11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

LIMITATIONS OF AN EQUIVALENT LINEARIZED METHOD ON VIBRATION ANALYSIS OF A FLEXIBLE CANTILEVER BEAM

BIN LI, WEN CAI, XIAOBING WANG AND WANYUAN DONG

Key Lab of FSND on Aircraft Structural Mechanics and Strength Technology,
Northwestern Polytechnical University, Xi'an P.R. China,
Email: leebin@nwpu.edu.cn, www.nwpu.edu.cn

Key Words: *Vibration, Equivalent linearized method, small amplitude hypothesis, nonlinear*

Abstract. Using an equivalent linearized method to analysis the vibrating behaviors of a flexible cantilever beam with large deformation is a very effective method, but the small amplitude hypothesis is the precondition of this method. This paper discusses the limitations of the equivalent linearized method proposed by D.G.Fertis through the experimental and theoretical investigations. A flexible cantilever beam with large deformation was designed and manufactured, and three frequency responses function (FRF) curves under three different vibrating amplitude levels were measured by the fixed frequency steady state exciting method. The comparisons between experimental and simulation results show that, when the ratio of tip dynamic displacement amplitude and static deformation amplitude is less than 10%, the variations of measured FRF curves are very small. So the system can be regarded as linear system, and the relative error of the proposed equivalent linearized method is less than or equal to 6%, this approximate method is effective. But when the ratio of tip dynamic displacement amplitude and static deformation amplitude is more than 20%, the value of the measured FRF at resonant frequency point is significantly reduced, and the relative error of the equivalent linearized method is more than 50%, and this method will be no longer suitable.

1 INTRODUCTION

Modern high-altitude long-endurance aircrafts have become a hot airborne platform for the military and civilian usage[1]. Because of the mission requirements, this kind of aircraft platforms is characterized by high-aspect-ratio wings. The design of high-aspect-ratio wings has become so lightweight and flexible that traditional (linear) design methods are no longer adequate. The wings may undergo large deformations during normal operating loads, exhibiting geometrically nonlinear behavior. Nonlinear vibration analysis methods and nonlinear aeroelastic methods are required to characterize structural nonlinear behaviors. Since 1970's, many experimental and theoretical researches have been conducted to analysis the nonlinear static and dynamic behaviors of flexible beam with large deformations [2~9], Demeter G. Fertis's book[10] provide a full view summary about these problems. Generally, nonlinear vibration analysis methods for flexible beams can be classified into two categories. First kind of method is called as a fully nonlinear method, such as total-Lagrange finite element method or updated Lagrange finite element method [11-12]. Second kind of method is called as an equivalent linearized method.

Equivalent linearization methods are efficient and approximate methods that analyses vibrating characteristics of a flexible beam with large deformation, but the small amplitude hypothesis is the precondition of this kind of methods. Unfortunately, there are few literatures to discuss the qualitative level of small amplitude hypothesis. On the other hand, the traditional fully nonlinear methods are usually hard to characterize explicitly the influence of geometric nonlinear properties on structure parameters, also can not satisfy with the requirements of the reduced-order models. The present paper considers the vibrating behaviours of a flexible cantilever beam with large deformation, and use Fertis's equivalent linearized method to calculate the vibrating response of this beam, and discussed the limitations of the equivalent linearized method through the experiment and simulation.

2 EQUIVALENT LINEARIZED METHOD

Consider a flexible cantilever beam shown in figure 1, it's cross section is uniform, A is the area of cross section, L is axis length, I is moment of area inertia, E is Young's modulus of beam material. Two loads are applied on the beam. $w_0=\rho g A$ is the self heavy load, where ρ is density of beam material, and g is gravity acceleration. P=mg is the tip load, which is applied by a lumped steel block. $y_s(x)$ is the static transverse deformation at x cross section, and Δ is the horizontal deformation under the action of w_0 and P. L_0 is the length of horizontal projection of the deformed beam, $L_0 = L - \Delta$. x is the new horizontal coordinate axis of the deformed beam, which zero point is located on the free end of beam. $x_0(x)$ is the arc length of the large deformed beam from x cross section to the free end.

Set the static deformed position as the new balance position, the beam will occurs small amplitude vibration, and write the vibrating deformation as $y_d(x,t)$.

