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3D ELLIPTICAL CRACK DEPTH ESTIMATION FROM 2D SURFACE DISPLACEMENT OBSERVATION

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Abstract. This paper presents a method to estimate the depth of an elliptical 3D crack from 2D observations on surface displacement fields. An iterative numerical procedure with least-squares error minimisation is developped to compare pseudo-experimental displacements fields with a FEM displacement fields database and estimate crack depth.

1 Introduction

The 3D geometry of a crack and mainly its depth has been investigated on surface with acoustic [1] or magnetic [2] effects but not yet with only 2D surface Digital Image Correlation type observations. Dealing with visual informations requires 3D observation methods such as tomography [3]. In this work is exposed a method to determine the depth of a 3D elliptical crack from 2D observation of displacement field on the surface.

The first step is to build a virtual database of surface reference displacement fields with the Finite Element Method softwares Zébulon and Z-cracks [4] for sampled relative depth vs. surface crack length. The database is constituted of reference fields U_s parameterized on depth parameter d. Displacement fields on surface are post-processed in a circular patch around the crack.

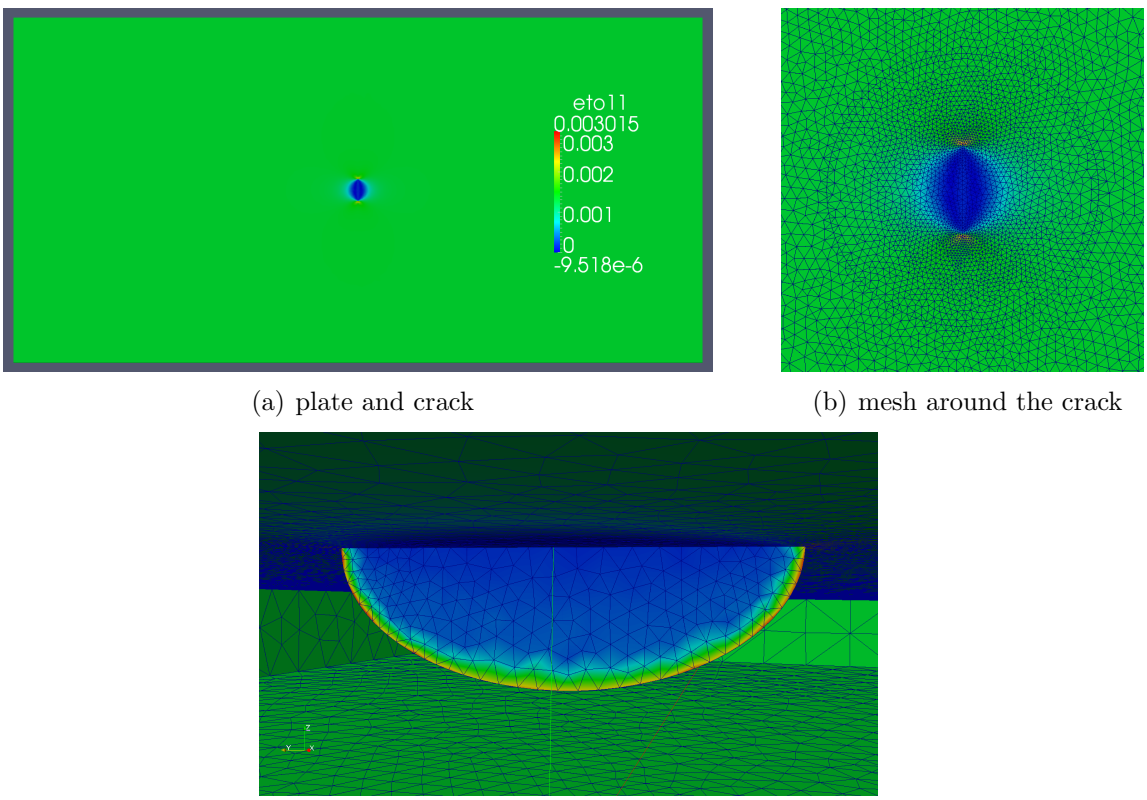
Secondly, the experimental field U_e is considered on surface around the crack in a circular patch identical to the ones on reference fields. It is compared to the reference database and an optimization process gives an estimated depth. The projection is a minimization algorithm for a least-squares error [5] between U_s and U_e. Two interpolations are computed to deal with various spacial discretizations and depth sampling.

