{(d)} = \mathrm{R}_{p,o}^{(d)}\,\hat{\mathbf{e}}_o^{(d)} \tag{17}$$

$$\mathbf{x}_{p(o)}^{(d)}(t) = \left(x^{(d)}(t) \quad y^{(d)}(t) \quad z_{p0(o)} + z^{(d)}(t)\right)^T \tag{18}$$

$$\boldsymbol{\theta}_{(o)}^{p(d)}(t) = \left(\phi^{(d)}(t) \quad \beta^{(d)}(t) \quad \gamma^{(d)}(t)\right)^T \tag{19}$$

$$\mathbf{x}_{Ti(o)}^{(d)} = \mathbf{x}_{p(o)}^{(d)} + \mathbf{R}_{p,o}^{T(d)}\mathbf{x}_{Ti(p)} \tag{20}$$

$$\mathbf{x}_{ti(o)}^{(d)} = \mathbf{x}_{p(o)}^{(d)} + \mathbf{R}_{p,o}^{T(d)}\mathbf{x}_{ti(p)} \tag{21}$$

$$\mathbf{v}_{Ti(o)}^{(d)} = \mathbf{R}_{p,o}^{T(d)}\left\{\mathbf{v}_{p(p)}^{(d)} + \boldsymbol{\omega}_{(p)}^{p(d)} \times \mathbf{x}_{Ti(p)}\right\} = \mathbf{R}_{p,o}^{T(d)}\mathbf{v}_{Ti(p)}^{(d)} \tag{22}$$

$$\mathbf{a}_{Ti(o)}^{(d)} = \mathbf{R}_{p,o}^{T(d)}\left\{\mathbf{a}_{p(p)}^{(d)} + \boldsymbol{\alpha}_{(p)}^{p(d)} \times \mathbf{x}_{Ti(p)} + 2\boldsymbol{\omega}_{(p)}^{p(d)} \times \mathbf{v}_{Ti(p)}^{(d)}\right\} = \mathbf{R}_{p,o}^{T(d)}\mathbf{a}_{Ti(p)}^{(d)} \tag{23}$$

The desired actuated leg length be

$$l_i^{a(d)} = \left|\mathbf{x}_{Ti(o)}^{(d)} - \mathbf{x}_{Bi(o)}\right| \tag{24}$$

3.3 Equations of inverse dynamic model

The inverse dynamic model helps to estimate the current input to the coils of the torque motors necessary for achieving the desired payload motion. In the equation stated below $F_{(Biz)}^{u(d)}$ is meant for desired actuating force along the screw axis, $\mathbf{F}_{ti}^{u(d)}$ is transverse force arises due to weight friction and inertia corresponding to the rotation of the screw joint axis about the point B^i.

$$\mathbf{F}_{ti}^{u(d)} = \mathbf{F}_{ti}^{u1(d)} + \mathbf{F}_{ti}^{u2(d)} \tag{25}$$

where (26)

$$\mathbf{F}_{ti}^{u1(d)} = -m^l \left[l^l \left\{ g i_{(Biz)}^{(d)}/2 + l^l \alpha_{(Bix)}^{ui(d)}/3 \right\} \hat{\boldsymbol{\imath}}_{Bi}^{(d)} + l^l \left\{ g j_{(Biz)}^{(d)}/2 + l^l \alpha_{(Biy)}^{ui(d)}/3 \right\} \hat{\boldsymbol{\jmath}}_{Bi}^{(d)} - m^l\, c^{Sl} \operatorname{sgn}\left(\omega_{(Biy)}^{ui(d)}\right) \left\{ g k_{(Biz)}^{(d)} + {\omega_{(Biy)}^{ui(d)}}^2 l^l/2 \right\} \hat{\boldsymbol{\imath}}_{Bi}^{(d)} \right] / (l_i^{a(d)} - l^u)$$