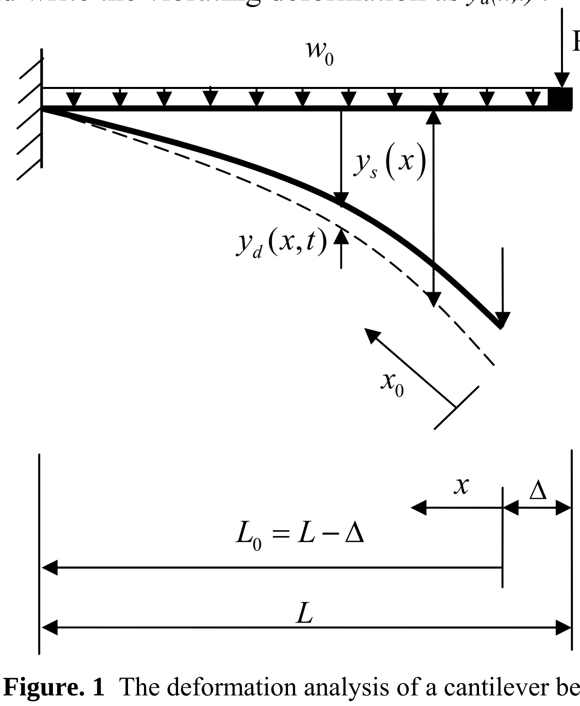

Figure. 1 The deformation analysis of a cantilever beam

According to the Fertis's method[10], $y_s(x)$ can be calculated by the solution of Eq.(1), here we omit the description of detail solving steps.

$$\frac{y_s'}{\left[1+\left(y_s'\right)^2\right]^{1/2}}=\frac{P}{12EIL}\begin{bmatrix}k\left(2x^3+3\Delta x^2-2L_0^3-3\Delta L_0^2\right)+\\6L\left(x^2-L_0^2\right)\end{bmatrix} \tag{1}$$

The differential equation of free vibration of deformed beam can be written as

$$\frac{d^2}{dx^2}\left\{EI\frac{y''}{\left[1+(y')^2\right]^{3/2}}\right\}=-\sqrt{1+(y')^2}\,(w_0+m(x_0)\frac{d^2y}{dt^2}) \;. \tag{2}$$

Here, $y(x,t)$ is total transverse displacement response of beam at x cross section. Assume vibration is small amplitude vibration around the new static balance position, and then we can have

$$y(x,t)=y_s(x)+y_d(x,t) \tag{3}$$

According to the previous small amplitude assumption, $\mathrm{y_s(x)>>y_d(x,t)}$, so following relationships also can be deduce:

$$y'(x,t)\approx y_s'(x)\,,\quad y''(x,t)\approx y_s''(x)+y_d''(x,t)\,,\ \frac{d^2y(x,t)}{dt^2}\approx\frac{d^2y_d(x,t)}{dt^2} \tag{4}$$

Substitute Eq.(4) to Eq.(2), and eliminate the relational static balance terms, a new differential equation of free vibration for the large deformed beam can be gotten, whose unknown variable is $y_d(x,t)$.

$$\frac{d^2}{dx^2}\left\{EI\frac{y_d''(x,t)}{\left[1+(y_s')^2\right]^{3/2}}\right\}=-\sqrt{1+(y_s')^2}\,m(x_0)\frac{d^2y_d(x,t)}{dt^2}. \tag{5}$$

Let $EI_e=\frac{EI}{\left[1+(y_s')^2\right]^{3/2}}$, $\quad m_e(x)=\sqrt{1+(y_s')^2}\,m(x_0)$, we can have

$$\frac{d^2}{dx^2}\left[EI_e y_d''(x,t)\right]+m_e(x)\frac{d^2y_d(x,t)}{dt^2}=0 \tag{6}$$

Eq.(6) is a normal differential equation of free vibration for a linear beam with equivalent variable cross section, and that can be solved by a usual finite element method. Now, the proposed equivalent linearized method has treated the effects of static large deformation on vibration properties of the beam as the variations of cross section moment of inertia and mass density distribution, so the physical significance of this method is much better clear.

3 VALIDATION OF EQUIVALENT LINEARIZED METHOD

We made a Matlab computing program to solve Eq.(6), and a literature[2] example was referred to verify the effectiveness of the proposed method and hand-made codes. The sizes of flexible beam mentioned in the literature[2] are listed here, the beam length is 20 inch, the width of cross section is 0.5 inch, and the thickness of beam cross section is 0.125 inch. The material of beam is 7075 aluminium alloy.

Table 1 show the comparison between the present calculated results and literature results. The computing results from the present method keep better consistency with the experimental results provided in the literature, this point verify the effectiveness of proposed equivalent linearized method and hand-made codes.

