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VEHICLE/BRIDGE INTERACTION DYNAMICS FOR HIGH SPEED RAIL SUSPENSION BRIDGES CONSIDERING MULTIPLE SUPPORT EXCITATIONS

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Abstract. In this study, a railway suspension bridge is modeled as a suspended beam and the train over it as a sequence of moving sprung masses. To investigate the bridge response subjected to simultaneous actions of moving load and earthquake-induced support motions, the total response of the suspended beam under ground motions can be decomposed into two parts: the *pseudo-static* response and the *inertia-dynamic* component, in which the *pseudo-static* displacement is analytically obtained by exerting the support movements on the suspended beam statically and the governing equations in terms of the *inertia-dynamic* component as well as moving oscillators are transformed into a set of nonlinearly coupled generalized equations by Galerkin's method. When conducting the dynamic response analysis of a suspended beam subject to moving vehicles and multiple support excitations, one needs to deal with nonlinear coupling vibration problems with time-dependent boundary conditions. Instead of solving the coupled equations for the *inertia-dynamic* generalized system containing *pseudo-static* support excitations and moving oscillators, this study treats all the nonlinear coupled terms as pseudo forces, and then solves the decoupled equations using Newmark's β method with an incremental-iterative approach that can take all the nonlinear coupling effects into account. Numerical investigations demonstrate that the present solution technique is available in conducting the dynamic interaction problem with support excitations. Moreover, non-uniform seismic inputs may amplify the both responses of the suspended beam and moving vehicles over it significantly. Such an effect is often neglected by the assumption of uniform seismic ground motions in conventional design of bridge structures.

1 INTRODUCTION

A suspension bridge possesses an advantage in spanning valleys, rivers for its characteristics of long span. However, the suspension bridge may be subjected to multiple support motion in seismic zone [1,2]. This issue would become an important role in affecting operation of high speed rail, especially for the running safety of a traveling train over it. When conducting the dynamic response analysis of a suspended beam subject to moving vehicles and multiple support excitations, one needs to deal with nonlinear coupling vibration problems with time-dependent boundary conditions. In this study, a railway suspension bridge is modeled as a suspended beam [3-6] and the train over it as a sequence of moving sprung masses. To investigate the bridge response subjected to simultaneous actions of moving load and earthquake-induced support motions, the total response of the suspended beam under ground motions can be decomposed into two parts: the *pseudo-static* response and the *inertia-dynamic* component [7,8], in which the *pseudo-static* displacement is analytically obtained by exerting the support movements on the suspended beam statically and the governing equations in terms of the *inertia-dynamic* component as well as moving oscillators are transformed into a set of nonlinearly coupled generalized equations by Galerkin's method [9]. Instead of solving the coupled equations for the *inertia-dynamic* generalized system containing *pseudo-static* support excitations and moving oscillators, this study treats all the nonlinear coupled terms as pseudo forces, and then solves the decoupled equations using Newmark's β method with an incremental-iterative approach that can take all the nonlinear coupling effects into account. Numerical investigations demonstrate that the present solution technique is available in conducting the dynamic interaction problem with support excitations. Moreover, the numerical demonstrations indicated that non-uniform seismic inputs may amplify the both responses of the suspended beam and moving vehicles over it significantly. Such an effect is often neglected by the assumption of uniform seismic ground motions in conventional design of bridge structures.

