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AN EFFICIENT APPROACH TO STUDY MULTI-LAYERED STRUCTURES WITH COHESIVE INTERFACES

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Abstract. A novel approach has been recently proposed by the author to study multilayered plate and shell structures with imperfect and cohesive interfaces and delaminations. Stress and displacement fields, which in these systems are characterized by large variations and discontinuities in the thickness direction, are fully described in the theory through a limited number of displacement unknown functions, independent of the number of layers and interfaces and equal to that of single layer theories for fully bonded structures. Applications which highlight the accuracy, range of applicability and limitations of the approach will be presented at the meeting.

1 INTRODUCTION

Delamination damage growth in multilayered plate and shell structures loaded dynamically is conveniently studied using fracture mechanics principles and a discrete-layer approach, which describes the system as an assemblage of layers joined by cohesive interfaces (e.g. [1-3]). The interfaces define all actual and potential fracture surfaces in the system and cohesive traction laws are introduced, which relate the interfacial tractions to the relative displacements between the layers, in order to describe all different nonlinear mechanisms taking place at the interfaces, e.g. material rupture, cohesive/bridging mechanisms, elastic contact. This approach leads to accurate solutions of the problem, however it may become computationally very expensive since the number of unknowns of the problem depends on the assumed kinematic description of the layers and on the number of layers chosen to discretize the system. If the First Order Shear Deformation theory is used to for the kinematic description of the layers in a plate or a cylindrical shell, the number of displacement unknowns is then 5×n (with *n* the number of layers).

A novel approach, based on a homogenization technique, has been recently proposed by the author to study the dynamic response of multilayered plates and shells with cohesive interfaces [4-6]. The model extends to systems with generally nonlinear cohesive interfaces a theory which was originally formulated for systems with linearly elastic interfaces [7-10]. As in a classical discrete-layer model, the system is decomposed into layers and cohesive interfaces, Fig. 1. A two length-scales displacement field is assumed, which is characterized by a global displacement (continuous and with continuous derivatives in the thickness) and local perturbations or enrichments (piece-wise linear with jumps at the interfaces). Through the a-priori imposition of interfacial continuity conditions, a homogenized displacement field is derived, which depends on the global displacement variables only. Hamilton principle of elastokinetics is then used to obtain the equilibrium equations.

The dynamic equilibrium equations depend on a limited number of unknowns, which is independent of the number of layers used to discretize the system. In a plate or a cylindrical shell, for instance, the number of displacement unknowns is reduced from 5×n, as in a classical discrete layer model, to 6. In [11] the theory has been compared with rigorous elasticity solutions and with the original models proposed in [7-10] for plates and shells with purely elastic interfaces. Omissions in the interfacial energy contributions in Hamilton equation in [7-10] were highlighted and revised formulations presented in the Appendices [11]. The accuracy and limitation of the approach have been further investigated in [12], where the model has been particularized to plates deforming in cylindrical bending and applied to investigate the effects of the number and position of the imperfect interfaces in unidirectionally reinforced plates and highly anisotropic multi-layered systems with linearly elastic cohesive interfaces.

The assumptions and formulation of the model are briefly recalled here and some applications are presented which highlight the capabilities of the proposed approach. At the meeting, preliminary applications of the theory to some simple fracture mechanics problems will be presented.

