11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
E. Oñate, J. Oliver and A. Huerta (Eds)

2.5D MODELING OF SOIL-STRUCTURE INTERACTION USING A COUPLED MFS-FEM FORMULATION

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Key Words: *Soil-structure interaction, frequency domain, MFS-FEM, 2.5D formulation.*

1 INTRODUCTION

Soil-structure interaction dynamic problems have been tackled by researchers using a variety of methods, ranging from analytical solutions (valid only for simple configurations) to complex numerical strategies, including coupled formulations between different numerical methods. An extensive review of such techniques can be found in review papers such as the recent one by Clouteau et al. [1].

Some recent contributions found in the literature, such as the ones by Godinho et al. [2] or by Alves-Costa et al. [3], propose the use of the FEM coupled with either the MFS or the BEM to efficiently and accurately model vibration propagation originated in a structure located within an infinite or semi-infinite elastic medium. These models allow incorporating all the physics of the linear propagation problem while keeping a quite elegant mathematical description of the physical system. The possibility of simulating long-distance wave propagation contrasts with other computationally alternatives, such as the use of FEM approaches [4].

An extension of the work presented by Godinho et al. [2] is now developed, presenting a coupled MFS-FEM formulation for the 2.5D soil-structure dynamic problem. The 2.5D approach is quite interesting in problems described by a constant cross section (two-dimensional, 2D) being actuated by a three-dimensional (3D) loading. Tadeu and Kausel [5] proposed the Green's functions for 2.5D elastodynamics problems, which can be used when modelling this type of problems within infinite elastic media. This kind of model is very adequate when dealing with the dynamics of transport infrastructures such as tunnels, roads or railroads, and surrounding media, since the requirement of invariability of the domain along the development direction is guaranteed by the geometry of the problem itself. The model now presented can easily be generalized to moving sources and integrated in a global model that comprises the problem of vibrations induced by traffic from the source to the receiver, as recently proposed by Lopes et al. [6].

In the following sections, the 2.5D formulation is briefly introduced and the coupled MFS-FEM model is described. Then, the proposed model is verified by solving two test cases and comparing its numerical results with reference solutions given either analytically and/or numerically. The final part of the paper comprises a set of numerical results, illustrating the capabilities of the 2.5D MFS-FEM formulation in the analysis of an elastic tunnel embedded in an infinite soil.

2 2.5D FORMULATION

Consider a physical system in which a solid inclusion, presenting a constant geometry along a longitudinal direction (the z direction), is embedded in a spatially homogeneous elastic medium, and being subjected to a harmonic dilatational point source, which is oscillating with an angular frequency ω. Since the system's geometry is infinite and does not change along the z direction, the 3D field may be determined by a summation of 2D solutions, corresponding to different wavenumbers,

$$k_\alpha = \sqrt{\frac{\omega^2}{\alpha^2} - k_{z\,m}^2}\ \text{, with } \operatorname{Im} k_\alpha < 0\ , \tag{1}$$

and α representing the dilatational wave velocity, $k_{z\,m}$ the axial wavenumber defined by $k_{z\,m} = \frac{2\pi}{L_{vs}} m$, (with $m = -M,...,0,...,M$), and L_{vs} the distance between virtual point sources, uniformly spaced along z.

Note that, in order to avoid any spatial contamination from the virtual sources, the distance L_{vs} must be sufficiently large, and that the case with $k_z = 0$ corresponds to the 2D problem.