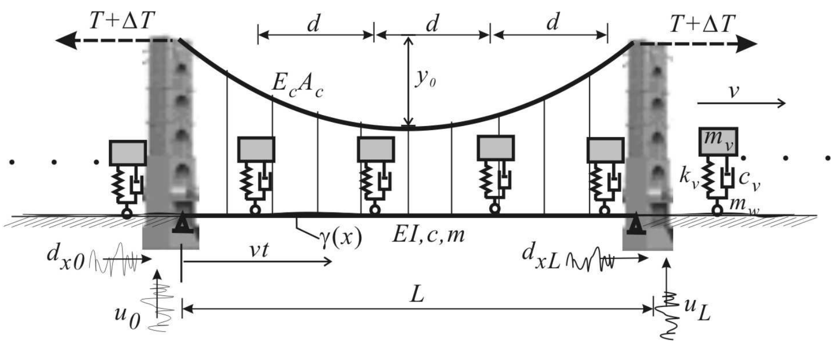


Fig. 1 Suspended beam under train loads and ground support motions

2 FORMULATION OF SUSPENSION BRIDGES

In this study, the dynamic behavior of a suspension bridge carrying a moving train is limited to vertical vibration of a single-span suspended beam with hinged supports. Based on the deflection theory of small deformation [3-6], basic simplifications for the analytical mode of suspended beam and moving train are outlined as follows: (1) The stiffening girder is modeled as a linear elastic Bernoulli-Euler beam with uniform cross section; (2) The rigid towers supporting the stiffening girder and suspension cable are assumed; (3) The train passing over the suspended beam a sequence of equidistant moving oscillators with identical properties (see Fig. 1).

2.1 Governing equations of motion

The governing equation of motion for the suspended beam undergoing differential support movements can be described as [8]

$$m\ddot{u}+c\dot{u}+EIu''''-(T+\Delta T_s)u''+\alpha\int_0^L u dx = p(x,t)+\frac{\alpha L}{2}[u_0+u_L]-\kappa\left(d_{xL}-d_{x0}\right), \quad (1)$$

$$u(0,t)=u_0, u(L,t)=u_L; u_x(0,t)=d_{x0}, u_x(L,t)=d_{xL}, EIu''(0,t)=EIu''(L,t)=0, \quad (2)$$

where $(\bullet)'=\partial(\bullet)/\partial x, (\dot{\bullet})=\partial(\bullet)/\partial t$, m = mass of the beam and cable per unit length along x-axis, c = damping coefficient, $u(x,t)$ = vertical deflection of the beam, EI = flexural rigidity of the beam, T = horizontal component in the initial cable tension (due to dead loads), $p(x,t)$ = loading function, and

$$\begin{aligned} &T=\frac{-mg}{y''}=\frac{mgL^2}{8y_0}, \Delta T_s=\frac{E_cA_c}{L_c}\left[\left(d_{xL}-d_{x0}\right)-\frac{4y_0}{L}\left(u_0+u_L\right)\right], \\ &\alpha=\left(\frac{8y_0}{L^2}\right)^2\frac{E_cA_c}{L_c}, \kappa=\left(\frac{8y_0}{L^2}\right)\frac{E_cA_c}{L_c}, L_c=\int_0^L\left(\frac{ds}{dx}\right)^3 dx=\int_0^L\left(\sqrt{1+y'^2}\right)^3 dx. \end{aligned} \quad (3)$$

with E_c = elastic modulus of the cable, A_c = area of the cable, L_c = the effective length of the cable. (u_0, u_L) and (d_{x0}, d_{xL}) represent the vertical and horizontal support movements at the left and right bridge supports, respectively. As shown in Fig. 1, each sprung mass unit is used to model either the front or rear half of a carriage, which is composed of a lumped mass supported by a spring-dashpot system. Let the oscillator model has the following properties: m_w = mass of wheel-set, m_v = lumped mass, c_v = damping coefficient, and k_v = stiffness coefficient. Consider the regular nature of sprung mass units shown in Fig.2, the load function $p(x,t)$ is given as [8]:

$$\begin{aligned} &p(x,t)=\sum_{k=1}^{N}\left(P-m_v\ddot{u}_{vk}-m_w\ddot{u}\right)\delta(x-x_k)\times\left[H(t-t_k)-H(t-t_k-L/v)\right], \\ &m_v\ddot{u}_{vk}+c_v\dot{u}_{vk}+k_vu_{vk}=f_{vk}, f_{vk}=\begin{cases} k_v[u(x_k,t)+\gamma(x_k)]+c_v\dot{u}(x_k,t) & 0\le x_k\le L \\ k_v[\gamma(x_k)+g_s(x_k)] \; x_k<0, & x_k>L \end{cases}, \end{aligned} \quad (4)$$

in which, P = -(m_v + m_w)g = lumped weight of a moving oscillator, δ = Dirac's delta function, $H(t)$ = unit step function, k =1, 2, 3, …, N-th moving load on the beam, t_k = arrival time of the k-th oscillator entering the beam, u_{vk} = vertical displacement of the k-th lumped mass, f_{vk} = interaction force existing between the beam and the wheel mass of the k-th moving oscillator, $\gamma(x_k)$ = track irregularity (vertical profile), and x_k = position of the k-th load along the rail line, as defined in Eq. (4). As shown in Eq. (4), $x_k<0$ represents the k-th oscillator is entering to the suspended beam, $0\le x_k\left(=v(t-t_k)\right)\le L$ running on the beam, and $x_k>L$ departing the beam.

