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DISCRETE AND CONTINUOUS MODELS FOR THE IN PLANE MODAL ANALYSIS OF MASONRY STRUCTURES

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Abstract. A modal analysis, developed in plane dynamics and linear elasticity, for periodic masonry structure is presented and validated both by means of a continuum modelling within the frame of the micropolar continuum theory and of a discrete model (DEM) within the frame of a molecular dynamic algorithm.
For running-bond masonry brickwork numerical micropolar models already exist [1,2] in static frameworks [2] and in dynamic frameworks [3,4].
Here the aim is twofold: i) a multi-scale modal analysis both at Representative Elementary Volume (REV) level -micro-scale- and at masonry panel level -macro-scale-; ii) a multi-model analysis both with continuum micro-structured and discrete models such as to evaluate sensitivity to masonry local microstructure and sensitivity to characteristic length of REV by reference to masonry panel size.
Two models are presented and compared. A discrete element model and a continuous micropolar model based on analytical homogenization procedures. Both models are based on the following assumptions: i) the structure is composed of rigid blocks; ii) the mortar joints are modelled as interfaces. The rigid block hypothesis is particularly suitable for historical masonry, in which stone blocks may be assumed as rigid bodies. Continuum homogenized model provides, in an analytical form, constitutive equivalent elastic functions, mass and inertia; discrete model describes masonry as a rigid skeleton such as to evaluate both its global and local behaviour.
A parametric analysis is carried out to investigate the effect of i) masonry texture (running versus header bond); ii) size of heterogeneity (block dimensions) respect panel dimensions. Modal analysis is hence carried on for a REV and different panels. Focus is on the sensitivity to heterogeneity size such as to verify models reliability and applicability field.

1 INTRODUCTION

Masonry is a composite material formed by bricks and mortar arranged more or less regularly and adopted for many centuries as structural material. Dynamic actions may represent the major risk of collapse of brickworks and, despite the progress achieved so far in science and mechanics, the assessment of their seismic performance remains a challenging task. Generally, masonry buildings may fail under dynamic actions following two different mechanisms: in-plane and out-of plane. The first one is characterized by shear deformations and fissures, while the second one may cause the tilting (or toppling) of entire portions of wall. Then, reliable physical and numerical models are worth of recommendation.

Particularity of masonry is that the size of heterogeneity (size of block) is not negligible with respect to the global size of structural element as in several composite materials. For this reason, in the last twenty years, several researchers developed models for the study of masonry or masonry-like materials adopting different approaches. A heterogeneous FE model may be adopted for modelling small single panels, but it is impracticable in a real scale context for computational limits. DEM is suitable for studying panels of consistent size and also for assemblage of panels, but it is limited by hypothesis of rigid blocks and mortar joint modelled as interfaces [5], that may be adopted for modelling historical masonry. Continuous equivalent models were proposed based on homogenization-identification procedures. Standard Cauchy continuous models were obtained applying periodic homogenization techniques and considering the elastic behaviour of both brick and mortar [5,6,7]. On the other hand, micropolar or higher order continua were also adopted for studying masonry in static frameworks [3,8,9]. These models were also extended to dynamic fields [3,4].

In the present work, in-plane modal analysis of masonry panels is carried on by adopting and comparing four models: i) DEM, ii) heterogeneous FEM, iii) continuous Cauchy model and iv) continuous micropolar model. Continuous models are based on analytical identification procedures. All models are based on rigid blocks hypothesis, whereas DEM and continuous models are both based on mortar joints modelled as elastic interfaces.

Modal analyses are carried out for investigating the effect of i) size of heterogeneity with respect to panel dimensions, ii) block dimension ratio. Furthermore, the opportunity to adopt a micropolar continuum model instead of a Cauchy one for the modal analysis of masonry panels is investigated. The heterogeneous FEM is adopted for verifying all models reliability and applicability field, together with results obtained by Brasile and Casciaro [10], assumed as benchmark.

2 DISCRETE MODEL

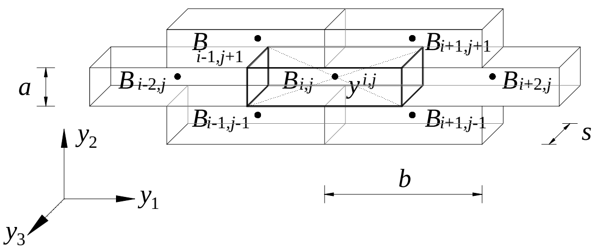

Figure 1: Masonry structure with running bond pattern.