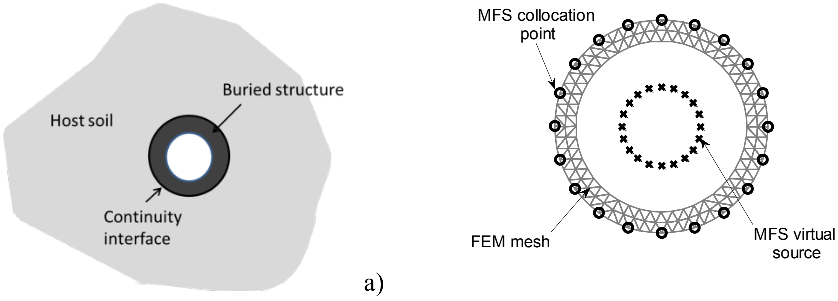
3 COUPLED MFS-FEM FORMULATION

An extension of the work presented by Godinho et al. [2], where a 2D model has been described, is now developed, with a coupled MFS-FEM formulation for the 2.5D soil-structure dynamic problem being now proposed. For that case, assuming null initial conditions, the frequency domain wave equation can be written as

$$(\lambda + 2\mu)\nabla\nabla\cdot\mathbf{u} - \mu\,\nabla\times\nabla\times\mathbf{u} + \omega^2\rho\,\mathbf{u} = 0 \tag{2}$$

where $\mathbf{u}$ is the displacement vector, λ and μ are the Lamé's constants and ρ is the mass density of the elastic medium, and β is its shear wave velocity.

Besides satisfying this governing equation at all points of both the structure and the soil, the adequate continuity and equilibrium conditions between these two parts of the model must be imposed along a continuity interface (see Figure 1).



a) b)

Figure 1: a) Schematic representation of the problem and b) its discretization with the MFS-FEM model.

A coupled model is proposed to solve this elastic 2.5D problem, with the meshless Method of Fundamental Solutions (MFS) being adopted to model the host unbounded medium and the Finite Element Method (FEM) used to discretize the embedded structure. These methods are now briefly introduced and the coupling procedure is referred.

3.1 The Method of Fundamental Solutions (MFS)

The MFS approximates the solution in the outer medium in terms of a linear combination of fundamental solutions of the governing differential equation. In fact, a set of NS fictitious sources is located outside the elastic domain (namely at points X_0^n, with $n = 1 \ldots NS$), in order to circumvent singularities, and then the displacement field at any point of the host medium can be written as a summation of the contribution of those virtual loads, with *a priori* unknown amplitudes A_j^n. Therefore, the displacement u_i at a generic point X, along direction $i = x, y, z$ is represented by:

$$u_i(X,k_z,\omega) = \sum_{n=1}^{NS}\sum_{j=1}^{3} G_{ij}\left(X, X_0^n, k_z, \omega\right) A_j^n\left(k_z,\omega\right) \tag{3}$$

where G_{ij} corresponds to the displacement generated at point X, along direction i, by a unit load acting along direction j at point X_0^n, which can be determined by using the appropriate fundamental solutions.

The required Green's functions $G_{ij}\left(X, X_0^n, k_z, \omega\right)$ for displacements in the x, y and z directions, with $i, j = x, y, z$, in an infinite homogeneous solid medium, were presented by Tadeu and Kausel [5], and can be shortly recapitulated as:

$$G_{xx}(X, X_0, k_z, \omega) = \frac{1}{4\mathrm{i}\,\rho\,\omega^2}\left[k_s^2\; H_{0\beta} - \frac{1}{r}B_1 + \gamma_x^2\; B_2\right] \tag{4a}$$

$$G_{yy}(X, X_0, k_z, \omega) = \frac{1}{4\mathrm{i}\,\rho\,\omega^2}\left[k_s^2\; H_{0\beta} - \frac{1}{r}B_1 + \gamma_y^2\; B_2\right] \tag{4b}$$

$$G_{zz}(X,X_0,k_z,\omega)=\frac{1}{4\mathrm{i}\,\rho\,\omega^2}\left[k_s^2\;H_{0\beta}-k_z^2\;B_0\right] \tag{4c}$$

$$G_{xy}(X,X_0,k_z,\omega)=G_{yx}(X,X_0,k_z,\omega)=\gamma_x\,\gamma_y\frac{1}{4\mathrm{i}\,\rho\,\omega^2}B_2 \tag{4d}$$

$$G_{xz}(X,X_0,k_z,\omega)=G_{zx}(X,X_0,k_z,\omega)=\mathrm{i}\,k_z\,\gamma_x\frac{1}{4\mathrm{i}\,\rho\,\omega^2}B_1 \tag{4e}$$