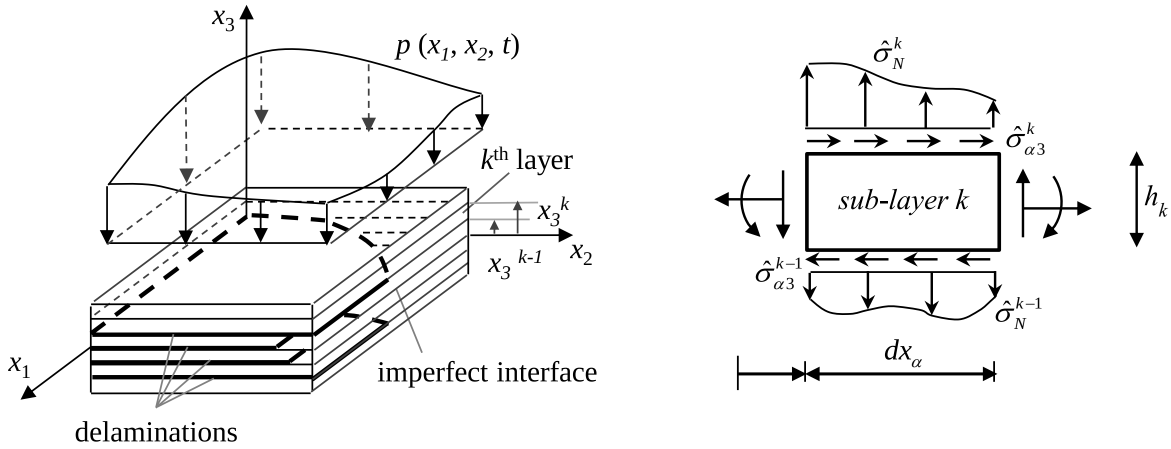

Figure 1. (a) Composite plate element showing discretization into sub-layers, cohesive interfaces and delaminations. (b) Interfacial tractions and counterparts of stress resultants and couples acting on layer *k*.

2 MODEL ASSUMPTIONS AND FORMULATION

Consider a multilayered plate of thickness h, Fig. 1. A system of Cartesian coordinates $x_1 - x_2 - x_3$ is introduced with the axis x_3 normal to the reference surface of the plate and measured from it. The plate has volume $\mathscr{V}$ and upper and lower external surfaces, $\mathscr{S}^+$ and $\mathscr{S}^-$; the lateral bounding surface, $\mathscr{B}$, is generated by the normal to the reference surface along its boundary curve $\mathscr{C}$. The plate consists of n layers exhibiting different mechanical properties and joined by $n-1$ interfaces. The layer *k*, where the index $k = 1,..,n$ is numbered from bottom to top, is defined by the coordinates $x_3{}^{k-1}$ and $x_3{}^k$ of its lower and upper interfaces, ${}^{(k)}\mathscr{S}^-$ and ${}^{(k)}\mathscr{S}^+$, and has thickness ${}^{(k)}h$ (the *k* superscript in brackets identifies affiliation with layer *k*). Each layer is linearly elastic, homogeneous and anisotropic with monoclinic symmetry about its mid-surface; the principal material axes of the layer do not necessarily coincide with the geometrical axes of the structure $x_1 - x_2$. The mass density is uniform and equal to ρ_m. The plate is subjected to time dependent distributed loads $\mathbf{p}(\mathbf{x},t)$ acting on $\mathscr{S}^+$, $\mathscr{S}^-$ and $\mathscr{B}$.

In tensorial index notation the constitutive equations for the layer k are ${}^{(k)}\sigma_{ij} = {}^{(k)}C_{ijhk}\,{}^{(k)}\varepsilon_{hk}$ and ${}^{(k)}\varepsilon_{ij} = {}^{(k)}A_{ijhk}\,{}^{(k)}\sigma_{hk}$, where ${}^{(k)}A_{\alpha 3\gamma 3}\,{}^{(k)}C_{\gamma 3\beta 3} = \delta_{\alpha\beta}/4$, with $\delta_{\alpha\beta}$ the Kronecker index. (the Einsteinian summation convention applies to repeated subscripts of tensor components, with Latin subscripts ranging from 1 to 3 and Greek subscripts from 1 to 2).

The displacement vector $\mathbf{v}(\mathbf{x},t)$ at time t of an arbitrary point of the plate at the coordinate $\mathbf{x} = \{x_1, x_2, x_3\}^T$ is $\mathbf{v} = \{v_1, v_2, v_3 = w\}^T$, with v_1, v_2 and $w = v_3$ the displacement components onto the reference surface of the plate.

The *n-1* interfaces are assumed to be imperfect and, following the assumptions of the spring-layer approach [7] for sliding interfaces and its extension to mixed-mode interfaces [10], as well as classical cohesive-crack approaches [1-3], interfacial traction laws are introduced which relate the interfacial normal and shear tractions, $\hat{\sigma}_{33}^k = \hat{\sigma}_N^k$, $\hat{\sigma}_{13}^k$ and $\hat{\sigma}_{23}^k$, acting along the surface of the layer k at the interface ${}^{(k)}\mathscr{S}^+$ with unit positive normal vector, to the interface relative displacements, defined by the vector $\hat{\mathbf{v}}^k = \{\hat{v}_1^k, \hat{v}_2^k, \hat{w}^k\}^T = {}^{(k+1)}\mathbf{v}(x_3{}^k) - {}^{(k)}\mathbf{v}(x_3{}^k)$, Fig. 1.b. The traction laws are typically assumed to be nonlinear, in order to describe different physical mechanisms, which may include cohesive/bridging mechanisms developed by trans-laminar reinforcements or other means, material rupture, elastic contact along delamination surfaces, … [1-3]. This formulation refers to linear non-proportional interface laws, given by:

$$\hat{\sigma}_{13}^k = K_{11}^k \hat{v}_1^k + K_{12}^k \hat{v}_2^k + t_1^k\,, \quad \hat{\sigma}_{23}^k = +K_{21}^k \hat{v}_1^k + K_{22}^k \hat{v}_2^k + t_2^k\,, \quad \hat{\sigma}_N^k = K_N^k \hat{w}^k + t_N^k\,, \tag{1}$$

or, in direct and inverse matrix forms,

$$\hat{\boldsymbol{\sigma}}^k = \mathbf{K}^k \hat{\mathbf{v}}^k + \mathbf{t}^k \text{ and } \hat{\mathbf{v}}^k = \mathbf{B}^k(\hat{\boldsymbol{\sigma}}^k - \mathbf{t}^k) \tag{2}$$

where $\mathbf{K}^k$ and $\mathbf{B}^k$ are the symmetrical interface stiffness and compliance matrices, $\hat{\boldsymbol{\sigma}}^k = \{\hat{\sigma}_{13}^k, \hat{\sigma}_{23}^k, \hat{\sigma}_{33}^k = \hat{\sigma}_N^k\}^T$, and $\mathbf{t}^k = \{t_1^k, t_2^k, t_3^k = t_N^k\}^T$ is a vector of constant interfacial tractions which are assumed to act for any $\hat{\mathbf{v}}^k \neq \mathbf{0}$. The interfacial traction laws in Eqs. (1) and (2) assume no coupling between in-plane and out of plane interface tractions, namely $K_{13}^k = K_{23}^k = 0$ and $K_{33}^k = K_N^k$, as it is normally assumed in the literature on cohesive cracks.

A purely elastic interface is described by $\mathbf{t}^k = \mathbf{0}$, perfectly bonded interfaces are defined by $\mathbf{t}^k = \mathbf{0}$ and $\mathbf{B}^k = 0$, which yields $\hat{\mathbf{v}}^k = \mathbf{0}$, and fully debonded interfaces by $\mathbf{t}^k = \mathbf{0}$ and $\mathbf{K}^k = 0$, which yields $\hat{\boldsymbol{\sigma}}^k = \mathbf{0}$. The classical spring-layer approaches for sliding interfaces [7-9], are obtained by imposing $B_N^k = 0$, which yields $\hat{w}^k = 0$. For $\mathbf{t}^k \neq \mathbf{0}$, the linear non proportional laws of Eqs. (1) and (2) could describe the bridging mechanisms developed by a through-thickness reinforcement, e.g. stitching, applied to a laminated composite [13]. As proposed in [4-6], the linear non proportional laws in Eqs (1,2) allows the treatment of interfaces with generally nonlinear cohesive traction laws, which can be approximated as piece-wise linear functions of the relative displacements and where Eq. (2) then defines an arbitrary branch *i* of the functions.

A two length-scales displacement field $\mathbf{v} = \{v_1, v_2, v_3 = w\}^T$ is assumed, which is analogous to that proposed by Librescu and Schmidt for multilayered shells in [10], and is given by the superposition of a global field and local perturbation terms (or enrichments):

$$
\begin{aligned}
v_\alpha(x_1,x_2,x_3,t) &= v_{0\alpha} + \varphi_\alpha x_3 + \sum_{k=1}^{n-1} \Omega_\alpha^k (x_3 - x_3{}^k) H^k + \sum_{k=1}^{n-1} \hat{v}_\alpha^k H^k \\
w(x_1,x_2,x_3,t) &= w_0 + \varphi_3 x_3 + \sum_{k=1}^{n-1} \Omega_3^k (x_3 - x_3{}^k) H^k + \sum_{k=1}^{n-1} \hat{w}^k H^k
\end{aligned}
\tag{3}
$$