In the present work, one leaf masonry panels having a running bond periodic masonry pattern are considered. Block dimensions are: *a* (height), *b* (width) and *s* (thickness). Assuming rigid block hypothesis, the displacement of the generic block $B_{i,j}$ (Fig. 1) is a rigid motion referred to the displacement of its centre $\mathbf{y}^{i,j}$ [5]:

$$\mathbf{u}^{i,j}(\mathbf{y}) = \mathbf{u}^{i,j} + \mathbf{\Omega}^{i,j}(\mathbf{y} - \mathbf{y}^{i,j}) \tag{1}$$

where $\mathbf{u}^{i,j} = \{u_1^{i,j}, u_2^{i,j}\}^T$ is the translation vector and $\mathbf{\Omega}^{i,j}$ is the rotation tensor, having one component $\omega_3^{i,j}$, of the block $B_{i,j}$. Considering the regularity of the masonry pattern (Fig. 1), the generic block $B_{i,j}$ is surrounded by six blocks $B_{i+k_1,j+k_2}$ by means of six interfaces Σ_{k_1,k_2}. If k_1 and k_2 are equal to ±1, Σ_{k_1,k_2} is an horizontal interface; whereas with k_1 = ±2 and k_2 = 0, Σ_{k_1,k_2} is a vertical interface. The interactions between the blocks through the interfaces are represented by elastic forces $f_1^{k_1,k_2}$, $f_2^{k_1,k_2}$ and couple $c_3^{k_1,k_2}$ that depend on $\Delta_1^{k_1,k_2}, \Delta_2^{k_1,k_2}$ defined as follows:

$$\begin{aligned} \Delta_1^{k_1,k_2} &= u_1^{i+k_1,j+k_2} - u_1^{i,j} + k_2 a(\omega_3^{i+k_1,j+k_2} + \omega_3^{i,j})/2 \\ \Delta_2^{k_1,k_2} &= u_2^{i+k_1,j+k_2} - u_2^{i,j} - k_1 b(\omega_3^{i+k_1,j+k_2} + \omega_3^{i,j})/4 \end{aligned} \tag{2}$$

and on relative rotation $\delta_3^{k_1,k_2}$ between two neighbouring blocks. Elastic forces and relative displacements related to the generic interface Σ_{k_1,k_2} may be collected in the following vectors:

$$\begin{aligned} \mathbf{f} &= \{ f_1^{k_1,k_2} \quad f_2^{k_1,k_2} \quad c_3^{k_1,k_2} \}^T \\ \mathbf{d} &= \{ \Delta_1^{k_1,k_2} \quad \Delta_2^{k_1,k_2} \quad \delta_3^{k_1,k_2} \}^T \end{aligned} \tag{3}$$

It is worth noting that vector **d** may be expressed in terms of the translation vectors **u** and the rotations ω_3 of the blocks in contact along the interface Σ_{k_1,k_2}. The constitutive relation that defines interaction between block $B_{i,j}$ and $B_{i+k_1,j+k_2}$ is $\mathbf{f} = \mathbf{K}\,\mathbf{d}$. The interfacial stiffness matrix is $\mathbf{K} = (1/e)[\mu\mathbf{I} + (\mu+\lambda)\mathbf{e}_\perp \otimes \mathbf{e}_\perp]$, where $\mathbf{e}_\perp$ is the versor orthogonal to plane of the interface, $e = e_v$ or e_h is the actual thickness of vertical or horizontal mortar joint and μ, λ are the Lamé's constants of the mortar. Hence, **K** can assume two different forms, for horizontal and vertical interfaces, respectively. In the following, instead of Lamé's constants, the bulk and the shear moduli (K and G, respectively) of mortar are adopted; moreover, horizontal and vertical mortar joints are assumed to have the same thickness ($e^v = e^h = e$) and the same elastic properties ($K_v = K_h = K$, $G_v = G_h = G$) that depend on mortar elastic modulus E^M and Poisson's ratio ν^M:

$$\begin{aligned} K &= E^M / [(1+\nu^M)(1-2\nu^M)] \\ G &= E^M / [2(1+\nu^M)] \end{aligned} \tag{4}$$

Further details of the discrete model may be found in [5,11,12].

2.1 Elastic energy and stiffness matrix

The elastic energy expended by the generic interface is given as follows:

$$\Pi^{k_1,k_2} = \frac{1}{2}\int_{\Sigma_{k_1,k_2}} \mathbf{f}^T\, \mathbf{d}\; dA = \frac{1}{2}\int_{\Sigma_{k_1,k_2}} \mathbf{d}^T \mathbf{K}\, \mathbf{d}\; dA = \frac{1}{2}\mathbf{d}^T \bar{\mathbf{K}}\, \mathbf{d} \tag{5}$$

where the stiffness matrix $\bar{\mathbf{K}}$ of the generic interface, accordingly to **K**, may assume two different forms for horizontal and vertical interfaces and it may be expressed in terms of the translation and rotations components of the blocks in contact along the interface Σ_{k_1,k_2}. The stiffness matrix of the entire panel is obtained by assembling the stiffness matrices of all the interfaces.