$$G_{yz}(X,X_0,k_z,\omega)=G_{zy}(X,X_0,k_z,\omega)=\mathrm{i}\,k_z\,\gamma_y\frac{1}{4\mathrm{i}\,\rho\,\omega^2}B_1 \tag{4f}$$

with $B_n=k_\beta^n H_{n\beta}-k_\alpha^n H_{n\alpha}$, $k_\beta=\sqrt{k_s^2-k_z^2}$ with $\mathrm{Im}\,k_\beta<0$, $k_s=\omega/\beta$, $\gamma_l=\partial r/\partial x_l=x_l/r$ with $l=x,y$, $H_{n\alpha}=H_n^{(2)}(k_\alpha r)$, $H_{n\beta}=H_n^{(2)}(k_\beta r)$ ($H_n^{(2)}$ is the Hankel function of second type and n^{th} order) and r stands for the distance between the source and field points.

Note that, by imposing the adequate boundary conditions, in this case, by prescribing continuity of normal and tangential displacements and stresses along the continuity physical interface between both elastic media, at a set of NS collocation points, a linear equation system with $2\times NS$ equations and $2\times NS$ unknowns may be established. The resolution of this equation system leads to the determination of the unknown amplitudes A_j^n, which may be used to compute the displacements at any point X of the host elastic medium.

3.2 The Finite Element Method (FEM)

The 2.5D finite elements approach is based on the concept of discretization of the domain along the cross-section of the problem, being the solution on the remaining direction evaluated making use of a Fourier transformation. So, assuming that the response of the structure is linear, the analysis can be carried out in the wavenumber/frequency domain. All variables, i.e., loads (action) and displacements (response), must be transformed to the wavenumber/frequency domain by means of a double Fourier transform, related both with the development direction and with time.

The mentioned approach allows establishing the equilibrium of the domain by means of the following equation, written in terms of nodal variables [7-9]:

$$\left(\int_x\int_y B^T(k_z)DB(k_z)\,\mathrm{d}x\,\mathrm{d}y-\omega^2\int_x\int_y N^T\rho\,N\,\mathrm{d}x\,\mathrm{d}y\right)u_n(k_z,\omega)=p_n(k_z,\omega) \tag{5}$$

Adopting the classic finite elements notation leads to,

$$\left[K\right]=\int_x\int_y B^T(k_z)\,D\,B(k_z)\,\mathrm{d}x\,\mathrm{d}y \tag{6}$$

and

$$\left[M\right]=\int_x\int_y N^T\rho\,N\,\mathrm{d}x\,\mathrm{d}y \tag{7}$$

where $[K]$ and $[M]$ are the stiffness and mass matrices, respectively.

As usual, matrix $[B]$ is derived from the product of the differential operator matrix $[L]$ (in the transformed domain) and matrix $[N]$. Since the direction z is transformed to the wavenumber domain, the derivatives with respect to k_z are analytically computed, as presented in the following expression:

$$[L]=\begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial y} & 0 & \mathrm{i}k_z \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & \mathrm{i}k_z & 0 \\ 0 & 0 & \mathrm{i}k_z & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}^T \tag{8}$$

The computational efficiency can be improved by dividing matrix $[K]$ into sub-matrices, independent of the wavenumber and frequency. This is achieved by considering the matrix $[B]$ as the sum of two matrices, whereby the numerical and analytical derivatives are separated. Equation (5) can then be replaced by:

$$\left([K]_1 + \mathrm{i}\,k_z[K]_2 + k_z^2[K]_3 - \omega^2[M]\right)u_n(k_z) = p_n(k_z) \tag{9}$$

3.3 Coupling the MFS and FEM models

To couple those two numerical methods, a direct coupling procedure is suggested, by imposing the continuity and equilibrium conditions at the boundary nodes,

$$\mathbf{u}_{FEM}(X\ ,\omega) - \mathbf{u}_{MFS}(X\ ,\omega) = 0 \tag{10a}$$

$$\mathbf{F}_{FEM}(X\ ,\omega) - \mathbf{F}_{MFS}(X\ ,\omega) = 0 \tag{10b}$$

The coupling strategy enables the use of non-matching discretization nodes for each sub-domain of the model: the host soil modelled by the meshless MFS and the structure discretized by the mesh-based FEM. This coupling procedure, also adopted in [2], is quite easy to implement, not requiring significant modifications of the MFS and FEM formulations, and leading to flexible, accurate and efficient analyses.