and

$$\mathbf{F}_{ti}^{u2(d)} = -\hat{\boldsymbol{\imath}}_{Bi}^{(d)}\, c^{Sl} \operatorname{sgn}\left(\omega_{(Biy)}^{ui(d)}\right) F_{(Biz)}^{u(d)} / (l_i^{a(d)} - l^u) \tag{27}$$

$$\sum_{i=1}^{6} \left\{ F_{(Biz)}^{u(d)} \hat{\boldsymbol{k}}_{Bi}^{(d)} + \mathbf{F}_{ti}^{u(d)} - \mathbf{B}_i^{u(d)} \right\} = m^p \left(\mathbf{a}_{p(o)}^{(d)} + g\hat{k} \right) \tag{28}$$

$$\boldsymbol{B}_i^{u(d)} = m^u \left[g\hat{k} + \left\{ \alpha_{(Biy)}^{ui(d)} \left(l_i^{a(d)} - l^u/2 \right) + \omega_{(Biy)}^{ui(d)} v_{(Biz)}^{ui(d)} \right\} \hat{\boldsymbol{\imath}}_{Bi}^{(d)} + \left\{ a_{(Biz)}^{ui(d)} + {\omega_{(Biy)}^{ui(d)}}^2 \left(l_i^{a(d)} - l^u/2 \right) \right\} \hat{\boldsymbol{k}}_{Bi}^{(d)} \right] \tag{29}$$

$$\sum_{i=1}^{6} \mathbf{x}_{Ti(p)}^{(d)} \times \left\{ F_{(Biz)}^{u(d)} \hat{\boldsymbol{k}}_{Bi}^{(d)} + \mathbf{F}_{ti}^{u(d)} - \mathbf{B}_i^{u(d)} \right\} - \sum_{i=1}^{6} \left\{ c^{Ul} \operatorname{sgn}\left(\acute{\omega}_{(Biy)}^{ui(d)} \right) \left(F_{(Biz)}^{u(d)} - \mathbf{B}_i^{u(d)} \cdot \hat{\boldsymbol{k}}_{Bi}^{(d)} \right) \hat{\boldsymbol{\imath}}_{Bi}^{(d)} \right\} = \mathbf{I}_{(p)}^{p} \boldsymbol{\alpha}_{(p)}^{p(d)} + \mathbf{P}^{(d)} \tag{30}$$

where $\acute{\omega}_{(Biy)}^{ui(d)}$ is the effective angular velocity about $\hat{\boldsymbol{\jmath}}_{Bi}^{(d)}$ and expressed by $\acute{\omega}_{(Biy)}^{ui(d)} = \omega_{(Biy)}^{ui(d)} - \boldsymbol{\omega}_{(p)}^{p(d)} \cdot \hat{\boldsymbol{\jmath}}_{Bi}^{(d)}$ c^{Sl} and c^{Ul} are the friction coeffiient indicator and has the unit in length and is of value of $1/10^{th}$ of the order of the radius of curvature of the contact surfaces of sphrerical and universal joints respectively.

$\mathbf{I}_{(p)}^{p} = diag\left(I_{(px)}^{p} \quad I_{(py)}^{p} \quad I_{(pz)}^{p} \right)$ and

$$\mathbf{P}^{(d)} = \left[\omega_{(py)}^{p(d)} \omega_{(pz)}^{p(d)} \left(I_{(py)}^{p} - I_{(pz)}^{p} \right) \quad \omega_{(pz)}^{p(d)} \omega_{(px)}^{p(d)} \left(I_{(pz)}^{p} - I_{(px)}^{p} \right) \quad \omega_{(px)}^{p(d)} \omega_{(py)}^{p(d)} \left(I_{(px)}^{p} - I_{(py)}^{p} \right) \right]^T \hat{\mathbf{e}}_p^{(d)} \tag{31}$$