2.2 Kinetic energy and mass matrix

Differently than the elastic energy, the kinetic energy of the discrete system is not defined at the interfacial level, but it involves directly the global displacements referred to the position $\mathbf{y}^{i,j}$ of the centre of each block and it is given by the sum of the kinetic energy of each block:

$$\Pi_{kin} = \sum \Pi_{kin}^{i,j} = \sum \frac{1}{2}\{m[(\dot{u}_1^{i,j})^2 + (\dot{u}_2^{i,j})^2] + J_3(\dot{\omega}_3^{i,j})^2\,]\} \tag{6}$$

where $m = \rho \times (a \cdot b \cdot s)$ is the mass of the block, ρ is its density, $J_3 = m(b^2 + a^2)/12$ is its polar inertia respect to y_3 axis and $\dot{u} = du/dt$. Mass and polar inertia may be corrected by taking into account the mortar joint thickness, especially if it is not negligible with respect to block plane dimensions *a*,*b*; then *m* and J_3 may be substituted by $m^* = \rho \times [(a + e_v)\cdot(b + e_h)\cdot s]$ and $J_3{*} = m{*}[(b + e_h)^2 + (a + e_v)^2]/12$. The mass matrix $\bar{\mathbf{M}}$ of the generic block may be highlighted into the expression of the kinetic energy:

$$\Pi_{kin}^{i,j} = \frac{1}{2}\{\dot{u}_1^{i,j} \quad \dot{u}_2^{i,j} \quad \dot{\omega}_3^{i,j}\} \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J_3 \end{bmatrix} \begin{Bmatrix} \dot{u}_1^{i,j} \\ \dot{u}_2^{i,j} \\ \dot{\omega}_3^{i,j} \end{Bmatrix} = \frac{1}{2}(\dot{\mathbf{u}}^{i,j})^T \bar{\mathbf{M}}\, \dot{\mathbf{u}}^{i,j} \tag{7}$$

and the mass matrix of the entire panel is obtained by assembling the mass matrices over the panel.

3 PLANE CONTINUUM MODEL

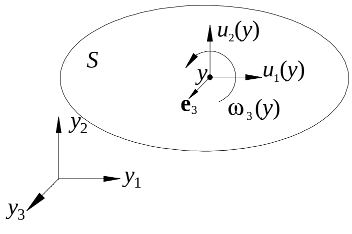

Figure 2: 2D continuum model.

A micropolar plane 2D model is defined, it is identified by *S* middle plane of body in a Euclidean coordinate system (y_1, y_2) of normal versor $\mathbf{e}_3$ along the y_3 coordinate direction

(Fig. 2). The kinematic descriptors of a generic point belonging to the 2D continuum are represented by the fields **u**(**y**), **Ω**(**y**), that are respectively the translation vector and rotation tensor of the generic point **y**. Translation **u**(**y**) has two components, $u_1(y_1, y_2), u_2(y_1, y_2)$, whereas the rotation **Ω**(**y**) is a skew tensor with one component defined as follows:

$$\mathbf{\Omega}(\mathbf{y}) = \begin{bmatrix} 0 & -\omega_3(\mathbf{y}) \\ \omega_3(\mathbf{y}) & 0 \end{bmatrix} \tag{8}$$

The generic displacements are described by the fields

$$\mathbf{u} : \mathrm{S} \rightarrow V; \;\; \mathbf{\Omega} : \mathrm{S} \rightarrow \mathrm{Skw}V \tag{9}$$

that completely describe the translation and the rotation of all points belonging to *S*. Following the notation of [13], the static counterpart is fully described by the field **N**, collecting the in-plane actions, and by the field **C**, representing the microcouple:

$$\mathbf{N} : \mathrm{S} \rightarrow V; \;\; \mathbf{C} : \mathrm{S} \rightarrow \mathrm{Skw}V \tag{10}$$

The balance equations for the in plane case, including body forces and couples (**b**,**B**) and inertial forces and moments $(\rho^{2D}\,\ddot{\mathbf{u}}, J_3^{2D}\,\ddot{\mathbf{\Omega}})$ are:

$$\begin{gathered} div\mathbf{N} + \mathbf{b} = \rho^{2D}\,\ddot{\mathbf{u}} \\ div\mathbf{C} - 2\mathrm{Skw}\mathbf{N} + \mathbf{B} = J_3^{2D}\,\ddot{\mathbf{\Omega}} \end{gathered} \tag{11}$$

where div is an operator defined on *S*, ρ^{2D} is the mass density of the equivalent continuum, J_3^{2D} is its polar inertia and $\ddot{\mathbf{u}} = d^2\mathbf{u}/dt^2$. For the continuum, set **N** and **C** actions, the infinitesimal potential energy and the kinetic energy on *S* may be written as:

$$\begin{gathered} \mathrm{d}\Pi = 1/2\,[\mathbf{N}\cdot(\mathrm{grad}\,\mathbf{u} + \mathbf{\Omega}) + \mathbf{C}\cdot(\mathrm{grad}\mathbf{\Omega})] \\ \mathrm{d}\Pi_{kin} = 1/2\,[\rho^{2D}\;\dot{\mathbf{u}}^T\dot{\mathbf{u}} + J_3^{2D}\;\dot{\omega}_3\;\dot{\omega}_3] \end{gathered} \tag{12}$$

where grad represents the gradient operator on *S*. If the adopted continuum follows Cauchy’s hypotheses, the in-plane couple is assumed equal to zero [2,8]. The corresponding infinitesimal potential energy and kinetic energy are given by:

$$\begin{gathered} \mathrm{d}\Pi^{\mathrm{C}} = 1/2\,[\mathbf{N}\cdot sym(\mathrm{grad}\,\mathbf{u})] \\ \mathrm{d}\Pi^{\mathrm{C}}_{kin} = 1/2\,[\rho^{2D}\;\dot{\mathbf{u}}^T\dot{\mathbf{u}}] \end{gathered} \tag{13}$$

4 MODEL FOR RIGID BLOCKS CONNECTED BY ELASTIC INTERFACES

A compatible identification procedure is carried on [11,14], assuming that the translation and rotation of the center of the block $B_{i,j}$ are equal to the same quantities in the center **y** chosen in the continuum model: $\mathbf{u}^{i,j}(\mathbf{y}^{i,j}) = \mathbf{u}(\mathbf{y}), \mathbf{\Omega}^{i,j}(\mathbf{y}^{i,j}) = \mathbf{\Omega}(\mathbf{y})$. For a chosen Representative Elementary Volume (REV) and a given class of regular displacements, the energy expended by contact actions at the interfaces and the kinetic energy in the discrete model (Eqs. 5,7) are imposed to be coincident to those in the continuous model (Eqs. 12). Assuming the geometry of the discrete system showed in Fig. 1, in the present work the REV chosen is characterized by four horizontal interfaces and a vertical interface at the centre of the Cartesian coordinate system (Fig. 3).

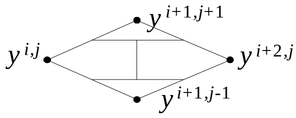

Figure 3: Representative Elementary Volume (REV) considered for the identification procedure.

Following [11], upper bounds for the strain energy of the equivalent medium may be obtained using a suitable kinematic field over the REV. Assuming **E** as the macroscopic in plane strain tensor in the equivalent medium, the continuum equivalent in plane tensor **A** and micro-couple tensor **L** are obtained by solving the following minimization problem:

$$1/2 \cdot \mathbf{E} \cdot [\mathbf{A} \cdot \mathbf{E}] + 1/2 \cdot \mathrm{grad}\,\boldsymbol{\Omega} \cdot [\mathbf{L} \cdot \mathrm{grad}\,\boldsymbol{\Omega}] = \min_{(\mathbf{U},\boldsymbol{\Omega} \in \mathrm{KC}(\mathbf{E},\mathrm{grad}\,\boldsymbol{\Omega}))} \zeta \tag{14}$$

where ζ is the strain energy averaged over the REV, **U** and **Ω** represent any strain-periodic rigid body displacement of the blocks, kinematically compatible with **E**. The set KC of **E**-kinematically compatible [**U**,**Ω**] is introduced:

$$\mathrm{KC}(\mathbf{E}, \mathrm{grad}\,\boldsymbol{\Omega}) = \{[\mathbf{U}, \boldsymbol{\Omega}], \mathbf{u}^{i,j} = \mathbf{E} \cdot \mathbf{y}^{i,j} + \mathbf{v}^{i,j}, \boldsymbol{\Omega}^{i,j} = \boldsymbol{\omega}^{i,j}, [\mathbf{v}, \boldsymbol{\omega}] \in L^2\} \tag{15}$$

where [**v**,**ω**] is the in-plane rigid displacement of block $B_{i,j}$ -respectively translation and rotation. Hence, uniform boundary displacement and rotation are applied to the REV.

The approximate equivalent elastic coefficients, denoted by A_{ijkl} and L_{ij} and the equivalent density and polar inertia must satisfy:

$$\begin{gathered} 1/2 \cdot A_{1111}(E_{11})^2 + 1/2 \cdot A_{2222}(E_{22})^2 + 1/2 \cdot A_{1212}(E_{12})^2 + 1/2 \cdot A_{2121}(E_{12})^2 + L_{13}\omega_{3,1}^2 + L_{23}\omega_{3,2}^2 \leq \zeta \\ 1/2 \cdot \rho^{2D}\,(\dot{u}_1^2 + \dot{u}_2^2) + J_3^{2D}\dot{\omega}_3^2 \leq \xi \end{gathered} \tag{16}$$

where $E_{12} = u_{1,2} + \omega_3$, $E_{21} = u_{2,1} - \omega_3$ and ξ is the kinetic energy averaged over the REV.