Table. 1 Comparison between present method and literature results

Tip mass load /kg	The method of literature[2]		The present method		Experimental results of literature[2]	
	tip static displacement/cm	1st natural frequency / Hz	tip static displacement/cm	1st natural frequency / Hz	tip static displacement/cm	1st natural frequency / Hz
0.4536	8.382	1.773	7.660	1.773	7.62	1.757
0.9072	15.91	1.33	14.72	1.259	15.24	1.254

4 EXPERIMENT DESIGN OF A FLEXIBLE BEAM

In this section, a new flexible beam will be designed and manufactured. Static deformation tests and vibrating response tests will be conducted, then the proposed equivalent linearized method will be used to calculate the vibrating responses for the experimental load case. The aim is to find the applicable range and limitations of the proposed equivalent linearized method.

The designed beam sample is a steel beam, it's length is 1.50m, the width of cross section is 20.19mm, and the thickness of beam cross section is 4.55mm. The mechanical properties of steel material are listed in Table 2.

Table. 2 The mechanical property parameter of 40 carbon structural steel

Modulus of elasticity	Density	Poisson ratio	σ_b/MPa	σ_s/MPa
196GPa	7800 kg/m3	0.3	569	333

Figure.2 The set-up of test

The established test set-up is shown in figure 2. First kind of test is to measure the static deformation of the beam under the action of self-heavy load and tip mass load. Second kind of test is to measure the natural frequencies and modes of the beam under different level tip mass load. Third kind of test is a vibrating response experiment. In the third kind of tests, a acceleration sensor is fixed in the tip end of the beam to pick up the vibrating response, and a electric-magnetic exciter is used to apply a steady exciting force on the beam. Three different vibrating amplitude levels will be measured. For each vibrating amplitude level, we will measure the frequency responses function (FRF) curve of the beam sample within it’s first natural frequency band by the fixed frequency steady state exciting method.

5 DISSCUSIONS OF COMPUTING RESULTS AND TEST RESULTS

5.1 Computation of static deformation

The present method and a fully nonlinear static finite element analysis (MSC/MARC)were used respectively to perform the computation of static deformation of the flexible beam. Two load cases were considered, in load case 1 just self heavy load was applied, and in load case 2, a tip mass load was applied. Table 3 show the computing results from two methods. The comparisons indicate that the method proposed in this paper is applicable for static deformation analysis.

Table3: The results of the deformation of the beam

Tip mass load/kg	The method of this paper		FEM(MARC)	
	Δ/mm	*Ys-ti displacement*/mm	Δ/mm	*Ys-tip displacement*/mm
0	7.7	142.1	7.7	142.1
0.311	24	247.3	24.1	248

5.2 Analysis of natural vibration characteristics

Table 4 lists the measured first three bending natural frequencies of the beam under different static deformation conditions. The column named without deformation means the flexible cantilever beam is mounted in vertical-down direction, and the self heavy load will just bring the axial force on the beam, so no transverse deformation is induced. The testing results show that the large static deformations can lead a light decrease of natural frequencies, but the percent decrease is less than 6%. The computing results from the present equivalent linearized method also show the same phenomenon with the experiments. The computing result of natural frequencies is consistent with the experimental results. This comparison also shows the validity of the present equivalent linearization method on the computation of natural frequencies of the flexible beam with large deformations.

Table 4: The first frequency comparison of the present methods and experimental results

Tip mass load /kg	Experimental results		Present method	
	$\omega_{1,2,3}$/Hz		$\omega_{1,2,3}$/Hz	
	Without deformation	With deformation	Without deformation	With deformation
0	1.69/10.20/29.25	1.65/10.31/28.69	1.70/10.68/29.91	1.67/10.42/29.16
0.311	1.19/8.75/25.88	1.12/8.45/25.03	1.15/8.87/26.69	1.12/8.21/24.53

5.3 Effects of vibrating amplitude on response propeties

Frequency responses function (FRF) curves of the flexible cantilever beam with a tip mass 0.311kg were measured at three different vibrating amplitude levels by the fixed frequency steady state exciting method, and Figure 3 and Table 5 show experimental results. Figure 3 tells us that the FRF curve under the vibration amplitude level of 12.5mm almost coincides with the FRF curve under the vibration amplitude level of 25mm. But under the vibrating amplitude level 50mm, the FRF value at resonant frequency point just is 66% of small amplitude cases. The experimental results indicate that when the vibrating amplitude is less than 25mm, the equivalent linearization method is applicable. For the 50mm vibration amplitude case, the equivalent linearization method is inapplicable, and this amplitude is over the range of small amplitude assumption.