4 FEEDFORWARD-FEEDBACK CONTROL MODEL

$$c_i = c_i^{(f)} + c_i^{(b)} \tag{32}$$

where net actuator current input is c_i, forward estimated current is $c_i^{(f)}$ and

$$c_i^{(b)} = k_P \left(l_i^{a(d)} - l_i^a \right) + k_P \int \left(l_i^{a(d)} - l_i^a \right) dt + k_D \left(v_{Biz}^{ui(d)} - v_{Biz}^{ui} \right) \tag{33}$$

and

$$T_i^m = K_t c_i \tag{34}$$

$$T_i^m - F_{(Biz)}^{u} r^b \tan\left[\tan^{-1}\{ p^b/(2\pi r^b) \} + \operatorname{sgn}\left(v_{(Biz)}^{ui} \right) \tan^{-1} \mu^b \right] = \left(I_{(Biz)}^{l} + I_{(Biz)}^{m} \right) \alpha_{(Biz)}^{li} + I_{(Biz)}^{u} \alpha_{(Biz)}^{ui} \tag{35}$$

with

$$\boldsymbol{\alpha}^{li}_{(Bi)} = \boldsymbol{\alpha}^{ui}_{(Bi)} + \boldsymbol{\alpha}^{mi}_{(Bi)} \tag{36}$$

$$\boldsymbol{\alpha}^{mi}_{(Bi)} = \left(\ddot{l}^{a}_{i}/p^{b}\right)\hat{\boldsymbol{k}}_{Bi} \tag{37}$$

where p^b is the pitch of the screw having friction coefficient μ^b.

5 PERFORMANCE SUMMARY

Following results have been obtained in the MATLAB-SIMULINK environment with the actuator and control data from [5]. Dotted lines in all the figures below indicate desired value.

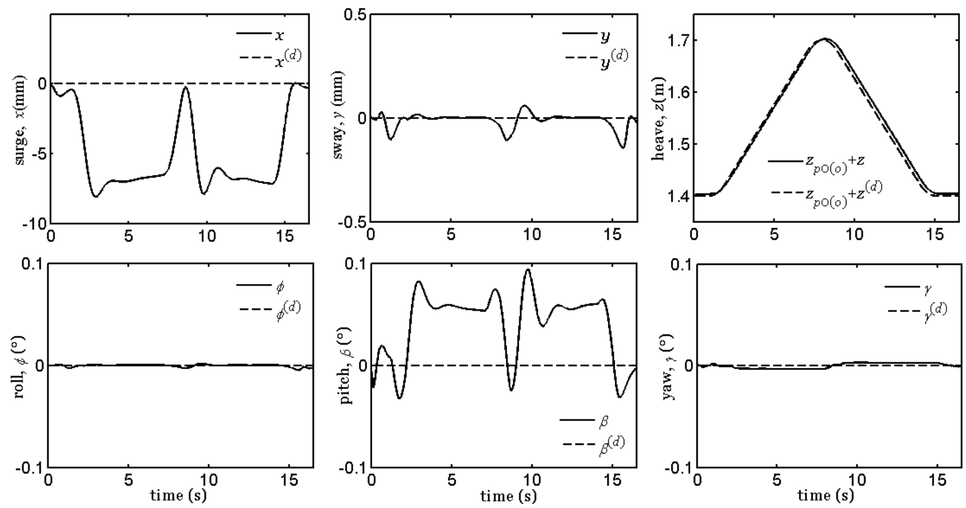

Figure 3a: All the pose of the platform center under the +300 mm peak heave demand