Following the procedure proposed above, it is possible to define the equivalent micropolar continuum. As well known, for the Cauchy continuum, Eqs. 16 become:

$$\begin{gathered} 1/2 \cdot A_{1111}(E_{11})^2 + 1/2 \cdot A_{2222}(E_{22})^2 + 2A_{1212}^{C}(E_{12}^{C})^2 \leq \zeta \\ 1/2 \cdot \rho^{2D}\,(\dot{u}_1^2 + \dot{u}_2^2) \leq \xi \end{gathered} \tag{17}$$

where $E_{12}^{C} = E_{21}^{C} = 1/2 \cdot (u_{1,2} + u_{2,1})$. The constitutive functions of the components of **N** and $\mathbf{C} \cdot \mathbf{e}$ on *S* plane are the same obtained in [14] adopting a discrete model and following the same compatible identification procedure.

$$\begin{gathered} \mathbf{N} = \mathbf{A} \cdot (grad\ \mathbf{u} + \boldsymbol{\Omega}) \\ \mathbf{C} \cdot \mathbf{e} = \mathbf{L} \cdot grad(\boldsymbol{\Omega}) \end{gathered} \tag{18}$$

The components of matrix **A** are given by the following expressions.

$$A_{1111} = [4K_v(e^h / a) + (b/a)G_h\ (e^v / a)] / [4(e^h / a)(e^v / b)] \tag{19}$$

$$A_{2222} = K_h / (e^h / a)$$
$$A_{1122} = 0$$
$$A_{1212} = G_h (a / e^h)$$
$$A_{2121} = K_h b^2 / (4a e^h) + G_v (b / e^v)$$

The equivalent moduli for the micropolar continuum are given by Eqs. 19, moreover it can be demonstrated that $A^{C}_{1212} = (A_{1212} \cdot A_{2121}) / (A_{1212} + A_{2121})$. The components of diagonal matrix **L** are given by:

$$L_{13} = (b^2 / 192)[16K_v a^2 / (b e^v) + K_h b^2 / (a e^h) + 12G_h a / e^h] \qquad (19)$$
$$L_{23} = b^2[K_h a / (48e^h)]$$

In the same manner, the identification of kinetic energy provides [3,15]:

$$\rho^{2D} = m / (a \cdot b \cdot s) = \rho \qquad (20)$$
$$J_3^{2D} = J_3 / (a \cdot b \cdot s) = \rho (a^2 + b^2) / 12$$

4.1 Finite Element formulation

In order to perform modal analysis of masonry panels represented by a micropolar continuum, an enriched triangular FE in plane stress state is adopted. Such FE may be obtained by simply adding a rotational degree of freedom to each node of the element and by discretizing rotations ω_3 into the element with the linear polynomials commonly adopted in standard triangular elements. This FE was studied in the past in [16], where linear analysis was presented and several patch tests were proposed. In the present work the stiffness matrix of the triangular element is equal to the one proposed in [16] and the elastic matrix is composed by matrices **A** and **L**. The mass matrix of the element is given by:

$$\mathbf{M}^{el} = s \int_T \mathbf{N}^{*T} \mathbf{I}_M \mathbf{N}^* \, dA \qquad (23)$$

Where $\mathbf{N}^*$ is the matrix of the shape functions, T is the area of the generic triangular element and $\mathbf{I}_M = \text{diag}\{\rho \quad \rho \quad J_3^{2D}\}$.

5 NUMERICAL EXAMPLES

A numerical campaign is carried on for evaluating the performances of both discrete and micropolar models, the latter adopted for the constitutive functions of the enriched FEM, in determining the natural frequencies and the corresponding modal shapes of masonry-like panels with different restraint conditions, varying block dimension ratio $\varepsilon = b/a$ and varying the size of heterogeneity. For the latter purpose, the scale factor $r = L/b = H/a$ is introduced in order to perform modal analysis by increasing the number of blocks along both plane directions, maintaining fixed panel overall dimensions (L,H) and block dimension ratio. The same scale factor was adopted in [14] for evaluating the effects of size of heterogeneity on linear elastic analysis of masonry panels.

Moreover, another purpose of this campaign consists in evaluating the opportunity of adopting a micropolar continuum instead of a traditional Cauchy continuum for determining in-plane natural frequencies and modal shapes of masonry-like panels. In order to validate the proposed models (discrete and micropolar continuum) the work of Brasile and Casciaro [10] is taken as benchmark solution for first; furthermore, a heterogeneous FEM is adopted in order to have reference solutions. The following list resumes the models considered:

1) Discrete Element Model (DEM);
2) Micropolar FE model (FEM microp.);
3) Benchmark solution (Brasile and Casciaro [10]);
4) Cauchy FE model (FEM Cauchy);
5) Heterogeneous FE model (FEM Hetero).