Table 5: Experiment results

Exciting Freq. /Hz	Vibration level:12.5±1mm				Vibration level:25±1mm				Vibration level: 50±1mm			
	Force Amp./N	Acc. Amp./g	Dis. Amp./mm	FRF mm/N	Force Amp./N	Acc. Amp./g	Dis. Amp./mm	FRF mm/N	Force Amp. /N	Acc. Amp./g	Dis. Amp./mm	FRF mm/N
1.05	30.75	0.0545	12.28	0.40	60.5	0.11	24.79	0.41	126	0.221	49.81	0.40
1.09	13.6	0.0595	12.44	0.92	27.15	0.12	25.10	0.92	50.5	0.238	49.78	0.99
1.10	9.15	0.0615	12.63	1.38	17.6	0.1215	24.95	1.42	36	0.241	49.49	1.37
1.11	4.65	0.0620	12.50	2.69	9.01	0.125	25.21	2.80	17.75	0.250	50.42	2.84
1.12	0.58	0.0625	12.38	21.35	1.23	0.126	24.96	20.29	3.55	0.252	49.92	14.06
1.13	4.55	0.0650	12.65	2.78	9.5	0.1315	25.59	2.69	20.35	0.254	49.43	2.43
1.14	9.01	0.0640	12.24	1.36	18.15	0.1305	24.95	1.37	40.5	0.263	50.29	1.24
1.15	13.8	0.0665	12.49	0.91	27.5	0.133	24.99	0.91	59.5	0.265	49.79	0.84
1.19	30	0.0725	12.72	0.42	58	0.144	25.27	0.44	129.5	0.285	50.01	0.39

Table 6: Comparison between experiment results and equivalent linearization method

Exciting Freq. /Hz	Vibration level:12.5±1mm				Vibration level:25±1mm				Vibration level: 50±1mm			
	Force Amp./N	Sim Dis. Amp./mm	Exp. Dis. Amp./mm	Relative Error%	Force Amp./N	Sim Dis. Amp./mm	Exp. Dis. Amp./mm	Relative Error%	Force Amp. /N	Sim Dis. Amp./mm	Exp. Dis. Amp./mm	Relative Error%
1.05	30.75	12.1	12.28	1.5	60.5	23.81	24.79	4.0	126	49.59	49.81	0.4
1.09	13.6	12.41	12.44	0.3	27.15	24.77	25.10	1.3	50.5	46.07	49.78	7.4
1.10	9.15	12.55	12.63	0.6	17.6	24.14	24.95	3.3	36	49.38	49.49	0.2
1.11	4.65	12.91	12.50	-3.2	9.01	24.98	25.21	0.9	17.75	49.27	50.42	2.3
1.12	0.58	12.43	12.38	-0.4	1.23	26.35	24.96	-5.6	3.55	76.06	49.92	-52.4
1.13	4.55	11.99	12.65	5.2	9.5	25.04	25.59	2.2	20.35	53.63	49.43	-8.5
1.14	9.01	12.51	12.24	-2.2	18.15	25.22	24.95	-1.1	40.5	56.28	50.29	-11.9
1.15	13.8	13.25	12.49	-6.0	27.5	26.4	24.99	-5.6	59.5	57.12	49.79	-14.7
1.19	30	13.36	12.72	-5.0	58	26.73	25.27	-5.8	129.5	59.68	50.01	-19.3

In order to validate above-mentioned conclusion, the equivalent linearization method is used to calculate the FRF curves of three experimental cases. The experimental load is taken as exciting input. The modal superposition method is chosen to calculate the responses. And the damping is assumed to be proportional. According to modal experimental results, the identified α and β are 0.0145 and 7.515e-6, respectively. Table6 and Figure 4 show the comparisons between the experimental and computing results. The comparison also indicates that the proposed equivalent linearization method is valid when the vibrating amplitude is less than 25mm. But when the amplitude achieves 50mm, the relative error is greater than 50%.