The terms on the right hand side of Eqs. (3) denote different contributions in the displacement representation: $v_{0\alpha} = v_{0\alpha}(x_\omega, t)$, $w_0 = w_0(x_\omega, t)$ and $\varphi_\alpha = \varphi_\alpha(x_\omega, t)$ define standard first order shear deformation theory terms, which are continuous with continuous derivatives in the thickness direction, C_3^1, and define the displacement components of a point on the reference surface and the rotations of the normal to the reference surface when the latter coincides with the mid-surface of the bottom layer; $\varphi_3 = \varphi_3(x_\omega, t)$ is the constant strain in the transverse direction, which is needed to capture, in the simplest way possible [10], the effect of transverse normal compressibility and to model opening and elastic contact along delaminations [4-6]; the third terms, with summations on the total number *n*-1 of interfaces and $H^k = H(x_3 - x_3{}^k) = \{0,\ \ x_3 < x_3{}^k;\, 1,\ \ x_3 \geq x_3{}^k\}$ the Heaviside's function, supply the zig-zag contributions [14-15], which are continuous in x_3 but with jumps in the first derivatives at the interfaces, C_3^0, and are necessary to satisfy continuity on normal and shear tractions at the interfaces in plates with arbitrary stacking sequences; the fourth terms, with summations on the number of cohesive interfaces *n*-1, define a discontinuous field and supply the contribution of the relative displacements (jumps) at the cohesive interfaces.

By imposing continuity of shear and normal tractions at the interfaces and the constitutive laws of the interfaces, the zigzag functions and the displacement jumps are defined as function of the global displacement variable. Details on the procedure can be found in [11]. The resulting homogenized displacement field in the kth layer is given by:

$$
\begin{aligned}
{}^{(k)}v_\alpha &= \underline{v_{0\alpha}} + \underline{\varphi_\alpha} x_3 + \left(\underline{w_{0,\omega}} + \underline{\varphi_\omega}\right) R_{S\alpha\omega}^k + \underline{\varphi_{3,\omega}} R_{N\alpha\omega}^k - \sum_{i=1}^{k-1} B_{\alpha\omega}^i t_\omega^i \\
{}^{(k)}w &= \underline{w_0} + \underline{\varphi_3} \left\{ \ x_3 + \sum_{i=1}^{k-1} \left[\ \Lambda_{33}^{(i)} (x_3 - x_3{}^i) + \Psi_{33}^i \ \right] \right\} - \sum_{i=1}^{k-1} B_N^i t_N^i
\end{aligned}
\tag{4}
$$

where:

$$
R_{S\alpha\omega}^k = R_{S\alpha\omega}^k(x_3) = \sum_{i=1}^{k-1} \left[\ \Lambda_{\alpha\omega}^{(1;i)} \left(x_3 - x_3{}^i\right) + \Psi_{\omega\alpha}^i \ \right] \tag{5}
$$

$$R^k_{N\alpha\omega} = R^k_{N\alpha\omega}(x_3) = \sum_{i=1}^{k-1}\left\{\left[\Lambda^{(1;i)}_{\alpha\omega} x_3{}^1 + \sum_{l=2}^{i} \Lambda^{(l;i)}_{\alpha\omega}\left(x_3{}^l - x_3{}^{l-1}\right)\left(1+\sum_{n=1}^{l-1}\Lambda^{(n)}_{33}\right) - \delta_{\alpha\omega}\Psi^i_{33}\right]\left(x_3 - x_3{}^i\right) + \Phi^i_{\omega\alpha}\right\}$$

Equations (4) highlight that the displacement field is fully defined by the 6 displacement variables which define the global part of the displacement, and are underlined in the equations. The equations depend on the elastic constants of the material, the layup and the geometry, through the terms with no lines $\Lambda^{(i)}_{33}$ and $\Lambda^{(1;i)}_{\alpha\omega}$, and on the parameters of the cohesive traction laws, through the terms with the curved lines on top, $B^i_{\alpha\omega}t^i_{\omega}$, $\Psi^i_{\omega\alpha}$, Ψ^i_{33} and $\Phi^i_{\omega\alpha}$. Expressions for these parameters can be found in [11]. In the case of linear interfacial laws, when $\mathbf{t}^k = \mathbf{0}$, the equations coincide with those obtained in [10] for a shallow shell; for perfectly bonded layers, all terms with the curved line on top vanish and the equations are those of classical first order zig-zag theory [14,15].

Hamilton principle of elastokinetics is used to derive the dynamic equilibrium equations in weak form. Dynamic equilibrium equations for multi-layered plates with arbitrary lay-ups can be found in [11] and equations for cylindrically bent plates in [12].

