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COMPUTATION OF THE EFFECTIVE MAGNETOSTRICTIVE COEFFICIENT OF MAGNETO-MECHANICALLY COUPLED COMPOSITES

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Abstract. Magnetostrictive composites are of high interest in the field of magneto-mechanically coupled applications. Such composites can enhance the effective mechanical properties and show advantages over homogeneous magnetostrictive materials. By combining the magnetoactive phase with polymers or metals, the composite is for instance mechanically more flexible, tougher under tensile loading or less brittle.

In this contribution we present a two-scale homogenization procedure for magnetostrictive composites involving a non-linear material behavior with dissipative magnetostriction. One focus is on a direct homogenization procedure which is implemented into the FE^2-method. With this homogenization approach, the effective properties of magnetostrictive composites can be determined under consideration of microscopic material properties. Different volume fractions of the phases and inclusion geometries of the magnetostrictive material influence the macroscopic magnetostriction behavior and are considered by using representative volume elements (RVEs). On the microscale we consider a constitutive rate-dependent model for non-linear magnetostriction. Numerical examples then demonstrate that the proposed formulation is capable of showing the characteristic ferromagnetic and field-induced strain hysteresis curves of magnetostrictive composites.

1 INTRODUCTION

In many technical applications the magnetostrictive material behavior is of high interest and the spectrum of applications could be enlarged by synthesizing composites with improved material properties. This enhancement can be achieved by combining the magnetoactive phase with polymers or metals, such that the composite is for instance mechanically more flexible, tougher under tensile loading or less brittle. However, this macroscopic response is highly dependent on the underlying microscopic properties. In order to explicitly account for the microscopic morphology of such composites, we make use of a scale transition in the framework of the FE^2-method. This homogenization procedure allows for the determination of macroscopic effective properties in consideration of attached representative volume elements ($\mathcal{RVE}$s) in each macroscopic integration point. This computational method is well-established in the context of purely mechanical problems [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. In order to obtain a material response with characteristic magnetization and magnetostriction, we implement a non-linear rate-dependent material model into the FE^2-method ([12]).

The variational theory of *micromagnetics*, that was established by Brown [13, 14] on the basis of earlier developments, e.g. by Landau and Lifschitz [15], is widely accepted as the standard framework for the modeling of magnetoelastic behavior involving crystallographic and magnetic microstructure evolution, see also [16]. Since this modeling approach typically requires the costly resolution of very small time and length-scales in numerical simulations, DeSimone and James [17] developed the *constrained theory of magnetoelasticity*, which simplifies the original micromagnetic theory. In particular, their theory assumes the so-called *large body limit*, in which the exchange energy, that for instance assigns energy to domain walls, is neglected on the macro-scale. The second central assumption of the constrained theory is known as the *high anisotropy limit*, which restricts the stored energy to exclusively consist of combinations of energetic minima w.r.t. the spontaneous strain and magnetization. On the other hand, purely *phenomenological* models have been suggested for the modeling of piezomagnetism and magnetostriction on the macro-scale [18, 19, 20, 21, 22, 12, 8, 23], in analogy to approaches that capture related phenomena in electromechanics. The theoretical framework for the modeling of *nonlinear*, *dissipative magnetostriction* followed in this work is in line with the latter approach. It is therefore assumed that all magnetizable and magnetostrictive phases of the considered composites, can themselves be modeled as macro-level continua, whose effective phase properties can be captured phenomenologically, without requiring a detailed knowledge of the underlying microstructure evolution.

The outline of the paper is as follows. In chapter 2 the theoretical framework for the two-scale homogenization procedure as well as the rate-dependent material model will be described. Chapter 3 contains numerical examples, which show that the model is capable to characterize the macroscopic magnetostrictive material behavior under consideration of microscopic properties. In the last chapter a small summary closes the paper.

2 THEORETICAL FRAMEWORK

In this chapter the theoretical basis of the FE^2-method as well as the constitutive framework for the modeling of magnetostrictive materials will be described. First, we will define the macroscopic as well as the microscopic boundary value problems for the performed scale-transition. Furthermore, the rate-dependent non-linear dissipative material model, for the characterization of the magnetostrictive material taken from [12], will be described.