2.2 Method of solution

As indicated in Eqs. (1) and (3), it is a partial integro-differential equation with time-dependent boundary condition. For this problem, this study divides the total deflection response $u(x,t)$ of the suspended beam into two parts: the *static* displacement $U(x,t)$ and the *dynamic* deflection $u_d(x,t)$ [7,8]

$$u(x,t) = U(x,t) + \sum_{n=1} q_n(t) \sin \frac{n\pi x}{L} \tag{5}$$

Here, $U(x,t)$ represents the structure deformation caused by the *relative* support displacements applied statically, and $q_n(t)$ means the generalized coordinate associated with the n-th assumed mode of the suspended beam. By this concept, the static displacement is given as:

$$U(x,t) = \left[u_0 + \left(u_L - u_0\right)\frac{x}{L} \right] + \left[\frac{d_{xL} - d_{x0}}{8y_0 / L} - (u_L + u_0) \right] \frac{\beta(x,t)}{\chi}, \tag{6}$$

where

$$\beta(x,t) = \begin{cases} 1 + \dfrac{\lambda^2 x(x-L)}{2} - \dfrac{\cosh \lambda(x - L/2)}{\cosh(\lambda L/2)} & \mathit{for} \quad T + \Delta T_s > 0 \\ 1 - \dfrac{|\lambda|^2 x(x-L)}{2} - \dfrac{\cos\left(|\lambda|(x - L/2)\right)}{\cos(|\lambda| L/2)} & \mathit{for} \quad T + \Delta T_s < 0 \end{cases}, \tag{7a}$$

$$\chi(t) = \begin{cases} \dfrac{(T+\Delta T_s)(\lambda L)^2}{\alpha L^3} + \dfrac{(\lambda L)^2}{12} + \dfrac{\tanh(\lambda L/2)}{\lambda L/2} - 1 & \mathit{for} \quad T + \Delta T_s > 0 \\ -\dfrac{(T+\Delta T_s)(|\lambda| L)^2}{\alpha L^3} - \dfrac{(|\lambda| L)^2}{12} + \dfrac{\tan(|\lambda| L/2)}{|\lambda| L/2} - 1 & \mathit{for} \quad T + \Delta T_s < 0 \end{cases}. \tag{7b}$$

and $\lambda^2 = (T + \Delta T)/EI$. The pseudo-static displacement shown in Eq. (6) reveals that the first term represents the *rigid body* displacement due to *vertical* support movements, and the second term means the *pseudo-static* natural deformation of the suspended beam caused by the *relative* support movements. It is emphasized that the effect of non-uniform horizontal seismic inputs is of importance to earthquake-induced vibration of suspension bridges since it is usually neglected in seismic design based on the assumption of *uniform* seismic ground motions. By Galerkin's method, the following generalized equation of motion for the n-th *dynamic* system of the suspended beam is given:

$$\begin{aligned} & m\ddot{q}_n + c\dot{q}_n + \left(\frac{n\pi}{L}\right)^2 \left[\left(\frac{n\pi}{L}\right)^2 EI + (T + \Delta T_s) \right] q_n + \Pi_n \\ & + \sum_{k=1}^{N} F_{vk}(\varpi_n, v, t) = \sum_{k=1}^{N} F_k(\varpi_n, v, t) - \frac{2}{n\pi}\left[m\ddot{\Upsilon}_n + c\dot{\Upsilon}_n \right], \end{aligned} \tag{8}$$

with the coupled equation of the k-th sprung mass unit in Eq. (4). Here,

$$\Pi_n = \frac{2\alpha L}{n\pi^2}(1-\cos n\pi)\left[\sum_{k=1}\frac{1}{k}(1-\cos k\pi)q_k\right], \tag{9a}$$