The method is evaluated considering 2D pseudo-experimental fields built from FEM simulations. The surface displacements are interpolated on a regular grid which mimics

a pixelised field and altered with a variable amplitude white noise. Several artificial experimental fields are generated for various grid size, noise amplitude and patch size.

2 FEM displacement fields database

In order to estimate elliptical crack depth from experimental data, we need to build a database of surface reference displacement fields for various crack depth. They are obtained with FEM simulations with Z-set software and particularly Zébulon solver and Z-cracks module. Elliptical cracks are sampled with various depth and introduced in a rectangular plate which dimensions are large enough to stand for an infinite plate and avoid finite size effects. For this work, all length are in mm and we chose a plate of length $L = 300$, width $l = 150$ and thickness $t = 15$. $\vec{x}$ direction corresponds to plate length, $\vec{y}$ to width and $\vec{z}$ to thickness. Crack length on surface is $2a = 10mm$ and crack depth d_s is sampled 50 times between 0.1 and 5mm which gives of a penny shape crack geometry for the maximal depth.



(a) plate and crack

(b) mesh around the crack

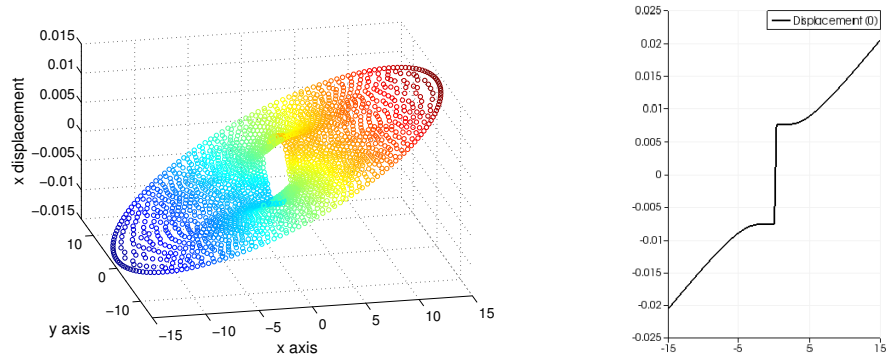
(c) inside view

Figure 1: ϵ_{xx} strain fields for a crack in a plate with $2a = 10$ and $d = 3$ from Zébulon & Z-cracks, visualisation with Paraview

Material parameters for FEM computations stand for a steel plate with Young's mod-

ulous $E = 210GPa$ and Poisson's ratio $\nu = 0.3$. Mechanical loading of tension in $\vec{x}$ direction is applied with displacements $u_x = \pm 0.2$ on lateral sides. It generates applied strain $\epsilon_{xx} = 1.33e^{-3}$ and stress $\sigma_{xx} = 280Mpa$. Simulation results are presented with strain fields on external surfaces of the material Fig.2.

Surface nodes in the circular patch around the crack which radius $R_p = 15$ are selected for post-processing. Displacement u_x and u_y in $\vec{x}$ and $\vec{y}$ directions are post-processed for sampled depth to build the database Fig.2(a). These fields are then normalized in regard to crack opening Fig.2(b) as loading stays in small displacement hypothesis. For all sampled depth value, crack opening is set to 0.01.