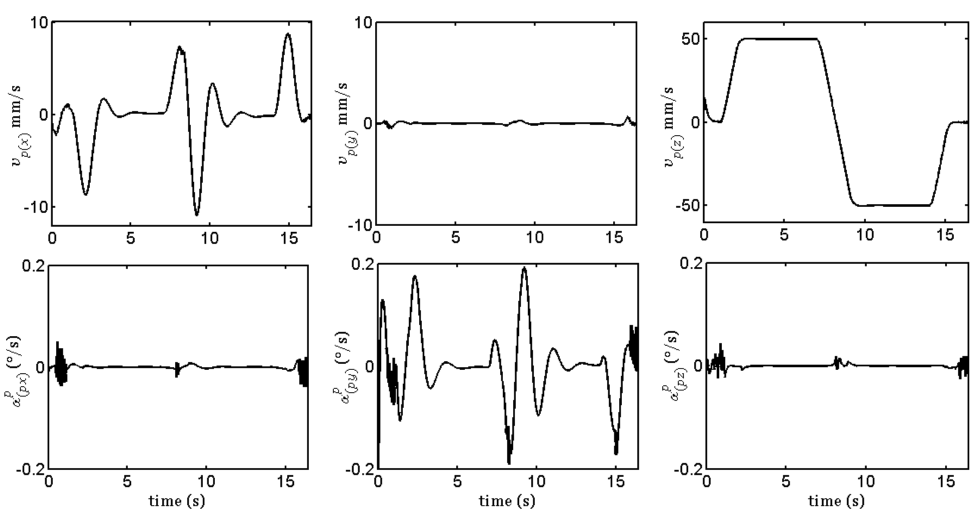

Figure 3b: All the velocities of the platform center under the +300 mm peak heave demand

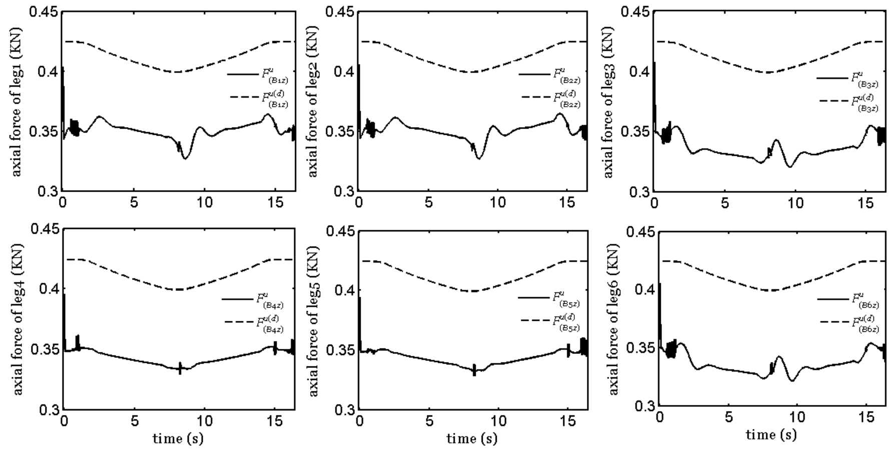

Figure 3c: All the leg force of the platform under the +300 mm peak heave demand

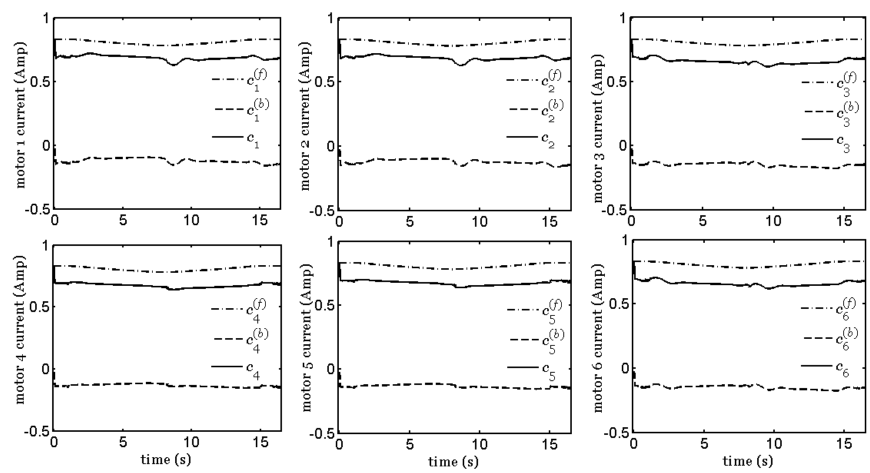

Figure 3d: Different currents that actuates the inline cylinder under the +300 mm peak heave demand