$$\Upsilon_n = \begin{cases} u_0 - u_L\cos n\pi + \varsigma_n\left(\dfrac{(\lambda L/n\pi)^4}{1+(\lambda L/n\pi)^2}\right) & for \;\; T+\Delta T_s > 0 \\ u_0 - u_L\cos n\pi + \varsigma_n\left(\dfrac{(|\lambda| L/n\pi)^4}{1-(|\lambda| L/n\pi)^2}\right) & for \;\; T+\Delta T_s < 0 \end{cases} \tag{9b}$$

$$\varsigma_n = \frac{(\cos n\pi - 1)}{\chi}\left[\frac{d_{xL}-d_{x0}}{8y_0/L} - \frac{(u_L+u_0)+(u_{cL}+u_{c0})}{2}\right]$$

and the generalized forces of $F_k(\varpi_n, v, t)$ with respect to the k-th sprung mass unit are respectively expressed as

$$\begin{aligned} &F_k(\varpi_n,v,t) = \frac{2P}{L}\psi_n(\varpi_n,t), \\ &F_{vk}(\varpi_n,v,t) = \frac{2}{L}\left[m_v\ddot{u}_{v,k} + m_w\left(\ddot{U}(x_k,t) + \sum_{n=1}\ddot{q}_n(t)\sin\frac{n\pi x_k}{L}\right)\right]\psi_n(\varpi_n,t), \\ &\psi_n(\varpi_n,t) = \sin\varpi_n(t-t_g-t_k)\left[H(t-t_g-t_k) - H\left(t-t_g-t_k-\frac{L}{v}\right)\right], \end{aligned} \tag{10}$$

and $\varpi_n(=n\pi v/L)$. Combining Eqs. (4) and (8) yields the following equation of motion for the vehicle/bridge interaction model

$$\begin{aligned} &[m_b(t)]\{\ddot{q}\} + [c_b]\{\dot{q}\} + [k_b]\{q\} = \{p\}, \\ &[m_v]\{\ddot{u}_v\} + [c_v]\{\dot{u}_v\} + [k_v]\{u_v\} = \{f_v\}, \end{aligned} \tag{11}$$

where $\{q\}$ = generalized coordinate vector of the suspended beam, $[m_b]$ = generalized beam mass matrix including the sprung masses moving on the suspended beam, $[c_b]$ = generalized beam damping matrix, $[k_b]$ = generalized beam stiffness matrix, $\{p\}$ = generalized force vector acting on the generalized beam system; $\{u_v\}$ = vehicle displacement vector, $\{f_v\}$ = exciting force vector, and $([k_c],[c_c],[m_c])$ = structural matrices of the vehicles corresponding to mass, damping, and stiffness.

3 STRATEGY FOR INCREMENTAL-ITERATIVE DYNAMIC ANALYSIS

To compute the dynamic response of vehicle-bridge interactions for a suspended beam undergoing support movements, an incremental-iterative procedure needs to be carried out [9]. The numerical procedure of incremental-iterative dynamic analysis conventionally involves three phases: predictor, corrector, and equilibrium checking. In performing the dynamic response analysis of structures containing seismic ground motions, two sets of structure responses have to be computed each for the pseudo-static response and for the inertia-dynamic response. The incremental-iterative procedure for nonlinear dynamic analysis of vehicle-bridge interaction system shaken by earthquakes is summarized as follows:

(1) Treat the pseudo-static displacement $U(x, t)$ derived in Section 2 as an exciting source of the equivalent dynamic force $(-m\ddot{U} - c\dot{U})$ to excite the inertia-dynamic response for the beam-oscillator interaction system. In this stage, the oscillators not traveling over the suspended beam are only subjected to the action of seismic ground motions;
(2) Transform the governing differential equation in Eq. (1) into a set of coupled equations of generalized system as Eq. (8) and then remove the coupled terms to the right hand side of Eq. (8) to form a set of uncoupled equations of motion;
(3) Discretize each of the uncoupled equations into an equivalent stiffness equations using Newmark's method;
(4) Perform the iterative procedure proposed to compute the *inertia-dynamic* response of the suspended beam and update the dynamic responses of moving oscillators at each iteration;
(5) Check the unbalanced forces to reach preset tolerances. As the root mean square of the sum of the generalized unbalanced forces is larger than the preset allowable values, go to step (4) to precede the next iteration for removing the unbalanced forces. Once the condition of convergence is satisfied, the total dynamic response of the suspended beam can be computed using Eq. (5) by combining the pseudo-static and inertia-dynamic parts of the beam response;
(6) Repeat the steps (4) and (5) for other time instants.