(a) u_x displacement in patch (b) u_x displacement

Figure 2: displacement in circular patch for depth $d = 3$ crack in displacement database

3 Numerical optimization procedure

The depth of the experimental crack is obtained by minimizing an error function between experimental displacement U_e and corrected displacement field U_{sc} from database. We chose a least-squares error between the two displacement fields.

$$Err\left(\lambda\right) = \left[U_{sc}\left(\lambda, X_e\right) - U_e\left(X_e\right)\right]^2 \tag{1}$$

in which λ stands for a set of optimisation variables with d being the estimated crack depth and (α, β, γ) being small displacement rigid body movement parameters such as

$$U_{sc}\left(\lambda\right) = U_s\left(d\right) + \alpha U_x + \beta U_y + \gamma U_z \tag{2}$$

with U_x and U_y being unitary displacements in $\vec{x}$ and $\vec{y}$ directions and U_z the appropriate expression of plane displacements for a $\vec{z}$ direction rotation. Solution field $U_s(d)$ that depends on estimated depth is interpolated on database solution fields U_s with sampled depth d_s.

Summation are executed using rows and columns and scalar products and notations are simplified throughout this document as long as there is no trouble of understanding.

Minimisation is performed using the stationnarity of error function in regard of optimization variables :

$$\min_{\lambda} \left(Err\left(\lambda\right)\right) \Leftrightarrow \frac{dErr\left(\lambda\right)}{d\lambda} = 0 \tag{3}$$

Through an iterative approach the problem is solved by minimising the error in regard to the increment $\delta\lambda$ to a given value λ :

$$\min_{\delta\lambda} \left(Err\left(\lambda^0 + \delta\lambda\right)\right) \Leftrightarrow \frac{dErr\left(\lambda + \delta\lambda\right)}{d\left(\delta\lambda\right)} = 0 \tag{4}$$

Using the firts order approximation expressed as

$$U_{sc}\left(\lambda + \delta\lambda\right) = U_{sc}\left(\lambda\right) + \frac{\partial U_{sc}\left(\lambda\right)}{\partial\lambda}\delta\lambda \tag{5}$$

the expression of minimisation equation becomes

$$\min_{\delta\lambda} \left(Err\left(\lambda + \delta\lambda\right)\right) \Leftrightarrow 2\left[\frac{\partial U_{sc}\left(\lambda\right)}{\partial\lambda}\delta\lambda + U_{sc}\left(\lambda\right) - U_e\right] \cdot \frac{\partial U_{sc}\left(\lambda\right)}{\partial\lambda} = 0 \tag{6}$$

The increment $\delta\lambda$ is obtained as

$$\delta\lambda = \frac{\left(U_{sc}\left(\lambda\right) - U_e\right) \cdot \frac{\partial U_{sc}(\lambda)}{\partial\lambda}}{\frac{\partial U_{sc}(\lambda)}{\partial\lambda}^2} \tag{7}$$

The next iteration value is estimated as

$$\lambda^0_{i+1} = \lambda^0_i + \delta\lambda_i \tag{8}$$

This expression is then declined for each variable d, α, β and γ.