2.1 Macroscopic boundary value problem

The considered body on the macroscale is denoted as $\mathcal{B} \subset \mathbb{R}^3$ and parameterized in space with the macroscopic coordinates $\overline{\boldsymbol{x}}$. Furthermore, the macroscopic displacement field is defined as $\overline{\boldsymbol{u}}$ and the macroscopic magnetic potential is denoted by $\overline{\phi}$. Depending on theses variables the basic kinematic and magnetic quantities are given by

$$\overline{\boldsymbol{\varepsilon}} = \mathrm{grad}^{sym}_{\overline{\boldsymbol{x}}} \overline{\boldsymbol{u}} \quad \text{and} \quad \overline{\boldsymbol{H}} = -\mathrm{grad}_{\overline{\boldsymbol{x}}} \overline{\phi}_m \,, \tag{1}$$

with the macroscopic symmetric linear strain tensor $\overline{\boldsymbol{\varepsilon}}$ as well as the macroscopic magnetic field vector $\overline{\boldsymbol{H}}$. The gradient operator with respect to the coordinates $\overline{\boldsymbol{x}}$ is denoted by $\mathrm{grad}_{\overline{\boldsymbol{x}}}$. The fundamental balance equations for the quasi-static case are given by the balance of linear momentum and Gauß's law of magnetostatics with

$$\mathrm{div}_{\overline{\boldsymbol{x}}} \overline{\boldsymbol{\sigma}} + \overline{\boldsymbol{f}} = \boldsymbol{0} \quad \text{and} \quad \mathrm{div}_{\overline{\boldsymbol{x}}} \overline{\boldsymbol{B}} = 0 \,, \tag{2}$$

where we introduce the Cauchy stress tensor $\boldsymbol{\sigma}$, the vector of body forces $\boldsymbol{f}$ and the vector of magnetic flux density $\boldsymbol{B}$. Furthermore, $\mathrm{div}_{\overline{\boldsymbol{x}}}$ denotes the divergence operator with respect the macroscopic coordinates.
On the macroscopic level, the mechanical Dirichlet and Neumann boundary conditions can be prescribed in terms of displacement and surface tractions $\overline{\boldsymbol{t}}$ as

$$\overline{\boldsymbol{u}} = \overline{\boldsymbol{u}}_b \quad \text{on } \partial\mathcal{B}_{\overline{u}} \quad \text{and} \quad \overline{\boldsymbol{t}} = \overline{\boldsymbol{\sigma}} \cdot \overline{\boldsymbol{n}} \quad \text{on } \partial\mathcal{B}_{\overline{\sigma}} \,, \tag{3}$$

where $\overline{\boldsymbol{n}}$ is the unit normal vector perpendicular to the body surface. The corresponding magnetic boundary conditions are described in terms of the magnetic potential as well as the magnetic surface charges ζ with

$$\overline{\phi} = \overline{\phi}_b \quad \text{on } \partial\mathcal{B}_{\overline{\phi}} \quad \text{and} \quad -\overline{\zeta} = \overline{\boldsymbol{B}} \cdot \overline{\boldsymbol{n}} \quad \text{on } \partial\mathcal{B}_{\overline{B}} \,. \tag{4}$$

By using the FE^2-method the macroscopic material response depending on the underlying BVP is not a result of a thermodynamic potential based constitutive relation on the macro-level. Instead a representative volume element ($\mathcal{RVE}$) is attached at each macroscopic integration point, which then gives the material response by solving a microscopic boundary value problem. In order to connect the macroscopic magneto-mechanical quantities $\{\overline{\boldsymbol{\varepsilon}}, \overline{\boldsymbol{\sigma}}, \overline{\boldsymbol{H}}, \overline{\boldsymbol{B}}\}$ to their corresponding microscopic quantities $\{\boldsymbol{\varepsilon}, \boldsymbol{\sigma}, \boldsymbol{H}, \boldsymbol{B}\}$, we define the macroscopic variables in terms of suitable surface integrals, see [24]. Volume averages of surface integrals are in the following abbreviated with $\langle \bullet \rangle_{\partial \mathcal{B}} := \frac{1}{V} \int_{\partial \mathcal{B}} \bullet \, da$, for volume integrals we introduce $\langle \bullet \rangle_{\mathcal{B}} := \frac{1}{V} \int_{\mathcal{B}} \bullet \, dv$. The macroscopic strains and stresses are defined as

$$\overline{\boldsymbol{\varepsilon}} = \langle \mathrm{sym}[\boldsymbol{u} \otimes \boldsymbol{n}] \rangle_{\partial RVE} = \langle \boldsymbol{\varepsilon} \rangle_{\mathcal{RVE}} \quad \text{and} \quad \overline{\boldsymbol{\sigma}} = \langle \mathrm{sym}[\boldsymbol{t} \otimes \boldsymbol{x}] \rangle_{\partial \mathcal{RVE}} = \langle \boldsymbol{\sigma} \rangle_{\mathcal{RVE}} \ , \tag{5}$$

with the microscopic displacement and traction vectors $\boldsymbol{u}$ and $\boldsymbol{t}$ as well as the unit outward normal of the $\mathcal{RVE}$ $\boldsymbol{n}$ on the boundary of the $\mathcal{RVE}$. Analogously, the macroscopic magnetic quantities are defined by

$$\overline{\boldsymbol{H}} = \langle -\phi \ \boldsymbol{n} \rangle_{\partial \mathcal{RVE}} = \langle \boldsymbol{H} \rangle_{\mathcal{RVE}} \quad \text{and} \quad \overline{\boldsymbol{B}} = \langle -\zeta \ \boldsymbol{x} \rangle_{\partial \mathcal{RVE}} = \langle \boldsymbol{B} \rangle_{\mathcal{RVE}}, \tag{6}$$

where we introduce the microscopic magnetic potential ϕ and the magnetic surface charges ζ on $\partial\mathcal{RVE}$.