3 APPLICATIONS

When the layers are perfectly bonded, namely when $\mathbf{B}^k = 0$ for all *k*s, the equations of the proposed model coincide with those of first order zigzag models formulated for fully bonded plates [14,15]. The accuracy of the approach to describe multilayered plates with complex layups under such assumption has been extensively investigated in the literature. Focus is placed here on the accuracy of the proposed theory for plates with imperfect interfaces and delaminations. To this aim, the results of the model will be compared with exact 2D elasticity solutions.

The model has been applied to a multilayered anisotropic plate with *n* layers and *n*-1 linearly elastic, slip-only interfaces ($\hat{\boldsymbol{\sigma}}^k = \mathbf{K}^k\hat{\mathbf{v}}^k$ with $B^k_N = 0$, Eq. (3)), Fig. 2. The plate is in cylindrical bending, simply supported at the edges and subjected to a static load $q = q_0 \sin\left(\pi x_2/L\right)$. The elastic constants of each transversely isotropic layer are E_L, E_T, G_{LT}, G_{TT} and ν_{LT}, ν_{TT}. The exact 2D elasticity solution of this problem was obtained by Pagano [16] for fully bonded multilayered plates ($\mathbf{B}^k = 0$); a strategy to extent the model to plates with linearly elastic interfaces was formulated in [17]. Both the exact 2D model and the proposed homogenized model [11] are solved in closed form.

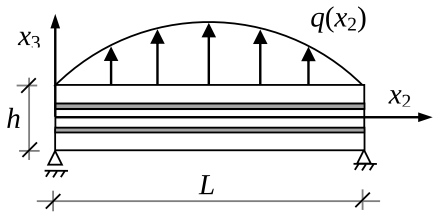

Fig. 2. Multilayered anisotropic plate with imperfect interfaces deforming in cylindrical bending.

Two different layups are examined:

(a) A unidirectional plate with fibers in the x2 direction and two equally spaced cohesive interfaces (0deg/0deg/0deg);

(b) A multilayered plate composed of three orthotropic layers (0deg/90deg/0deg) with n = 3 and two equally spaced interfaces.

Within the proposed homogenized approach, and if the interfaces have the same constitutive laws, the behavior of the unidirectional plate (a) is controlled by 2 dimensionless groups, the dimensionless shear stiffness, $\bar{G}_{LT}/\bar{E}_L\,(L/h)^2$, and the dimensionless interfacial stiffness, $K_2 h/\bar{E}_L\,(L/h)^2$ (or interfacial compliance $(h/B_2\bar{E}_L)(L/h)^2$), where $\bar{E}_L = E_L/(1-\nu_{LT}\nu_{TL})$ is the stiffness coefficient modified to account for negligible normal stresses in the x_3 direction.

Figure 3 shows longitudinal displacements and transverse shear stresses in the thickness of the plate at $x_2 = 0$ for three values of the dimensionless interfacial stiffness: (a,b) a fully bonded plate, (e,f) a fully debonded plate and (c-d) an intermediate case. The diagrams compare the exact solutions with the solutions of the proposed I order model. The shear stresses in the approximate model have been obtained a posteriori from equilibrium. The diagrams refer to $\bar{G}_{LT}/\bar{E}_L\,(L/h)^2$ = 0.5 and could represent a highly anisotropic plate with L/h = 5, and $E_T = E_L/25$, $G_{LT} = E_L/50$, and $\nu_{LT} = \nu_{TT} = 0.25$, e.g. a thick unidirectionally reinforced plate with a high degree of orthotropy. The results of the proposed model virtually coincide with the exact theory. Relative errors on the value of the interfacial tractions are always below 10% but for the smallest interfacial stiffnesses, $K_2 h/\bar{E}_L\,(L/h)^2 < 0.2$, where the error increases to a maximum of 20%. Relative errors on maximum values of shear and normal stresses are very low for all interfacial stiffness values.