4 MODEL VERIFICATION

The proposed 2.5D MFS-FEM model is first verified by analyzing its results for the case of an infinite homogeneous elastic medium and then for a system configuration with a solid inclusion embedded in a different infinite medium.

4.1 Homogeneous elastic medium

In the first verification case, the coupled 2.5D MFS-FEM method is verified by comparing its results with the corresponding fundamental solutions, available for the case of a homogeneous unbounded elastic medium [5], and also performing a comparison with the ones given by a 2.5D FEM model with a PML [4]. The infinite medium represents a homogeneous soil with the following properties: a Young Modulus of $3.09\times10^{8}\ \mathrm{Pa}$, a Poisson coefficient of 0.3 and a density of $1900\ \mathrm{kg/m^3}$. This full-space is excited by a vertical unit load ($P_0 = 1\,\mathrm{e}^{\mathrm{i}\omega t}$), located at a point of coordinates $(x=0\ \mathrm{m};\ y=-20\ \mathrm{m};\ z=0\ \mathrm{m})$, which is harmonic with respect to the longitudinal direction z and that is oscillating with a frequency of $40\ \mathrm{Hz}$.

For this case, some results are presented in Figure 2, considering varying values of the axial wavenumber (k_z), in the range $[0;2\ \mathrm{rad/m}]$. These results correspond to the real and imaginary parts of the vertical displacement response for the selected frequency, computed at a numerical receiver located at a point of coordinates $(x=0\ \mathrm{m};\ y=0\ \mathrm{m};\ z=0\ \mathrm{m})$. In order to use the proposed 2.5D MFS-FEM model, the same elastic properties were ascribed to the host solid medium and to a circular subdomain with radius of $3\ \mathrm{m}$, centered at the receiver, discretized with the FEM and coupled to the MFS for modeling the outer field.

Analyzing the results shown in Figure 2, a perfect agreement is observed between the proposed MFS-FEM model and both the fundamental solution for the full-space and the numerical results obtained by a FEM-PML model.

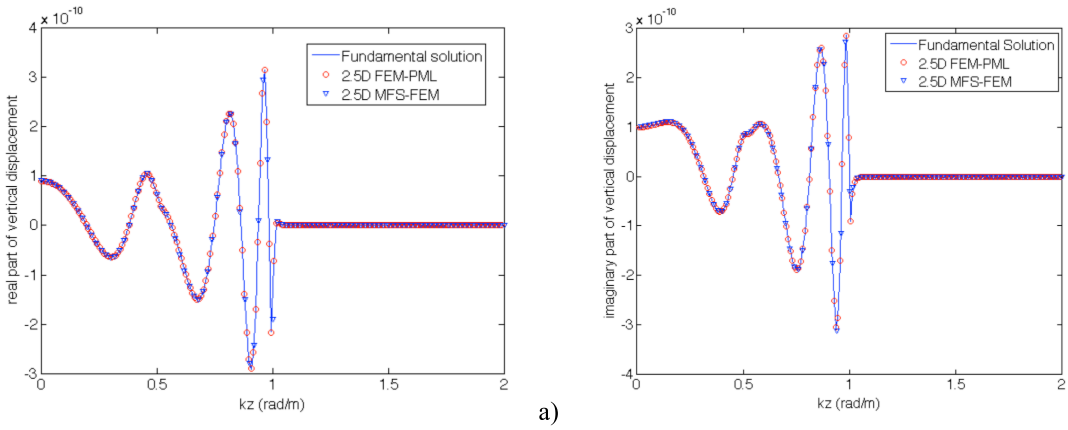


Figure 2: Comparison against the fundamental solution for a homogeneous medium: a) real part and b) imaginary part of the vertical displacement response for a 40 Hz excitation.