Detailed information for nonlinear VBI dynamic analysis is available in references [6].

Table 1. Properties and natural frequencies of the suspended beam.

L (m)	EI (kN-m^2)	E_cA_c (kN)	m (t/m)	c (kN-s/m/m)	y_0 (m) [y_0/L]	E_cA_c/L_e (kN/m)	Ω_1 (Hz)	Ω_2 (Hz)
125	2.3x10^8	6.0x10^7	16	4.61	12.5 [0.10]	4.437 x10^5	1.55	1.73

Table 2. Properties of moving oscillator and resonant speeds.

N	d (m)	P (kN)	m_w (t)	m_v (t)	c_v (kN-s/m)	k_v (kN/m)	$v_{res,1}$ (km/h)	$v_{res,2}$ (km/h)
16	27	340	4.7	30.0	5.2	157	152	168

4 NUMERICAL ILLUSTRATIONS

Figure 1 shows a series of moving oscillators with equal intervals d are crossing a single-span suspended beam at constant speed v. The properties of the suspended beam and sprung mass unit are listed in Tables 1 and 2, respectively. In Table 1, the symbol of Ω_i represents the ith modal frequency. As shown in Table 1, the first natural frequency of anti-symmetric mode of the suspended beam is lower than that one of symmetric bending mode. It means that the cable tension in the suspended beam due to dead loads can offer a strengthening effect on the first symmetric bending mode.

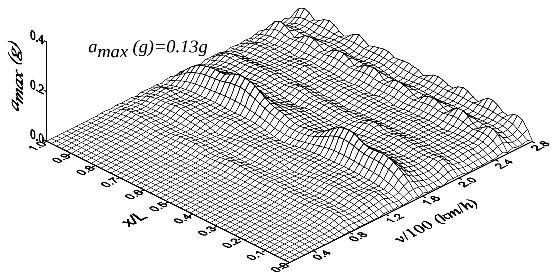


Fig. 2 a_{max}-v-x/L plot of the suspended beam due to moving loads

For the purpose of illustrating the maximum acceleration response distributed along the beam span, a three-dimensional (3D) plot for the maximum acceleration response (a_{max}) along the beam span (x/L) under the action of moving loads against various moving speeds (v) has been drawn in Fig. 2. Such a 3D plot will be called a_{max}-v-x/L plot in the following examples. As can be seen from the two resonant peaks, the maximum acceleration response of the suspended beam at the speed of 152 km/h is governed by the anti-symmetrical modes that have been excited. Moreover, the maximum vertical accelerations of sprung masses passing the suspended beam with various speeds have been plotted in Fig. 3. The acceleration response of sprung masses for moving oscillators reaches a maximum value in the vicinity of 152 km/h due to the train-induced resonant response of the suspended beam.

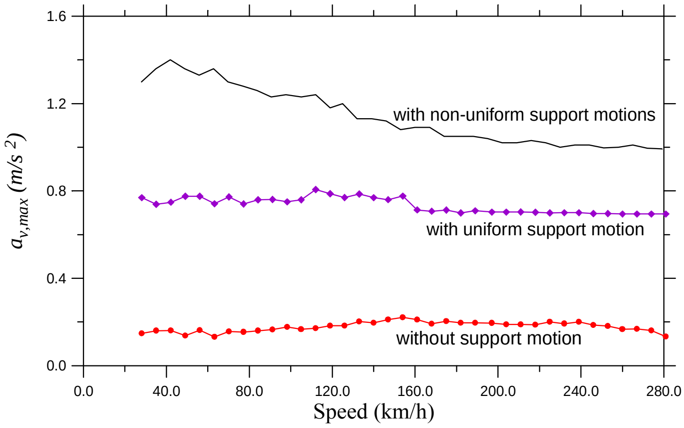


Fig. 3 Maximum acceleration of moving sprung masses.