Figure 4 shows the transverse displacements normalized to the exact displacements of a fully debonded plate for $\bar{G}_{LT}/\bar{E}_L\,(L/h)^2$ = 4 and 0.5 and the layup (a). The displacements of the proposed model have been derived assuming a shear factor coefficient K^2=5/6 (see [11] for a discussion on this point). The proposed model well reproduces the exact results but for the smaller values of interfacial stiffness. The discrepancy derives from the imposed continuity on the transverse shear tractions at the interfaces: when the interfacial stiffness goes to zero, the model predicts the same behavior in the transverse shear stresses. While vertical equilibrium is satisfied at all sections, and the stress field is properly described by imposing equilibrium conditions a posteriori (Fig. 3) [11], the transverse displacements are underestimated since the shear strains progressively reduce. This effect, which disappear when the dimensionless group $\bar{G}_{LT}/\bar{E}_L\,(L/h)^2$ is large, gives higher discrepancies for lower values of $\bar{G}_{LT}/\bar{E}_L\,(L/h)^2$. When the interfacial stiffness tends to zero, the solution of the proposed model with K^2=5/6 tends to the solution of two superposed Kirchhoff–Love plates free to slide over each other's, while the exact 2D solution is well described by the approximate solution of two superposed Mindlin-Reissner plates. The exact 2D solution however is fully recovered by the results of the proposed model if the shear strains in the plate are incremented by "fictitious" strains associated to the internal resultant which is needed to satisfy equilibrium [11]. This observation substantiates the idea of modifying the shear correction factor K^2 to account for the presence of imperfect interfaces (work is in progress on this problem).