4.2 Elastic medium with embedded solid structure

For the second verification case, a tunnel embedded on a full-space is considered. The external radius of the tunnel is $3.0\ \mathrm{m}$, being internally lined with $0.25\ \mathrm{m}$ of concrete (Figure 3a). The unbounded medium exhibits a density of $2000\ \mathrm{kg/m^3}$, and enables the propagation of elastic waves with velocities of $\alpha = 944\ \mathrm{m/s}$ and $\beta = 944\ \mathrm{m/s}$, while the elastic properties of the concrete lining correspond to a Young Modulus of $50\times10^{9}\ \mathrm{Pa}$, a Poisson coefficient of 0.3 and a density of $2500\ \mathrm{kg/m^3}$. A vertical unit load is applied on the tunnel invert, at coordinates $(x=0\ \mathrm{m};\ y=-2.75\ \mathrm{m};\ z=0\ \mathrm{m})$, oscillating with a varying frequency f. For verification purposes, the results in terms of the vertical displacements were computed at three observation points, namely R1, R2 and R3, located along the vertical xOy plane ($z=0\ \mathrm{m}$), at coordinates $(x=0\ \mathrm{m};\ y=20\ \mathrm{m})$, $(x=0\ \mathrm{m};\ y=10\ \mathrm{m})$ and $(x=10\ \mathrm{m};\ y=0\ \mathrm{m})$, in the frequency range up to $80\ \mathrm{Hz}$.

Reference results were evaluated by the 2.5D FEM model with a PML referred above [4] and making use of the semi-analytical model PiP developed by Hussein and Hunt [10]. With the proposed coupled 2.5D MFS-FEM approach, the problem was modelled by discretizing the solid ring structure with the FEM and a 240 elements mesh, and the outer infinite medium using the MFS, with non-coincident positions of the interface nodes for the two coupled methods. The number of collocation points for the MFS was set equal to 50, and the same number of virtual sources was adopted. Figure 3b illustrates the adopted MFS-FEM discretization along a vertical plane, with the FEM mesh and the MFS virtual sources and collocation points.

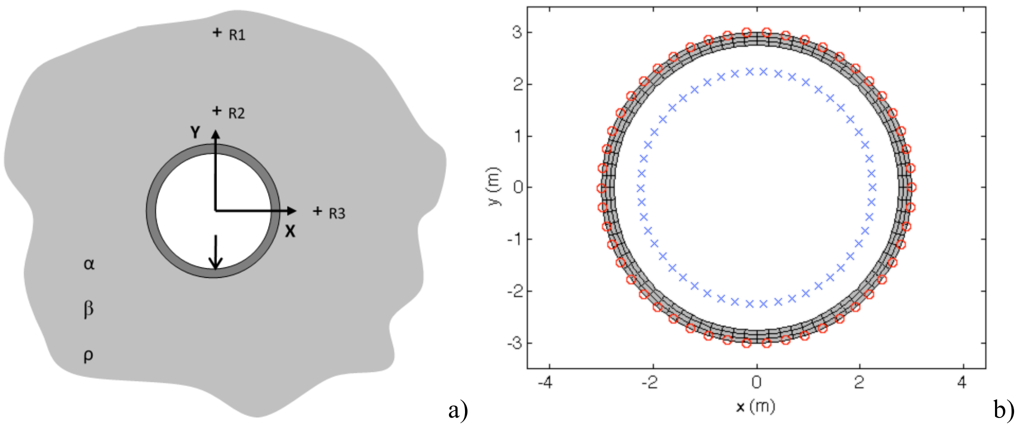


Figure 3: a) Schematic representation, on the vertical plane $z = 0.0$ m , of the verification model with a solid structure in an elastic medium; b) FEM discretization mesh, and MFS virtual sources (x x x) and collocation points (o o o).