2.2 Microscopic boundary value problem

The considered microscopic volume element is denoted with $\mathcal{RVE} \subset \mathbb{R}^3$ and parameterized in the microscopic coordinates $\boldsymbol{x}$. The strains and magnetic fields are given as

$$\boldsymbol{\varepsilon} = \mathrm{grad}^{sym}_{\boldsymbol{x}} \boldsymbol{u} \quad \text{and} \quad \boldsymbol{H} = -\mathrm{grad}_{\boldsymbol{x}} \phi_m \, , \tag{7}$$

respectively. Neglecting body forces and the density of free charge carriers, the balance of linear momentum as well as Gauß's law of magnetostatics result in

$$\mathrm{div}_{\boldsymbol{x}} \boldsymbol{\sigma} = \boldsymbol{0} \quad \text{in } \mathcal{RVE} \quad \text{and} \quad \mathrm{div}_{\boldsymbol{x}} \boldsymbol{B} = 0 \quad \text{in } \mathcal{RVE} \ , \tag{8}$$

The RVE is driven by boundary conditions that arise as a consequence of macroscopic fields. These boundary conditions have to ensure energetic consistency of the scales and can be derived by exploring a *scale transition*. For this purpose, we first decompose the microscopic fields into constant macroscopic fields $\bar{\bullet}$ and fluctuation parts $\widetilde{\bullet}$ as $\boldsymbol{\xi} = \overline{\boldsymbol{\xi}} + \widetilde{\boldsymbol{\xi}}$ with $\boldsymbol{\xi} := [\boldsymbol{\varepsilon}, \boldsymbol{\sigma}, \boldsymbol{H}, \boldsymbol{B}]$. Energetic consistency is then ensured by postulating a generalized macrohomogeneity condition of the form

$$\overline{\boldsymbol{\sigma}} : \dot{\overline{\boldsymbol{\varepsilon}}} - \overline{\boldsymbol{B}} \cdot \dot{\overline{\boldsymbol{H}}} = \langle \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} - \boldsymbol{B} \cdot \dot{\boldsymbol{H}} \rangle_{\mathcal{RVE}}, \tag{9}$$

in this context we refer to [25]. The above Hill-Mandel-condition can be subdivided into a mechanical and magnetic part as

$$\mathcal{P}_{mech} = \overline{\boldsymbol{\sigma}} : \dot{\overline{\boldsymbol{\varepsilon}}} - \langle \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} \rangle_{\mathcal{RVE}} = 0 \quad \text{and} \quad \mathcal{P}_{mag} = \overline{\boldsymbol{B}} \cdot \dot{\overline{\boldsymbol{H}}} - \langle \boldsymbol{B} : \dot{\boldsymbol{H}} \rangle_{\mathcal{RVE}} = 0 \tag{10}$$

which have to be fulfilled independently. In order to derive appropriate constraint conditions we reformulate equation (10) as

$$\mathcal{P}_{mech} = -\langle (\boldsymbol{\sigma} - \overline{\boldsymbol{\sigma}}) : (\dot{\boldsymbol{\varepsilon}} - \dot{\overline{\boldsymbol{\varepsilon}}}) \rangle_{\mathcal{RVE}} \quad \text{and} \quad \mathcal{P}_{mag} = -\langle (\boldsymbol{B} - \overline{\boldsymbol{B}}) \cdot (\dot{\boldsymbol{H}} - \dot{\overline{\boldsymbol{H}}}) \rangle_{\mathcal{RVE}} \tag{11}$$

The simplest solution that fulfills $\mathcal{P}_{mech} = 0$ and $\mathcal{P}_{mag} = 0$ is given by the assumption of equality of the microscopic and macroscopic quantities in all points of the microscale with $\boldsymbol{\sigma} = \overline{\boldsymbol{\sigma}} =$ const., or $\dot{\boldsymbol{\varepsilon}} = \dot{\overline{\boldsymbol{\varepsilon}}} =$ const., and $\boldsymbol{B} = \overline{\boldsymbol{B}} =$ const., or $\boldsymbol{H} = \overline{\boldsymbol{H}} =$ const., respectively. To obtain more suitable boundary conditions we reformulate the previous expression to

$$\mathcal{P}_{mech} = -\langle (\boldsymbol{t} - \overline{\boldsymbol{\sigma}} \cdot \boldsymbol{n}) : (\dot{\boldsymbol{u}} - \dot{\overline{\boldsymbol{\varepsilon}}} \cdot \boldsymbol{x}) \rangle_{\mathcal{RVE}} \quad \text{and} \quad \mathcal{P}_{mag} = -\langle (\zeta + \overline{\boldsymbol{B}} \cdot \boldsymbol{n})(\dot{\phi} + \dot{\overline{\boldsymbol{H}}} \cdot \boldsymbol{x}) \rangle_{\mathcal{RVE}} \; . \quad (12)$$