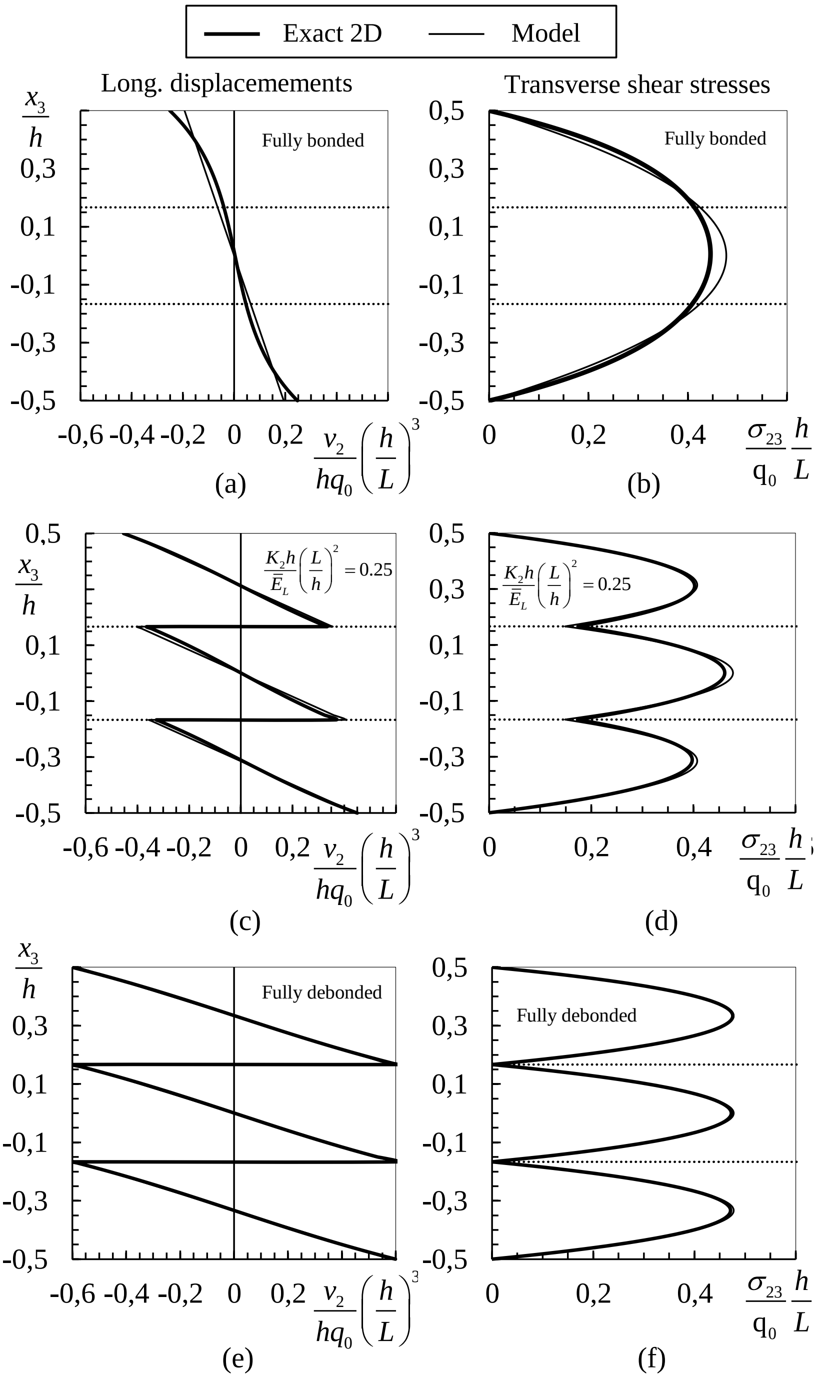

Fig. 3. Dimensionless longitudinal displacements and transverse shear stresses through thickness in the unidirectional three-layer plate (0/0/0) with $G_{LT}/\bar{E}_L\,(L/h)^2 = 0.5$ at $x_2 = 0$. Transverse shear stresses determined from equilibrium. (a,b) Fully bonded, (e,f) fully debonded, (c,d) intermediate bonding with $K_2 h/\bar{E}_L\,(L/h)^2 = 0.25$.

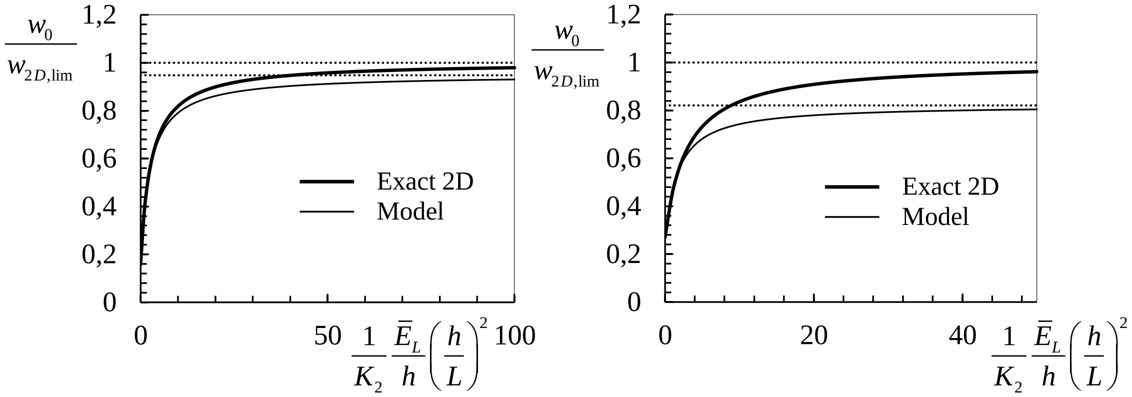

Fig. 4. Transverse displacements normalized to those of a fully debonded plate in the unidirectional three-layer system (0/0/0) with (a) $\bar{G}_{LT}/\bar{E}_L\,(L/h)^2 = 4$ and (b) $\bar{G}_{LT}/\bar{E}_L\,(L/h)^2 = 0.5$ on varying the interfacial stiffness (decreasing interfacial stiffness from left to right).