Figure 4 presents the comparison of the computed vertical displacement results by the three methodologies, at receivers R1, R2 and R3. One may observe a very close agreement between the proposed numerical results (2.5D MFS-FEM) and both the semi-analytical results by the PiP model and the numerical results determined using the 2.5D FEM-PML model.

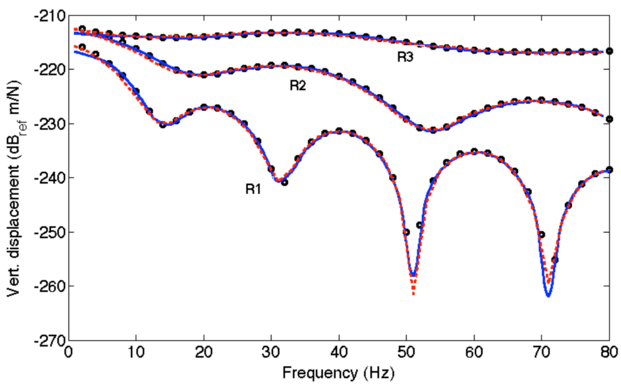


Figure 4: Vertical displacement transfer function at receivers R1, R2 and R3: (———) 2.5D FEM-PML model, (– – – –) PiP model and (o o o o) 2.5 MFS-FEM model responses.

5 NUMERICAL APPLICATION

The verifications presented herein reveal the good accuracy of the proposed 2.5D MFS-FEM coupled model, indicating that it can be used to address more complex geometries. In the present section, the authors make use of an identical system configuration presented in the previous section, with a concrete ring structure embedded in an elastic infinite medium, for the analysis of a soil-structure interaction 2.5D problem. A harmonic vertical load described above excites the inner surface of the concrete tunnel, vibrating at the vertical plane $z = 0.0\,\mathrm{m}$.

A grid of numerical receivers is now considered in the host soil, leading to an observation region, in each vertical plane, from coordinates $(x = -10\,\mathrm{m};\, y = -10\,\mathrm{m})$ to $(x = 10\,\mathrm{m};\, y = 10\,\mathrm{m})$. To illustrate the applicability of the 2.5D model, in the first set of results, the computed real part of the norm of the displacement field can be observed in Figure 5, along 4 different vertical planes, ranging from $z = 0.0\,\mathrm{m}$ to $z = 9.0\,\mathrm{m}$, for a selected frequency of excitation of $60\,\mathrm{Hz}$.