The heterogeneous FEM (5) is represented by a standard 2D FEM where quadrilateral elements in plane stress state are used and mortar joints and elastic blocks are distinguished by adopting different elastic parameters. In particular, the elastic modulus of the blocks is assumed 10^4 times larger than that of mortar in order to simulate the infinite rigidity of blocks with respect to mortar joints. A quite rough mesh refinement (Fig. 4a) is adopted in order to avoid a huge number of degrees of freedom involved in the analysis of panels with a large number of blocks. Micropolar (2) and Cauchy (4) FE models are defined by subdividing panel length and height into $n_{el,1}$ and $n_{el,2}$ subdivisions, respectively, and dividing each rectangle by its diagonals (Fig. 4b); then 4 $n_{el,1} \times n_{el,2}$ triangular elements are defined in order to obtain a symmetric mesh.

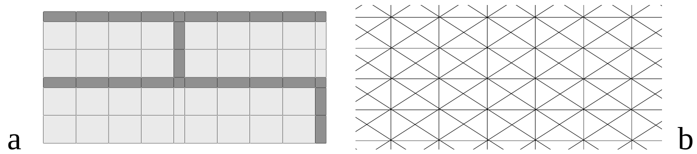

Figure 4: (a) detail of the heterogeneous FEM, (b) detail of micropolar and Cauchy FEMs.

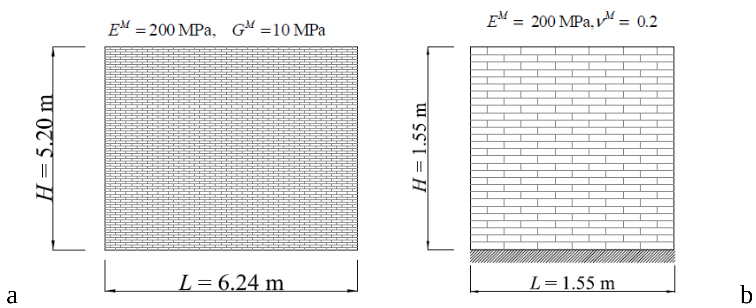

Figure 5: (a) rectangular panel studied by Brasile and Casciaro [10], (b) square panel having $r = 6$.

5.1 Rectangular panel

The masonry panel defined in [10] is considered (Fig. 5a). Modal analysis is performed for both free panel and panel with fixed base. Fig. 6 shows the first four eigenpairs of the free panel obtained with DEM (first row) and micropolar FEM (second row), whereas Fig. 7 shows the same results referred to the panel with fixed base. As it is shown in Tab. 1, vibration periods given by DEM -err. (1)- are in very good agreement with benchmark results -T (3)-, if the panel is free, whereas differences are larger for the panel with fixed base due to the different block boundary conditions adopted by the models (the model in [10] adopts boundary conditions along joints, whereas the present DEM has boundary conditions at block centres). The micropolar FEM -err (2)- appears to be quite more far from benchmark model with respect to the DEM, however differences are acceptable.

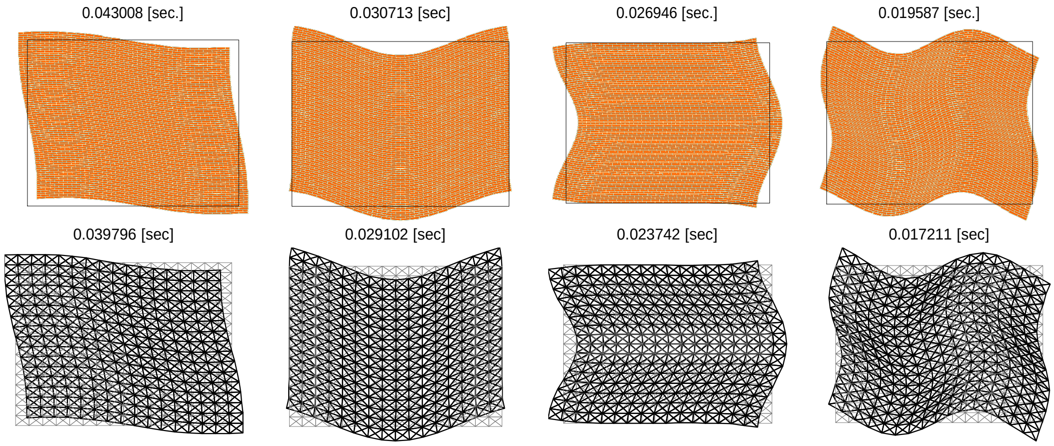

Figure 6: First four eigenparis of a free rectangular panel, DEM (first row), micropolar FEM (second row).