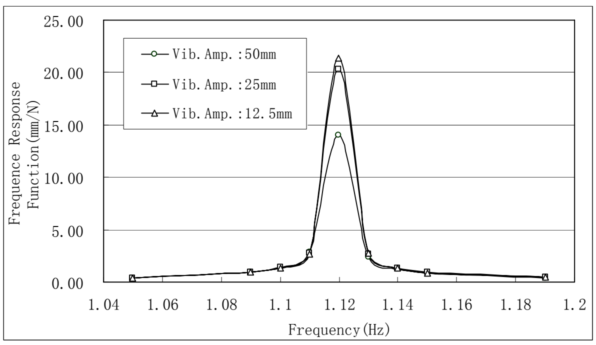

Figure 3 Measured FRF curves

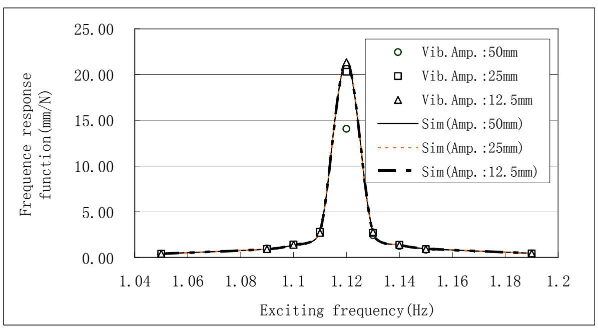

Figure 4 FRF Comparisons between experiment and equivalent linearization method

6 CONCLUSION

The present paper considered the vibrating behaviors of a flexible cantilever beam with large deformation, and developed an equivalent linearized method to calculate the vibrating response of this beam, and discussed the limitations of the equivalent linearized method through the experiment and simulation. The comparisons between experimental and simulation results show that, when the ratio of tip dynamical displacement amplitude to static deformation amplitude is less than 10%, the changes of measured FRF curves are very small. So the system can be regarded as a linear system, and the relative error of the equivalent linearized method is less than 6%, and this approximate modelling method is feasible. But when the ratio of tip dynamic displacement amplitude to static deformation amplitude is more than 20%, the value of the measured FRF at resonant frequency point is significantly decreased, and the relative error of the equivalent linearized method is more than 50%, and this method will not longer suitable.

Acknowledgement: This work was supported by the National Natural Science Foundation of China (Grant No. 11172238), and “111” Project (Grant No. B07050).

REFERENCES

[1] C. E. S. Cesnik, P.J. Senatore, W.H.Su. X-HALE: A Very Flexible UAV for Nonlinear Aeroelastic Tests[C]// 51st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference. Orlando, Florida: MICHIGAN UNIV ANN ARBOR DEPT OF AEROSPACE ENGINEERING, 2010: 12-15.

[2] Hodges D.H., Dowell E.H.. Non-linear equation of motion for the elastic bending and torsion of twisted non-uniform rotor blade[R]. NASA TN D-7818, 1974.

[3] Rose A, Friedman P. The Nonlinear Behavior of Elastic Slender Straight Beams Undergoing Small Strains and Moderate Rotation[J], Journal of Applied Mechanics, 1979, 46(1): 161-168.

[4] Dowell E.H, Traybar J, Hodges D.H. An Experimental-theoretical Correlation Study of Non-linear Bending and Torsion Deformation of a Cantilever Beam[J], Journal of Sound and Vibration, 1977, 50(4): 533-544.

[5] Crespo da Silva M R M, Glynn C C. Nonlinear flexural-flexural-torsional dynamics of inextensional beams-I: Equations of motion[J], Journal of Structural Mechanics , 1978, 6(4) :437-448.

[6] Crespo da Silva M R M, Glynn C C. Nonlinear flexural-flexural-torsional dynamics of inextensional beams-II:Forced motions[J]. Journal of Structural Mechanics , 1978, 6(4) :449-461.

[7] Howard E, Hinnant, Dewey H Hodges. Non- linear Analysis of a Cantilever Beam[J], AIAA JOURNAL, 1988, 26(12): 1521-1527.

[8] P.J.A.Minguet. Static and dynamic behavior of composite helicopter rotor blades under large defections[D]. Phd Dissertation, Massachusetts Institute of Technology, 1989.

[9] T. Kim, J. Dugundji. Nonlinear Large Amplitude Vibration of Composite Helicopter Blade at Large Static Deflection[J]. AIAA JOURNAL, 1993, 31(5): 938~946.

[10] Demeter G. Fertis. Nonlinear Mechanics [M]. US: CRC Press LLC, 1999.

[11] P.F.Pai, T.J.Anderson, E.A.Wheater. Large-deformation tests and total-Lagrange finite -element analyses of flexible beams[J]. International Journal of Solids and Structures，2000, 37(21) : 2951-2980.

[12] Wu, G., He, X., Pai, P. F. Geometrically Exact 3D Beam Element for Arbitrary Large Rigid-Elastic Deformation Analysis of Aerospace Structures[J], Finite Elements in Analysis and Design, 2011, 47: 402-412.

[13] D. M. Tang, E. H. Dowell. Experimental and Theoretical Study for Nonlinear Aeroelastic Behavior of a Flexible Rotor Blade[J], AIAA JOURNAL 1993, 31(6): 1133-1142.