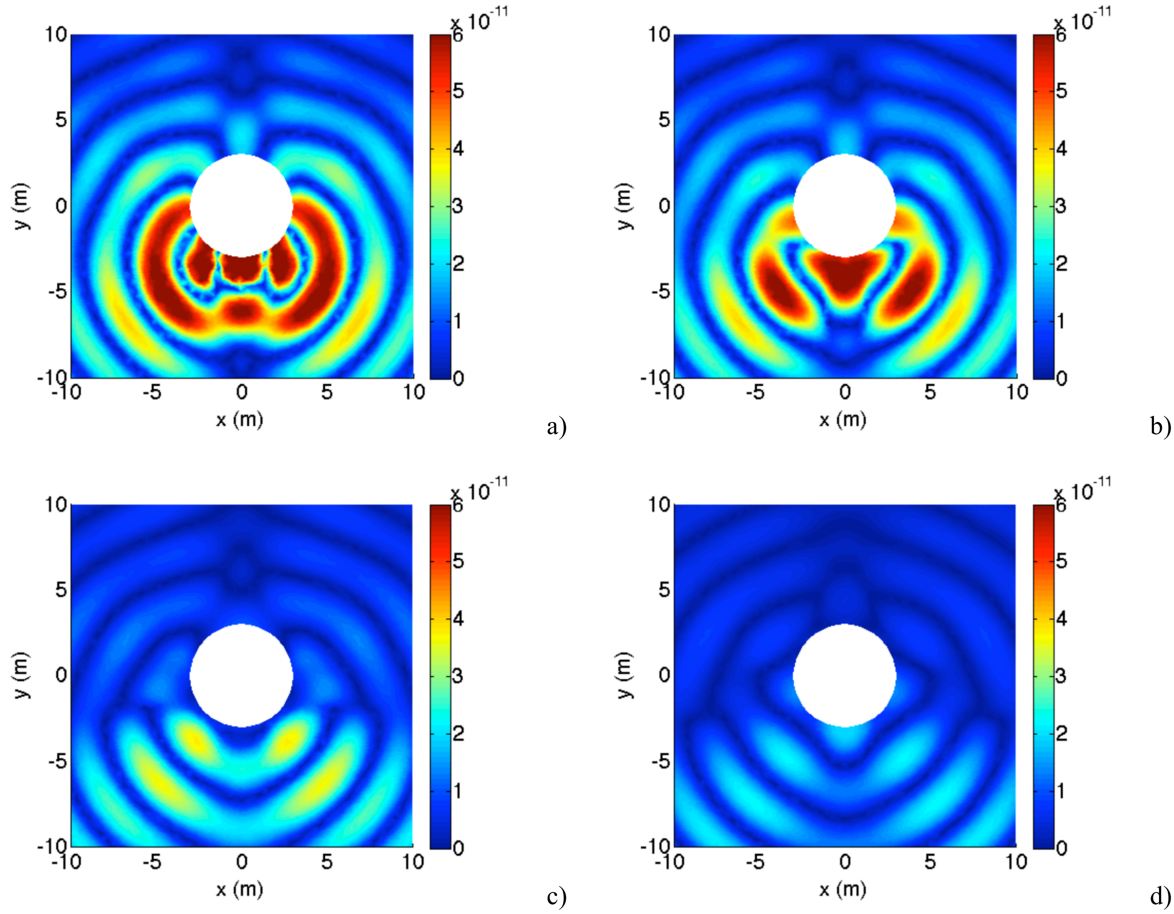


Figure 5: Real part of the field of displacements, for a harmonic point load with the frequency of excitation of $60\,\mathrm{Hz}$, along different vertical planes: a) $z = 0.0\,\mathrm{m}$; b) $z = 3.0\,\mathrm{m}$; c) $z = 6.0\,\mathrm{m}$; d) $z = 9.0\,\mathrm{m}$.

As expected, when vertical planes farther away from the load plane are considered, the vertical displacement field is observed with lower amplitude values, although maintaining the same radiated patterns from the embedded structure.

Additionally, a second set of results is illustrated in Figure 6, with the amplitude of the real part of the norm of the displacement field being plotted along a vertical plane xOy that corresponds to $z = 5.0\ \mathrm{m}$, for different frequencies of excitation of the harmonic point load, in the range from $20\ \mathrm{Hz}$ up to $80\ \mathrm{Hz}$.