For the derivation of the latter equation we made use of the divergence theorem, the balance of linear momentum, the Cauchy theorem $\boldsymbol{t} = \boldsymbol{\sigma} \cdot \boldsymbol{n}$, Gauß's law of magnetostatics and the relation $\zeta = -\boldsymbol{B} \cdot \boldsymbol{n}$. From this, $\mathcal{P}_{mech} = 0$ is fulfilled with the mechanical Neumann and Dirichlet boundary conditions on the $\mathcal{RVE}$

$$\boldsymbol{t} = \overline{\boldsymbol{\sigma}} \cdot \boldsymbol{n} \quad \text{or} \quad \dot{\boldsymbol{u}} = \dot{\overline{\boldsymbol{\varepsilon}}} \cdot \boldsymbol{x} \quad \text{on } \partial\mathcal{RVE} \; . \quad (13)$$

The corresponding magnetic Neumann and Dirichlet boundary conditions which fulfill $\mathcal{P}_{mag} = 0$ are defined as

$$\zeta = -\overline{\boldsymbol{B}} \cdot \boldsymbol{n} \quad \text{or} \quad \dot{\phi} = -\dot{\overline{\boldsymbol{H}}} \cdot \boldsymbol{x} \quad \text{on } \partial\mathcal{RVE} \quad (14)$$

Boundary conditions satisfying the relations $\mathcal{P}_{mech} = 0$ as well as $\mathcal{P}_{mag} = 0$ are the following periodic boundary conditions

$$\widetilde{\boldsymbol{u}}(\boldsymbol{x}^+) = \widetilde{\boldsymbol{u}}(\boldsymbol{x}^-) \; , \; \boldsymbol{t}(\boldsymbol{x}^+) = -\boldsymbol{t}(\boldsymbol{x}^-) \quad \text{and} \quad \widetilde{\varphi}(\boldsymbol{x}^+) = \widetilde{\varphi}(\boldsymbol{x}^-) \; , \; \zeta(\boldsymbol{x}^+) = -\zeta(\boldsymbol{x}^-) \; , \quad (15)$$

where $\boldsymbol{x}^+$ and $\boldsymbol{x}^-$ denote points on opposite faces of a periodic unit cell and $\widetilde{\boldsymbol{u}}$ and $\widetilde{\phi}$ are fluctuations of the mechanical displacements and the magnetic potential.

2.3 Material Model for Nonlinear Dissipative Magnetostriction

For the modeling of the magnetostrictive material on the microscale we use the constitutive model proposed by Miehe, Kiefer and Rosato [12, 8], which will briefly be summarized in this section. For reasons of consistency with the formulation of the direct numerical homogenization framework, the model equations are presented in classical, rather than the original incremental variational, format.

In the considered approach, a reversible, linear piezomagnetic response is assumed to exist at a given state of remanent magnetization $\boldsymbol{M}$. Nonlinearity and dissipation enter the formulation through their direct association with the evolution of this vector-valued internal state variable. More specifically, in analogy to Perzyna-type overstress models in viscoplasticity, the *dissipation function*[1]

$$\phi_\eta(\dot{\boldsymbol{M}}) = \sup_{\mathcal{H}} \left[\mathcal{H} \cdot \dot{\boldsymbol{M}} - \frac{H_c}{\eta(n+1)} \langle f(\mathcal{H}) \rangle^{n+1} \right] \quad (16)$$

[1]Here, $\langle x \rangle := \frac{1}{2}(x + |x|)$ denotes the *ramp function* in Macaulay bracket notation.

is introduced to govern the evolution of the effective remanent magnetization vector. Expression (16) can be interpreted as the approximate penalty-type solution of the nonlinear inequality-constrained maximization problem

$$\phi(\dot{\boldsymbol{M}}) = \sup_{\mathcal{H}\in\mathbb{E}} \left[\mathcal{H} \cdot \dot{\boldsymbol{M}} \right] \ , \tag{17}$$

where $\mathbb{E} := \{\mathcal{H} \,|\, f(\mathcal{H}) := |\mathcal{H}|/H_c - 1 < 0\}$ defines the reversible range in terms of the thermodynamic driving force

$$\mathcal{H} := -\partial_{\boldsymbol{M}} \psi(\boldsymbol{\varepsilon}, \boldsymbol{H}, \boldsymbol{M}) \ , \tag{18}$$

with critical threshold value H_c. In this viscous regularization of the rate-dependent dissipation function, the constants $\eta > 0$ and $n > 0$ are material parameters associated with the viscosity of the magnetostrictive response. For $n < 1$ one obtains the phenomenological characteristics of a nonlinear Norton-Bailey-type creep response. For $\eta \to 0$ the rate-independent case is recovered.