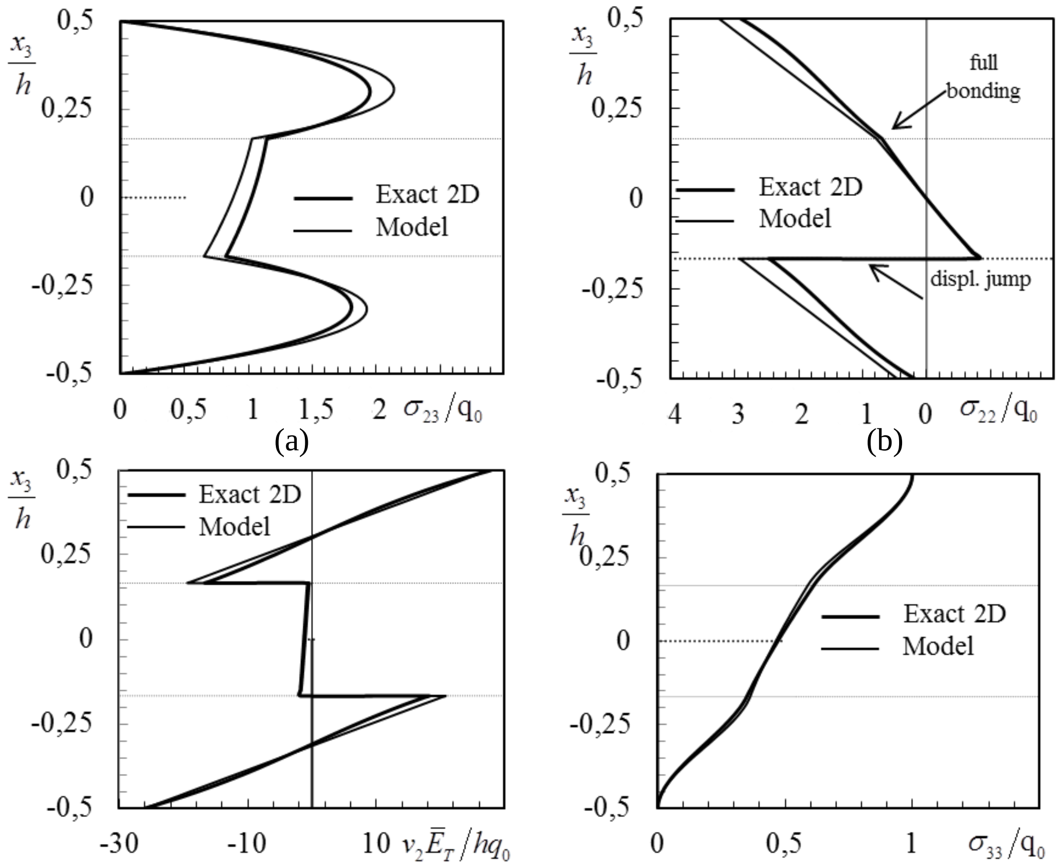

Fig. 5. Transverse shear stresses (a), longitudinal displacements (b), normal stresses and transverse normal stresses through thickness in the three-layer plate (0/90/0), with L/h = 4, at $x_2 = 0$ (a,b) and $x_2 = L/2$ (c,d). Transverse shear/normal stresses determined from equilibrium. Lower interface, intermediate bonding with $B_2\bar{E}_T/h = 4$; upper interface, $B_2\bar{E}_T/h = 4\cdot 10^{-4} \approx 0$ (fully bonded,). Elastic constants: $E_T = E_L/25$, $G_{LT} = E_L/50$, $G_{TT} = E_L/125$ and $\nu_{LT} = \nu_{TT} = 0.25$ [16] (B_2 = interfacial sliding compliance).

The behavior of the three-layer plate (b) is controlled by 4 dimensionless groups, the dimensionless shear stiffnesses, $\bar{G}_{LT}/\bar{E}_L\,(L/h)^2$ and $\bar{G}_{TT}/\bar{E}_L\,(L/h)^2$, transverse stiffness, $\bar{E}_T/\bar{E}_L\,(L/h)^2$, and interfacial stiffness, $K_2 h/\bar{E}_L\,(L/h)^2$. Figures 5 shows longitudinal displacements, transverse shear and normal stresses and longitudinal stresses in the thickness of the plate for the material and stacking sequence studied in the original model by Pagano [16] (perfect interfaces) and in [7] (imperfect interfaces) $E_T = E_L/25$, $G_{LT} = E_L/50$, $G_{TT} = E_L/125$ and $\nu_{LT} = \nu_{TT} = 0.25$. The solutions of the model proposed in this paper are compared with the exact solutions for a plate with L/h = 4, a fully bonded upper interface and an imperfect lower interface with $B_2\bar{E}_T/h = 4$. The proposed model well describes the highly discontinuous displacement field and is able to capture the large variations observed in all stress components.

4 CONCLUSIONS

A novel mechanical model has been formulated to study plates with imperfect and cohesive interfaces and delaminations [11,12]. Dynamic equilibrium equations have been derived for linear non-proportional cohesive traction laws. The equations depend on only six unknown displacement variables, independently of the number of layers and cohesive interfaces which have been used to discretize the system. The model can be applied, following classical discrete-layer approaches, to study plates with generally nonlinear cohesive interfaces. This can be done by approximating the nonlinear cohesive traction laws as piecewise linear functions of the relative displacements at the interfaces. The dynamic equilibrium equations derived here will then describe the generic *ith* branch of the functions. At the meeting, preliminary applications of the theory to some simple fracture mechnics problems will be presented.

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