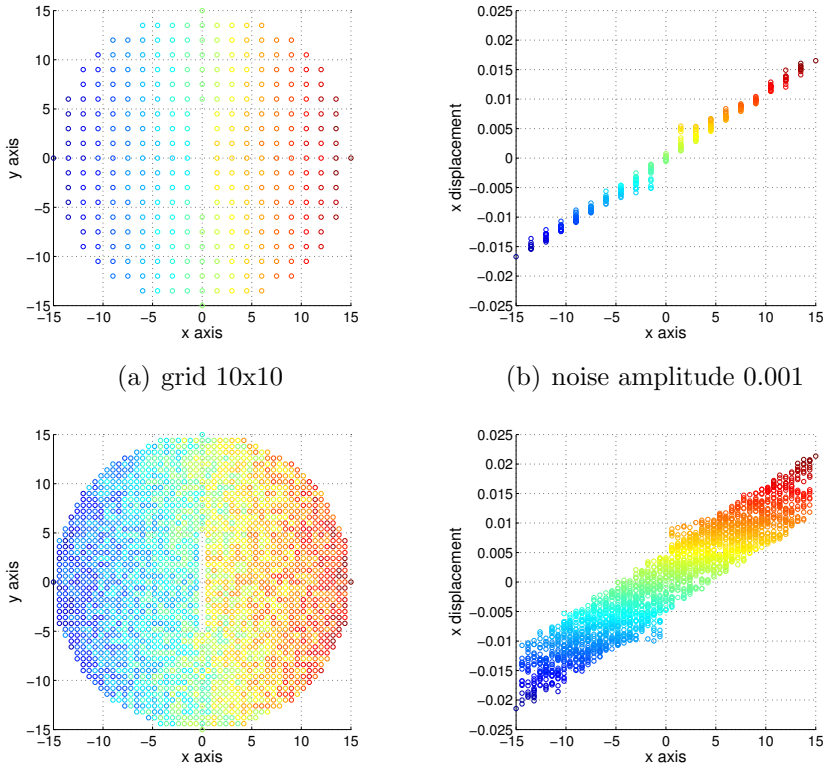
The iterative procedure is stopped when the estimated depth is stabilized and the relative error is smaller than a given tolerance.

$$\frac{\delta d}{d^0} < 1e^{-3} \tag{9}$$

4 Application cases and validation

4.1 Pseudo-experimental displacement fields

As experimental displacement fields are not yet available for elliptical cracks, we decided to create pseudo-experimental ones. A solution field is interpolated on a regular grid in a circular patch around the crack with stands for a pixelised field, the value discribing the grid is the number of points in a radius along axis. Various grid size are considered as shown Fig.3(a) and 3(c). The interpolated displacement field is then scattered with a white noise with various amplitude Fig.3(b) and 3(d). Both modifications lead to artificial fields that mimic experimental observations.



(a) grid 10x10

(b) noise amplitude 0.001

(c) grid 25x25

(d) noise amplitude 0.01

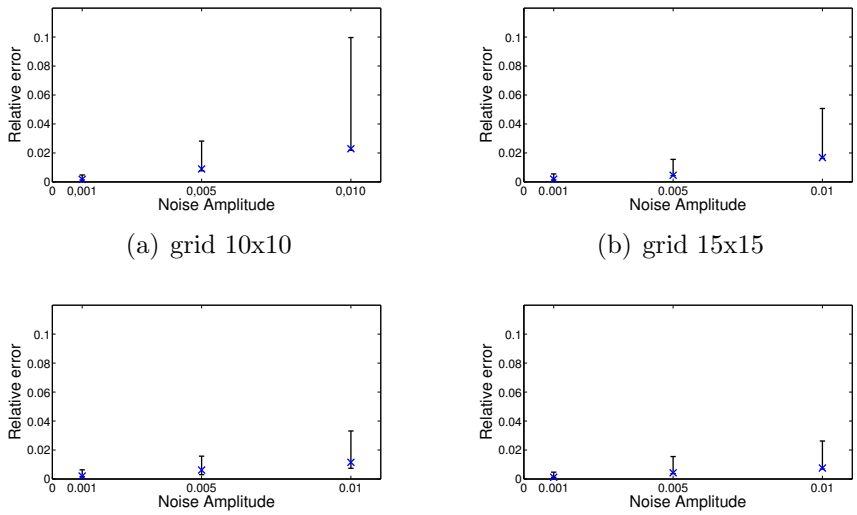
Figure 3: Various interpolation grid size and noise amplitude for pseudo-experimental u_x direction displacement fields

4.2 Grid size and noise amplitude effect on depth diagnostic accuracy

Various grid size and noise amplitude combination were tested to evaluate the performance of the optimisation algorithm. Grid size is sampled in [10,15,20,25], noise amplitude in sampled in [0.001,0.005,0.01]. The highest value of noise amplitude is chosen to be the same order as crack opening.