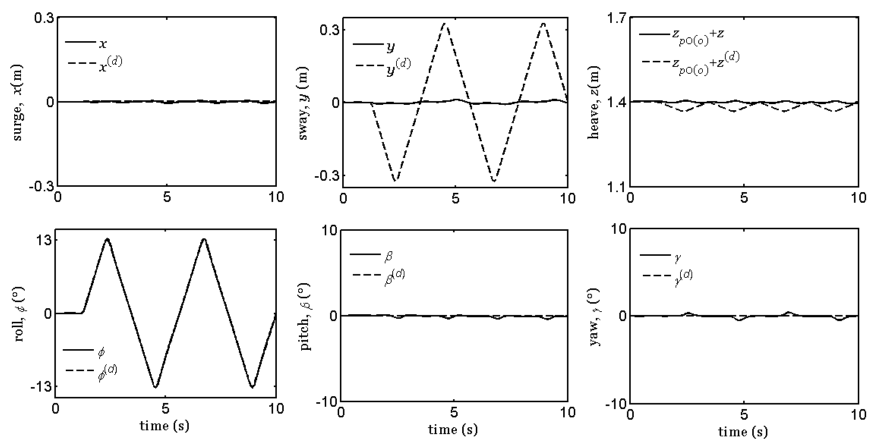

Figure 4a: All the pose of the platform center under the cyclic ±13^0 roll demand

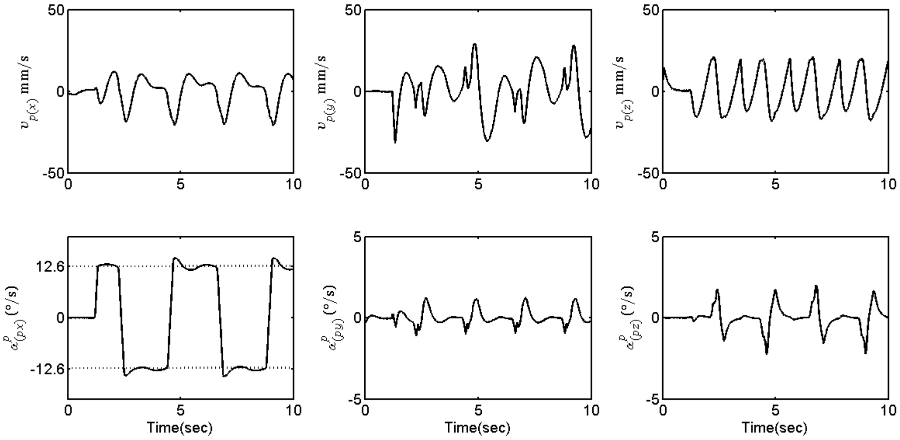

Figure 4b: All the velocities of the platform center under the cyclic ±13^{0}roll demand

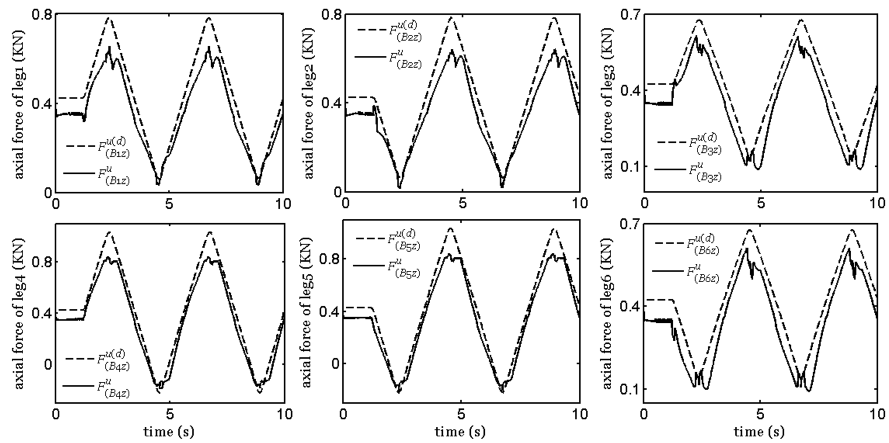

Figure 4c: All the leg force of the platform center under the cyclic ±13^0 roll demand