The necessary condition associated with the maximum principle (16) yields the evolution equation

$$\dot{\boldsymbol{M}} = \frac{H_c}{\eta} \left\langle f(\mathcal{H}) \right\rangle^n \partial_{\mathcal{H}} f(\mathcal{H}) = \frac{1}{\eta} \left\langle \frac{|\mathcal{H}|}{H_c} - 1 \right\rangle^n \frac{\mathcal{H}}{|\mathcal{H}|} \ . \tag{19}$$

Based on the implicit Euler integration of (19), in combination with (18), one may define the nonlinear residual expression

$$\boldsymbol{r}_{n+1}(\boldsymbol{v}_{n+1}) := \begin{bmatrix} \mathcal{H}_{n+1} + \partial_{\boldsymbol{M}} \psi_{n+1} \\ \boldsymbol{M}_{n+1} - \boldsymbol{M}_n - \dfrac{\Delta t}{\eta} \left(\dfrac{|\mathcal{H}_{n+1}|}{H_c} - 1 \right)^n \dfrac{\mathcal{H}_{n+1}}{|\mathcal{H}_{n+1}|} \end{bmatrix} = \boldsymbol{0} \ , \tag{20}$$

with $\Delta t := t_{n+1} - t_n$, in terms of the unknowns $\boldsymbol{v}_{n+1} := [\boldsymbol{M}_{n+1}, \mathcal{H}_{n+1}]^T$. Computing the nonlinear and loading history-dependent evolution of the remanent magnetization thus reduces to the nonlinear root-finding problem (20). A summary of all central constitutive relations for the Miehe, Kiefer and Rosato dissipative magnetostriction model is given in Table 1[2]. For more details on the model formulation, the reader is again referred to [12].

A typical set of material parameters adapted for Galfenol is listed in Table 2.

[2]Notational conventions:

i) The generalized dyadic product notation used here is defined through the relations

$$(\boldsymbol{A}\overline{\otimes}\boldsymbol{B}) : \boldsymbol{C} = \boldsymbol{A} \cdot \boldsymbol{C} \cdot \boldsymbol{B}^T \quad \Rightarrow \quad [\boldsymbol{A}\overline{\otimes}\boldsymbol{B}]_{ijkl} = A_{ik} B_{jl}$$
$$(\boldsymbol{A}\underline{\otimes}\boldsymbol{B}) : \boldsymbol{C} = \boldsymbol{A} \cdot \boldsymbol{C}^T \cdot \boldsymbol{B}^T \quad \Rightarrow \quad [\boldsymbol{A}\underline{\otimes}\boldsymbol{B}]_{ijkl} = A_{il} B_{jk} \ .$$

ii) The symmetrization operator for a third-order tensor w.r.t. the first two base vectors is defined as

$$\overset{12}{\text{sym}}[\mathrm{a}] := \frac{1}{2}(\mathrm{a} + \mathrm{a}^T) \quad \Rightarrow \quad [\mathrm{a}]_{ijk} = \frac{1}{2}(a_{ijk} + a_{jik}) \ .$$

Table 1: Constitutive relations for the rate-dependent dissipative magnetostriction model.

1. *Magnetic enthalpy function:*

$$\psi(\boldsymbol{\varepsilon},\boldsymbol{H},\boldsymbol{M}) = \frac{1}{2}(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^r(\boldsymbol{M})):\mathbb{C}:(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^r(\boldsymbol{M})) - (\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^r(\boldsymbol{M})):\frac{|\boldsymbol{M}|}{M_s}\boldsymbol{q}(\boldsymbol{a})\cdot\boldsymbol{H} - \frac{1}{2}\boldsymbol{H}\cdot\boldsymbol{\beta}\cdot\boldsymbol{H} - \boldsymbol{H}\cdot\boldsymbol{M} + \psi_{mag}(\boldsymbol{M})$$

with the *elastic stiffness*, *piezomagnetic coupling*, and *permeability* tensors

$$\mathbb{C} = \lambda\boldsymbol{1}\otimes\boldsymbol{1} + 2\mu\mathbb{I}^{sym}$$
$$\boldsymbol{q} = \overset{12}{\mathrm{sym}}\,[\alpha_0\boldsymbol{a}\otimes\boldsymbol{a}\otimes\boldsymbol{a} + \alpha_\perp\boldsymbol{1}\otimes\boldsymbol{a} + \alpha_=\boldsymbol{a}\otimes\boldsymbol{1}]$$
$$\boldsymbol{\beta} := \mu_m\boldsymbol{1}$$

where $\boldsymbol{a} := \boldsymbol{M}/|\boldsymbol{M}|$, and the *remanent strain* caused by remanent magnetization

$$\boldsymbol{\varepsilon}^r(\boldsymbol{M}) = \frac{3}{2}\varepsilon_s\frac{|\boldsymbol{M}|}{M_s}\,\mathrm{dev}[\boldsymbol{a}\otimes\boldsymbol{a}]$$