For each combination of noise and grid size, 10 realisations of depth estimation are computed. Results are presented Fig. 4 with average accuracy and dispersion intervals.

Excellent results are obtained with average relative error inferior to 3% for the estimation of crack depth for a noise with magnitude as high as crack opening. Two intuitive aspects are here spotted : estimation of crack depth is better with a finer grid and a smaller noise, it naturally corresponds to a higher resolution and robust observation of displacements.



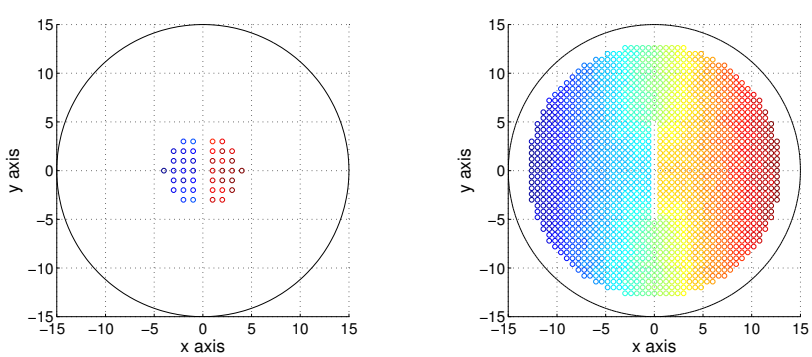
(a) grid 10x10 (b) grid 15x15

(c) grid 20x20 (d) grid 25x25

Figure 4: Relative error for depth estimation of a 5mm length and 2.3mm depth crack with 0.01 mm crack opening using a 30mm diameter 2D patch

4.3 Patch size convergence on depth diagnostic

As experimental crack patch radius was initially chosen equal to database radius patch, we found interesting to evaluate the performance of depth estimation in regard to experimental crack patch size. Noise amplitude is taken equal to 0.001, grid size is sampled in [15,20,25] and patch radius r_p in [3:15] as examples are shown Fig. 5.



(a) grid 15x15, r_p=4 (b) grid 25x25, r_p=13

Figure 5: Various combination of grid size and patch radius

The interesting result is that depth prediction is accurate with a patch diameter superior to crack length on surface Fig.6. We assess that the local displacements around crack tip on surface has to be taken into account.

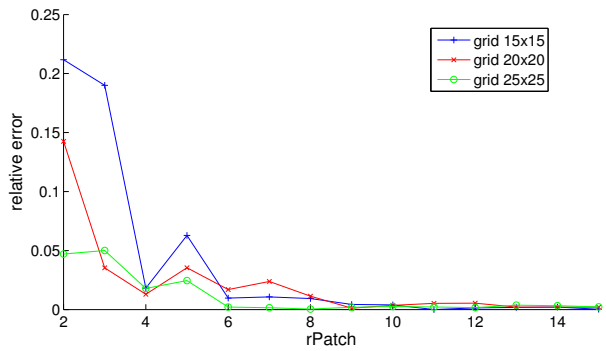


Figure 6: Convergence of depth estimation vs. patch size for various grid size

5 Conclusions and prospects

A reference displacement field database was obtained from FEM simulation with Zébulon and Z-cracks software. An iterative optimization procedure is developped to estimate crack depth from a least square error between experimental and reference displacement fields. The formulation allows a correction of small rigid body movement for crack orientation and position unprecision. Pseudo-experimental displacement were build to overcome the lack of experimental data. The identification procedure gave very good results with relative errors inferior to 3% with various combinations of spatial resolution, noise on displacement and surface patch size.

This depth estimation procedure will laterly be applied to multiple cracking configurations and its performances confronted to interaction between cracks. The further developpement of the numerical approach will allow to take into account unprecision on experimental crack opening and length.

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