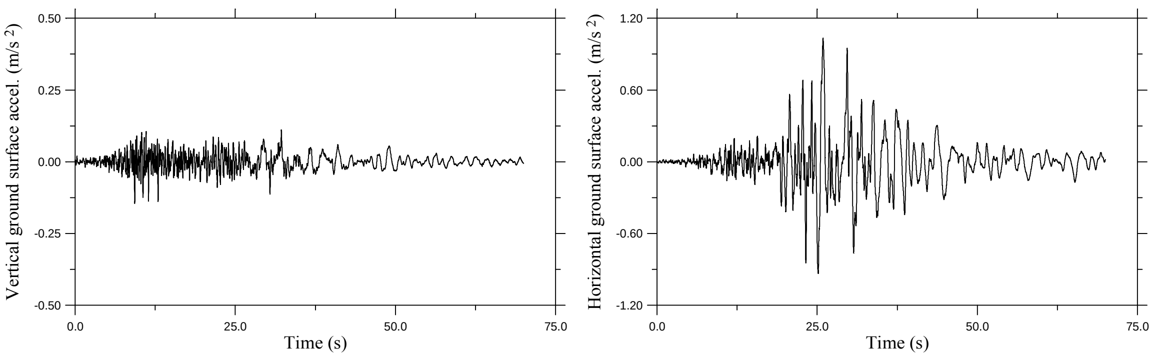


Fig. 4 Histograms of ground acceleration of TAP003 Station: (a) vertical, (b) NS horizontal.

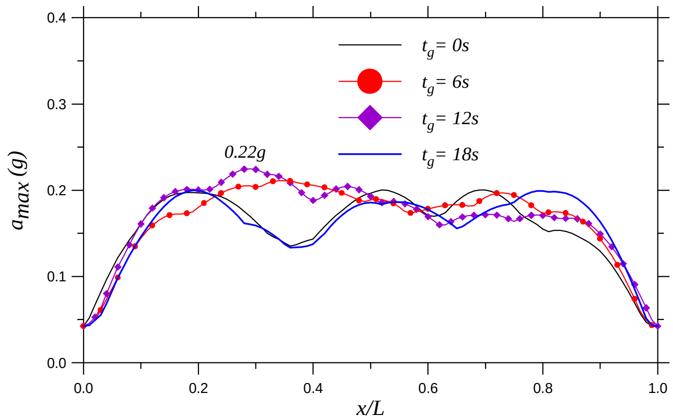
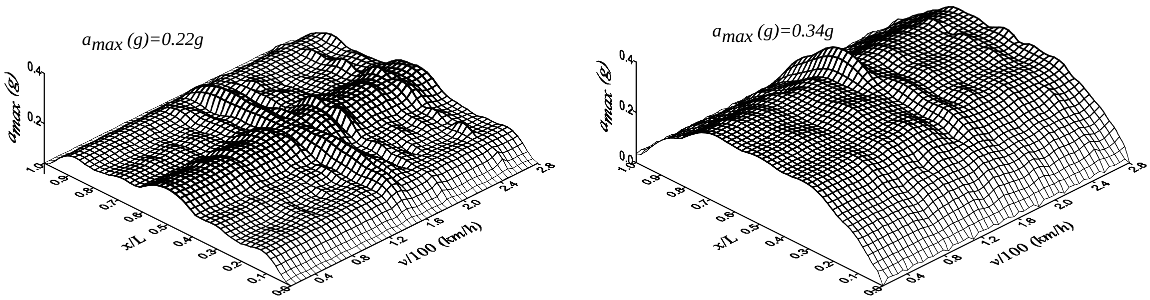


Fig. 5 Effect of time lags of moving oscillators on beam response.



(a) (b)

Fig. 6 a_{max}-v-x/L plot: (a) vertical uniform support motion; (b) multiple support motions.