2. *Stresses and magnetic induction and driving force:*

$$\boldsymbol{\sigma} = \partial_{\boldsymbol{\varepsilon}}\psi = \mathbb{C}:(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^r(\boldsymbol{M})) - \frac{|\boldsymbol{M}|}{M_s}\boldsymbol{q}(\boldsymbol{a})\cdot\boldsymbol{H}$$
$$-\boldsymbol{B} = \partial_{\boldsymbol{H}}\psi = -\frac{|\boldsymbol{M}|}{M_s}\boldsymbol{q}^T(\boldsymbol{a}):(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^r(\boldsymbol{M})) - \boldsymbol{\beta}\cdot\boldsymbol{H} - \boldsymbol{M}$$
$$-\mathcal{H} := \partial_{\boldsymbol{M}}\psi = -\boldsymbol{\sigma}:\partial_{\boldsymbol{M}}\boldsymbol{\varepsilon}^r(\boldsymbol{a}) - \frac{1}{M_s}\big((\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^r(\boldsymbol{M})):\boldsymbol{q}(\boldsymbol{a})\cdot\boldsymbol{H}\big)\boldsymbol{a} - \frac{1}{M_s}(\mathbb{H}(\boldsymbol{a})\cdot\boldsymbol{H}):(\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}^r(\boldsymbol{M})) - \boldsymbol{H} + \partial_{\boldsymbol{M}}\psi_{mag}$$

The expressions for $\partial_{\boldsymbol{M}}\boldsymbol{\varepsilon}^r(\boldsymbol{a})$ and $\mathbb{H}(\boldsymbol{a})$ take the particular form

$$\partial_{\boldsymbol{M}}\boldsymbol{\varepsilon}^r(\boldsymbol{a}) = \frac{3}{2}\frac{\varepsilon_s}{M_s}\left(-\boldsymbol{a}\otimes\boldsymbol{a}\otimes\boldsymbol{a} + 2\,\overset{12}{\mathrm{sym}}[\boldsymbol{a}\otimes\boldsymbol{1}] - \frac{1}{3}\boldsymbol{1}\otimes\boldsymbol{a}\right)$$
$$\mathbb{H}(\boldsymbol{a}) = \alpha_0\Big(-3\,\boldsymbol{a}\otimes\boldsymbol{a}\otimes\boldsymbol{a}\otimes\boldsymbol{a} + \boldsymbol{1}\,\underline{\otimes}\,(\boldsymbol{a}\otimes\boldsymbol{a}) + (\boldsymbol{a}\otimes\boldsymbol{a})\,\overline{\otimes}\,\boldsymbol{1} + (\boldsymbol{a}\otimes\boldsymbol{a})\otimes\boldsymbol{1}\Big) + \alpha_\perp\Big(\boldsymbol{1}\otimes\boldsymbol{1} - \boldsymbol{1}\otimes\boldsymbol{a}\otimes\boldsymbol{a}\Big) + \alpha_=\Big(\mathbb{I}^{sym} - \overset{12}{\mathrm{sym}}[\boldsymbol{a}\otimes\boldsymbol{1}]\otimes\boldsymbol{a}\Big)$$

and the hysteresis shape is governed by

$$\partial_{\boldsymbol{M}}\psi_{mag} = \partial_{|\boldsymbol{M}|}\psi_{mag}\,\boldsymbol{a} = \frac{1}{2c}\ln\left(\frac{1+\dfrac{|\boldsymbol{M}|}{M_s}}{1-\dfrac{|\boldsymbol{M}|}{M_s}}\right)\boldsymbol{a}$$

Table continued on next page

Table 1: Constitutive relations for the rate-dependent dissipative magnetostriction model.

Table continued from previous page

3. *Update relation for the internal variable and conjugate force:*

   (a) For $|\mathcal{H}_{n+1}| < H_c$ the response is reversible and *piezomagnetic*, and thus

$$\mathcal{H}_{n+1} = \mathcal{H}_n \ , \quad \boldsymbol{M}_{n+1} = \boldsymbol{M}_n$$