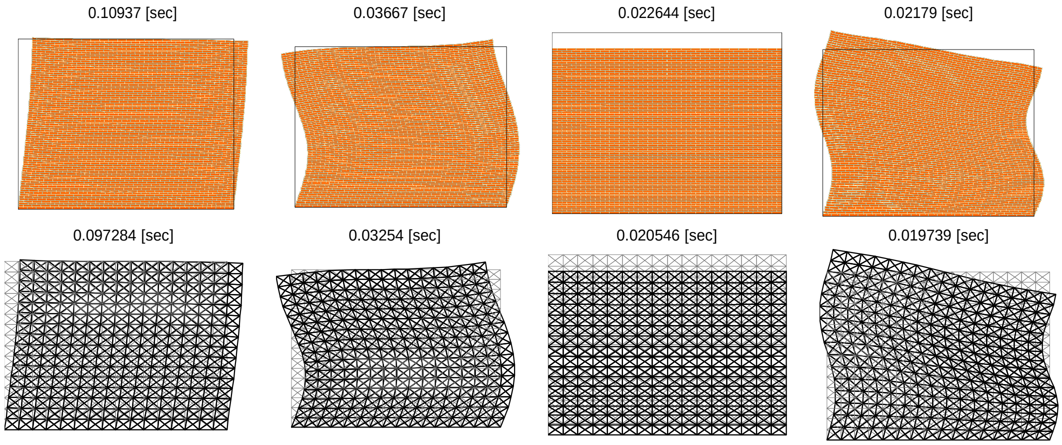

Figure 7: First four eigenparis of a rectangular panel with fixed base, DEM (first row), micropolar FEM (second row).

Table 1: Vibration periods of the rectangular panel [10], comparison with DEM (1) and micropolar FEM (2).

	free panel			panel with fixed base		
mode	T (3) [sec]	err.(1) [%]	err.(2) [%]	T (3) [sec]	err.(1) [%]	err.(2) [%]
1	0.0425	1.20	-7.47	0.1183	-7.55	-11.05
2	0.0302	1.70	-5.25	0.0395	-7.16	-11.26
3	0.0267	0.92	-11.89	0.0230	-1.55	-9.26
4	0.0192	2.02	-12.14	0.0226	-3.58	-9.41

5.2 Square panel – sensitivity to size of heterogeneity

A square panel made by UNI bricks (b = 250 mm, s = 120 mm, a = 55 mm) with standard mortar joint thickness ($e_v = e_h = e$ = 10 mm) is considered. Mass density assumed for blocks is ρ = 1800 kg/m^3. The panel is fixed at the base and its dimensions are $L = H$ = 1550 mm. Fig. 5b shows the case characterized by r = 6 and Fig. 7 shows the first three eigenpairs obtained with the DEM. First and second mode shapes represent a flexural deformation, whereas the second one is a pure vertical deformation. Figs. 8 show separately the first three vibration periods of the panel obtained with different models and increasing r. As expected, results obtained with the Cauchy model are not influenced by the scale factor, whereas results given by other models converge to those given by the Cauchy one. The behaviour of micropolar model converge to that of Cauchy model faster with respect to the DEM if flexural mode shapes are considered, whereas the second mode shape, a simple vertical deformation, is not influenced by r. The heterogeneous FEM appear to be more rigid than other models due to the rough mesh refinement adopted, anyway for this reason, differences are acceptable.

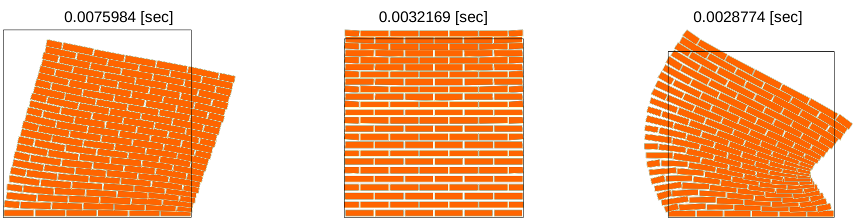

Figure 7: First three eigenparis of a square panel with fixed base and r = 6 modelled by DEM.