4.1 Uniform support motion

To investigate the influence of seismic ground excitations on train-induced vibration of suspension bridges, the far-field ground motions of TAP003 station recorded at the free-field station of Taipei during the 1999 Chi-Chi Earthquake in Taiwan [9] are used to simulate the seismic support inputs exerting the suspended beam. The histograms of acceleration of the ground motions, containing both the NS horizontal and vertical components, have been plotted in Figs. 4(a) and 4(b), respectively. As can be seen from the ground acceleration records depicted in Fig. 4(a), the intensive zone of vertical ground accelerations occurs early compared to that of horizontal components in Fig. 4(b) due to the fact that the primarily wave (P-wave) produced by earthquakes travels faster than the shear one (S-wave).

Let us consider the special case of *uniform* support motion, i.e., $u_0 = u_L$, $d_0 = d_L$, with various time lags of t_g for the train loads entering the suspended beam at the first resonant speed of $v_{res,1}$ (=152km/h), Fig. 5 shows the maximum acceleration amplitude in a_{max}-x/L plot will occur at the critical time lag of 12s, which is just inside the intensive zone of ground motions depicted in Fig. 4(a). In the following examples, the critical time of 12s, therefore, will be used as the time lag for the moving loads to start entering the suspended beam after earthquakes shake the vehicle-bridge system. Besides, as shown in the a_{max}-v-x/L 3D plot of

Fig. 6(a) and the corresponding maximum acceleration response of sprung masses in Fig. 3, the inclusion of uniform ground motions can totally amplify the acceleration amplitudes of the suspended beam and sprung masses even though the seismic support inputs are of far-field ground motions. Moreover, from the acceleration amplitudes plotted in Fig. 6(a), most of the symmetric modes are excited by the uniform ground support motion.

4.2 Multiple *support motions*

Due to possibly different soil conditions at local site of bridge towers depicted in Fig. 1, let us assume that the intensities of seismic ground inputs tranmitting into the right support have comparative attenuation, say, $u_L = 0.8u_0$, $d_L = 0.7d_0$, compared to those into the left one. Then the suspended beam will undergo the action of multiple-support excitations during seismic ground motions. Figure 10 shows the a_{max}-v-x/L 3D plot for the train-induced vibration of the suspended beam shaken by the seismic excitations. By comparing the maximum acceleration amplitudes in Fig. 3 with those in Fig. 6(b), the amplification effect of multiple support excitations involving horizontal and vertical components is rather significant on the stiffening girder response. Especially in the vicinity of three-quarters span of the suspended beam at the first resonant speed $v_{res,1}$, there exists a noticeable peak acceleration amplitude (= 0.34g). The reason is that as the last following loads of the train loadings pass over the three-quarters span with the resonant speed $v_{res,1}$, the time-consuming is about $t_g+[(N-1)d+0.75L]/v_{res,1}$ = 24s. It is just inside the intensive zone of horizontal seismic inputs at the histograms of ground acceleration shown in Fig. 6(b). Therefore, the last moving sprung mass experiences a fierce vibration transmitted from the vibrating beam.

Moreover, the maximum accelerations of sprung masses traveling over the suspension bridge shaken by the non-uniform ground excitations have been plotted in Fig. 3 as well. As expected, the maximum acceleration responses of sprung masses are totally amplified, but the magnification effect of multiple support excitations on the maximum response of sprung masses becomes noticeable significance at low speeds. The reason is attributed to the fact that a series of sprung masses moving with low speeds (< 80 km/h) need spend much time crossing the suspended beam so that they will experience the action of intensive horizontal support excitations around the time of 25s.

5 CONCLUDING REMARKS

Considering non-uniform characteristics of multiple support excitations, the dynamic interaction responses of a single-span suspended beam subject to moving oscillators have been carried out using a *pseudo-decomposition* concept and a rigorous incremental-iterative procedure involving the three phases of *predictor, corrector,* and *equilibrium-checking*. From this study, the following conclusions are reached:

1. Instead of updating the structural matrices with time-dependent coupled nature for the beam-oscillator generalized systems at each time step, this study treats all the coupled terms as pseudo-forces and converts the coupled equations into a set of equivalent uncoupled equations. Then, an incremental-iterative procedure for nonlinear dynamic analysis is employed to solve these equivalent equations in an accurate way.
2. From the exact solution for the *pseudo-static* response, it indicates that non-uniform horizontal seismic inputs may amplify the response of suspended beams significantly.

Such an effect is often neglected by the assumption of uniform seismic ground motions in conventional design of bridge structures.

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