   (b) For $|\mathcal{H}_{n+1}| \geq H_c$, the iterative Newton-Raphson solution of

$$\boldsymbol{r}_{n+1}(\boldsymbol{v}_{n+1}) = \begin{bmatrix} \boldsymbol{r}_{1,n+1} \\ \boldsymbol{r}_{2,n+1} \end{bmatrix} := \begin{bmatrix} \mathcal{H}_{n+1} + \partial_{\boldsymbol{M}} \psi_{n+1} \\ \boldsymbol{M}_{n+1} - \boldsymbol{M}_n - \dfrac{\Delta t}{\eta} \left(\dfrac{|\mathcal{H}_{n+1}|}{H_c} - 1 \right)^m \boldsymbol{n}_{n+1} \end{bmatrix} = \boldsymbol{0}$$

   with $\boldsymbol{v}_{n+1} := [\boldsymbol{M}_{n+1}, \mathcal{M}_{n+1}]^T$ and $\boldsymbol{n}_{n+1} := \mathcal{H}_{n+1}/|\mathcal{H}_{n+1}|$, yields the update relation

$$\boldsymbol{v}_{n+1} \Leftarrow \boldsymbol{v}_{n+1} - \boldsymbol{j}_{n+1}^{-1} \boldsymbol{r}_{n+1}$$

   The associated Jacobian $\boldsymbol{j}_{n+1} := \partial_{\boldsymbol{v}} \boldsymbol{r}_{n+1}$ is given by

$$\boldsymbol{j}_{n+1} := \begin{bmatrix} \partial^2_{\boldsymbol{MM}} \psi_{n+1} & \boldsymbol{1} \\ \boldsymbol{1} & \boldsymbol{J}_{22} \end{bmatrix}$$

   where

$$\boldsymbol{J}_{22} := -\frac{n \Delta t}{\eta H_c} \left(\frac{|\mathcal{H}_{n+1}|}{H_c} - 1 \right)^{n-1} \boldsymbol{N}_{n+1} - \frac{\Delta t}{\eta |\mathcal{H}_{n+1}|} \left(\frac{|\mathcal{H}_{n+1}|}{H_c} - 1 \right)^{n} (\boldsymbol{1} - \boldsymbol{N}_{n+1})$$

   with $\boldsymbol{N}_{n+1} := \boldsymbol{n}_{n+1} \otimes \boldsymbol{n}_{n+1}$

4. *Global algorithmic tangent:*

$$\hat{\mathbb{C}}_{n+1} = \partial^2_{\boldsymbol{pp}} \psi_{n+1} - \begin{bmatrix} \partial^2_{\boldsymbol{pM}} \psi_{n+1} \\ \partial^2_{\boldsymbol{p}\mathcal{H}} \psi_{n+1} \end{bmatrix}^T \boldsymbol{j}_{n+1}^{-1} \begin{bmatrix} \partial_{\boldsymbol{p}} \boldsymbol{r}_{1,n+1} \\ \partial_{\boldsymbol{p}} \boldsymbol{r}_{2,n+1} \end{bmatrix}$$

   with the compact notation $\boldsymbol{p} := [\boldsymbol{\varepsilon}, \boldsymbol{H}]^T$

5. *Material Parameters:*

$$\boldsymbol{\kappa} := \{\lambda, \mu, \alpha_0, \alpha_\perp, \alpha_=, \mu_M, H_c, M_s, \varepsilon_s, c, \eta, n\}$$

3 NUMERICAL EXAMPLES

This chapter gives an example for the application of the direct homogenization procedure to two-scale magnetomechanical boundary value problems. More specifically, we consider a two-dimensional macroscopic body consisting of a porous magnetostrictive material (Galfenol) with the material parameters listed in table 2.

Table 2: Material Parameters adapted for $Fe_{0.81}Ga_{0.19}$ (Galfenol) [26].

No.	Param.	Unit	Name	Galfenol
1	λ	N/mm^2	Lamé parameter	$15.0 \cdot 10^3$
2	μ	N/mm^2	Lamé parameter	$10.0 \cdot 10^3$
3	α_0	N/(kA mm)	piezo-magnetic coefficient	73.3
4	$\alpha_\perp$	N/(kA mm)	piezo-magnetic coefficient	180.0
5	$\alpha_=$	N/(kA mm)	piezo-magnetic coefficient	−121.8
6	μ_M	N/kA^2	ferromagnetic permeability	4.5
7	H_c	kA/mm	coercive field strength	0.002
8	M_s	N/(kA mm)	sat. remanent magnetization	1.48
9	ε_s	—	saturation remanent strain	0.0245%
10	c	kA^2/N	hysteresis shape parameter	$0.3 \cdot 10^3$
11	η	(kA mm)/(N s)	viscosity coefficient	10^{-6}
12	n	—	viscosity exponent	2

The boundary value problem is characterized by an applied magnetic potential on the left and right boundary, where the magnetic potential on the right boundary is time dependent with a loading frequency of 0.25 Hz, see Figure 1.