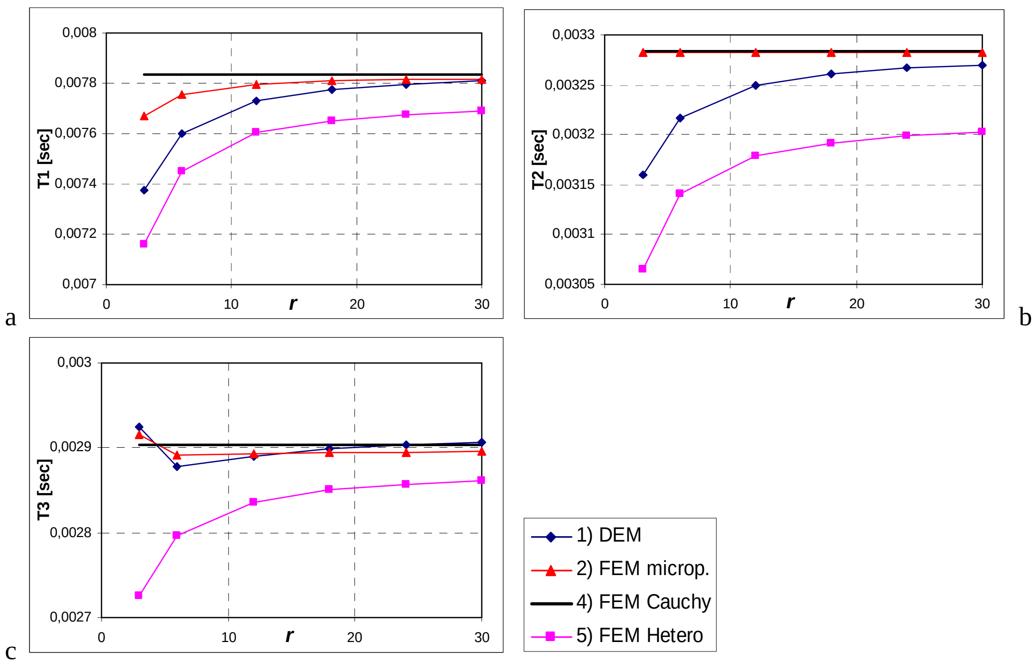

Figure 8: First three eigenparis of a square panel with fixed base and increasing r.

5.3 Square panel – sensitivity to block dimension ratio

Here the effect of block dimension ratio $\varepsilon = b/a$ on modal shapes and vibration periods is taken into account by considering the square panel with fixed base, maintaining block height a fixed and varying its width b. For instance, the panel in Fig. 5b is characterized by $\varepsilon = 5$ and represent a 'running bond' texture pattern, whereas a panel with $\varepsilon = 2$ may represent a 'header bond' texture pattern. Fig. 9a shows the first three vibration periods of the square panel for increasing ε obtained with DEM (1), whereas Fig. 9b shows differences of micropolar FEM (2) with respect to DEM results. It is clear that periods tend to converge to constant values for increasing ε and errors committed by adopting the micropolar FEM decrease for increasing ε, except for the second eigenpair, characterized by a vertical deformation that is not well approximated by the micropolar model with respect to the other eigenpairs, as it has been seen in the previous example.

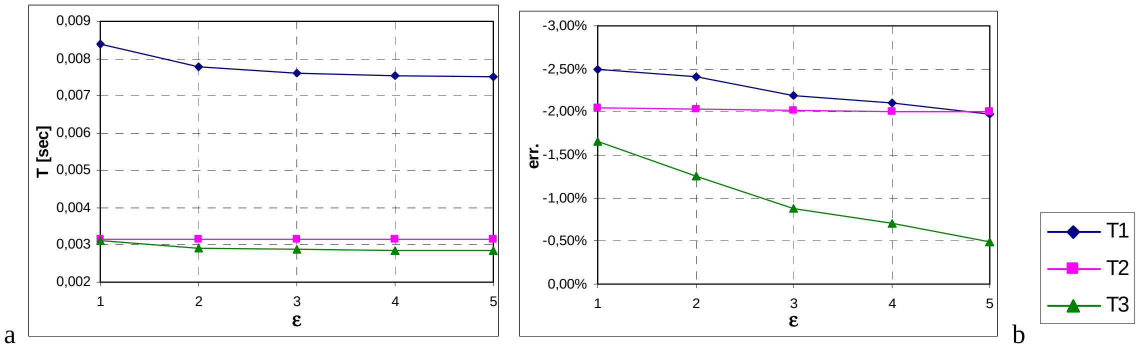

Figure 9: First three eigenparis of a square panel with fixed base and increasing ε determined with DEM (a), errors of micropolar FEM with respect to DEM (b).

12 CONCLUSIONS

- In the present work, an existing DEM has been extended to the field of in-plane modal analysis of masonry panels with regular texture, such model turned out to be simple and effective if compared with existing results and with an heterogeneous FEM.
- A micropolar model has been taken into account for modelling masonry behaviour, such model turned out to be suitable for determining panel vibration periods and modal shapes with respect to a traditional Cauchy model especially if size of heterogeneity is larger than panel dimensions. More generally, both DEM and micropolar FEM converged to Cauchy solutions for increasing number of blocks.
- Vibration periods and modal shapes have turned out to be slightly influenced by block dimension ratio.
- The modal analysis performed with DEM allowed to define easily the stiffness matrix of a panel with regular texture and rigid block hypothesis, further developments may regard the extension of the model to out of plane modal analysis and nonlinear dynamic analysis.

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