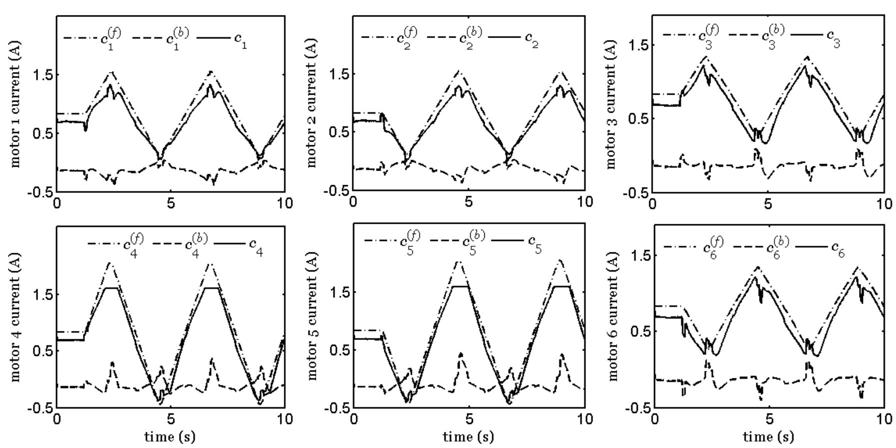

Figure 4d: Different currents that actuates the inline cylinder under the cyclic ±13^0 roll demand

6 CONCLUSIONS

- It is observed that with the improved inverse model the coupled surge motion is as low as 2.5% of the pure heave demand under no additional coupling compensation. Again for pure roll demand the coupling compensation is found mandatory to reduce the coupled sway motion completely.
- The conventional PD feedback controller is found to works efficiently to cater the un-modeled dynamics in presence of friction at all the joints of each leg.
- The feedforward control current found to dominate over that of the feedback control current in both the cases, thus the enhanced inverse model effectively improved the feedforward control effort.

REFERENCES

[1] Dasgupta, B.; Mruthyunjaya, T.S.: Closed-form dynamic equations of the general Stewart platform through the Newton–Euler approach. *Mechanisms ane Machines Theory* (1998)**33**(7) : 993–1012.

[2] Dasgupta, B.; Mruthyunjaya, T.S.: A Newton–Euler formulation for the inverse dynamics of the Stewart platform m0anipulator. *Mechanisms and Machines Theory* (1998)**33**(7) :1135–1152.

[3] Fichter, E. F.: A Stewart platform-based manipulator: general theory and practical construction. *International Journal of Robotics Research* (1986)**5**(2) :157-182

[4] Halder, B.; Saha, R.; Sanyal, D.: Control-Integrated Design by Theoretical Simulation for a Torque-Actuated 6-SBU Stewart Platform. *Int. J. on Manufacturing and Material Science* (2011)**2**(1) :13–21.

[5] Halder, B.; Saha, R.; Sanyal, D.: Tracking Error Compensation of a 6-SBU Stewart Platform. *Proceedings of the 2nd Joint International Conference on Multibody System Dynamics,* May 29-June 1, 2012, Stuttgart,germany.

[6] Liu, K.; Fitzgerald, J.; Lewis, F.L.: Kinematic analysis of a Stewart platform manipulator. *IEEE Transactions on Industrial Electronics*, (1993)**40**(2) :282–293.

[7] Merlet, J. P.: Direct kinematics of parallel manipulators. *IEEE Transactions on Robotics and Automation*, (1993)**9**(6) :842–845.

[8] Stewart, D.: A platform with six degrees of freedom. *Proceedings of Institute of Mechanical Engineering*, (1965)**180**(1) :371–386.