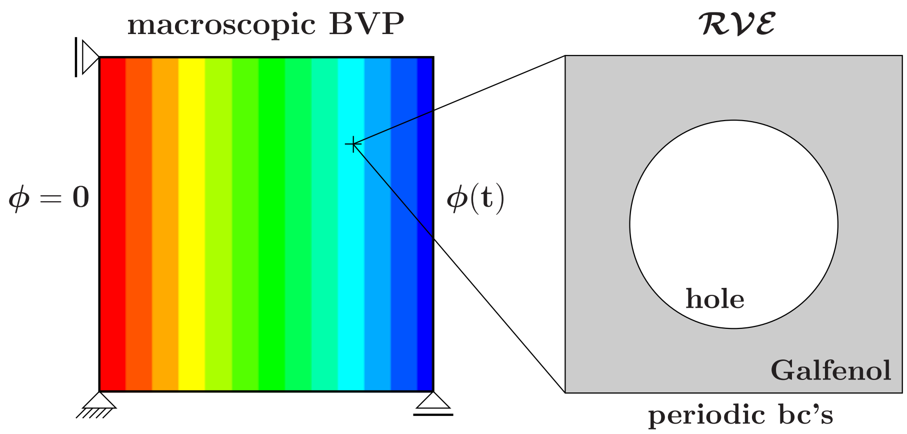
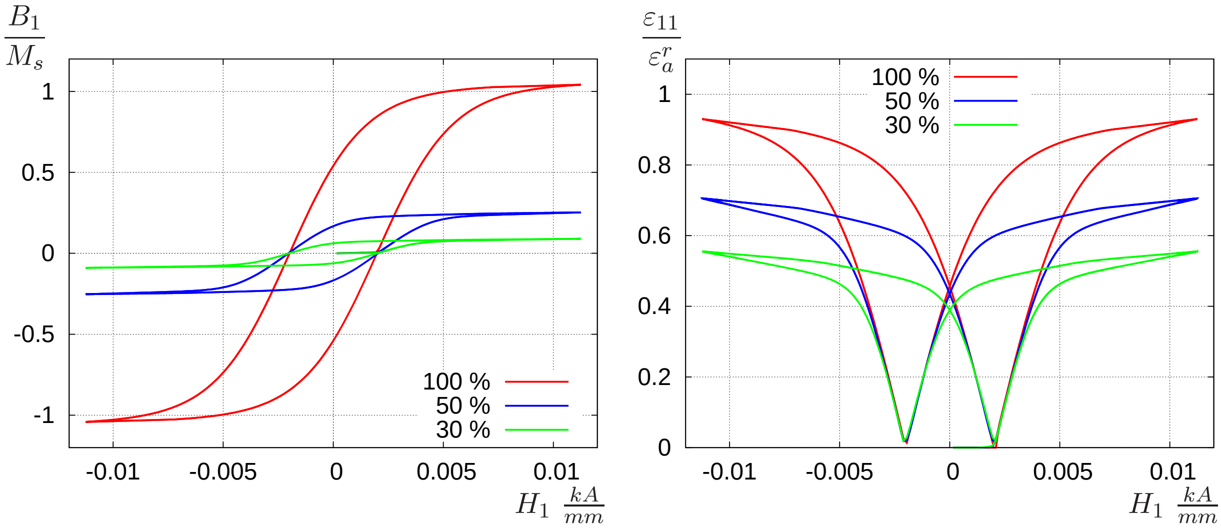


Figure 1: Macroscopic magnetomechanical boundary value problem with attached $\mathcal{RVE}$.

In order to obtain the macroscopic material response a two-dimensional $\mathcal{RVE}$ is attached at each macroscopic integration point. In the following the results of three different $\mathcal{RVE}$s are shown, which consist of a magnetostrictive matrix with a centered circular hole. They differ in the volume fraction of the magnetostrictive matrix phase. To obtain a reference solution of the pure magnetostrictive material response we first consider an $\mathcal{RVE}$ without porosity. Further $\mathcal{RVE}$s consist of 50% as well as 30% volume fraction of the matrix material. The determined effective magnetization and strain hysteresis curves are shown in Figure 3.



It is observed, that the maximum value of magnetic flux density as well as the amount of magnetostriction decrease due to the lower volume fraction of the magnetic material Galfenol.

4 CONCLUSIONS

In the present contribution, we have presented a framework for the two-scale homogenization of nonlinear, dissipative magneto-mechanically coupled boundary value problems. The transition between the micro- and macroscale was implemented into the FE^2-method, which allows for the determination of effective macroscopic properties in consideration of microscopic representative volume elements. For the simulation of the magnetostrictive material with its characteristic hysteresis curves, we used a non-linear material model with dissipative magnetostriction. In the numerical examples we attached two-dimensional microstructures at each macroscopic integration points with different volume fractions of the magnetostrictive matrix material. Determined effective properties showed that the model is capable of describing the typical hysteresis behavior of the magnetic flux density and the magnetostrictive strain curve. In the future work, this material model can be used for the determination of effective magneto-electric coupling coefficients. Due to the non-linear material behavior in synthesized composites, the implemented model can significantly enhance the simulation of two-phase composites and can decrease the deviation between experimentally measured and numerically simulated effective magneto-electric properties.

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