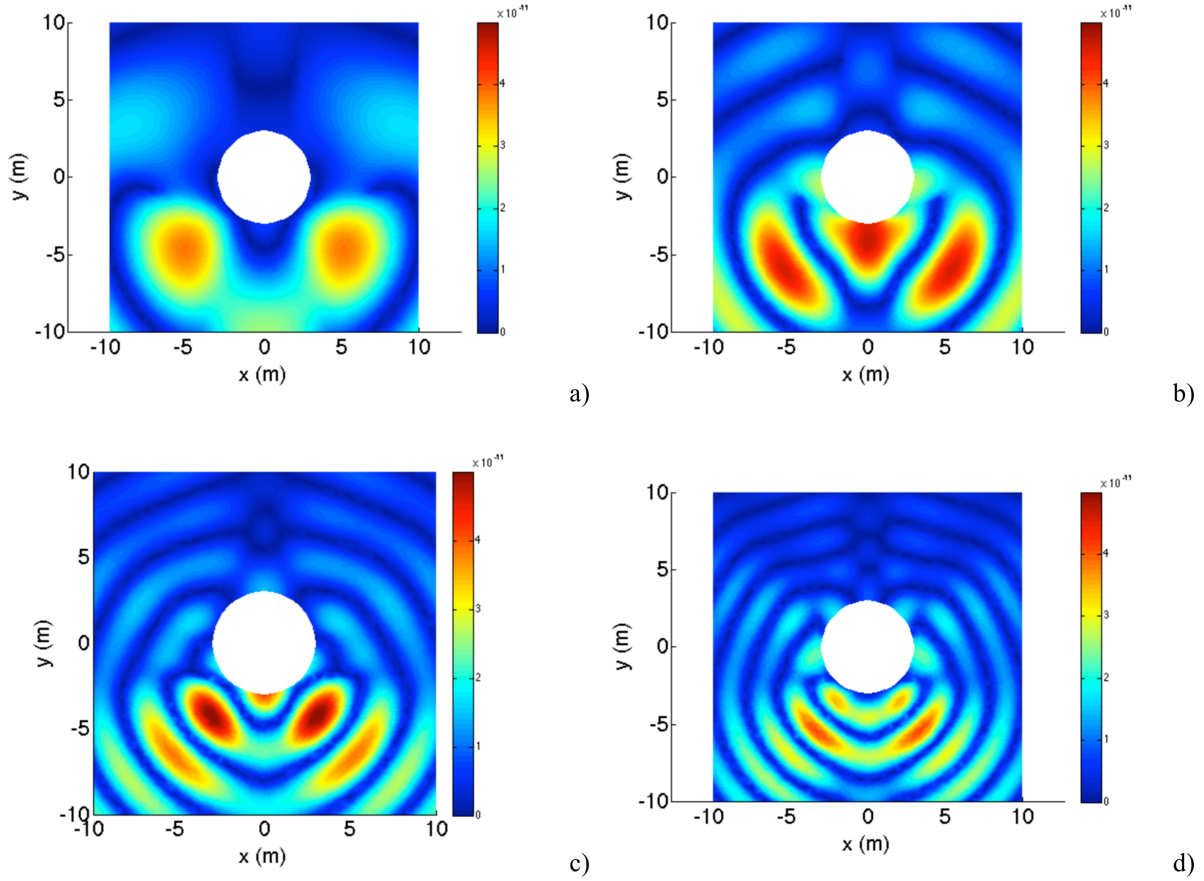


Figure 6: Real part of the field of displacements, along the vertical plane xOy corresponding to $z = 5.0\ \mathrm{m}$, for different frequencies of excitation of the harmonic point load: a) $f = 20\ \mathrm{Hz}$; b) $f = 40\ \mathrm{Hz}$; c) $f = 60\ \mathrm{Hz}$; d) $f = 80\ \mathrm{Hz}$.

Observing these plots, the increasing radiation pattern complexity is perceived, as the excitation frequency is incremented and the corresponding wavelengths are reduced.

6 FINAL REMARKS

Following some previous works by the authors, a coupled MFS-FEM model for the 2.5D soil-structure dynamic problem was presented. In different types of problems, the 2.5D formulation is an interesting choice, namely when the system geometry is kept constant along one longitudinal direction and a 3D load is applied, presenting an incremented efficiency when compared to the more usual 3D approach. In the proposed 2.5D model, the FEM is used for discretizing an elastic structure, while the MFS models the evolution of the outer displacement field. It should be mentioned that the proposed method is quite simple to implement, and proved to be accurate and computationally efficient, and thus the authors believe that it can be a promising tool in the analysis of soil-structure interaction dynamic problems.

The integration of the model herein presented on a holistic model for the prediction of vibrations induced by traffic is a simple step, which allows to achieve a powerful tool for the analysis of the vibration problem from the source (vehicle-structure dynamic interaction) to the receiver (buildings close to the transport infrastructure).

ACKNOWLDEGMENT

The financial support by FCT ("Fundação para a Ciência e a Tecnologia") and COMPETE, through project PTDC/ECM/114505/2009, is greatly acknowledged.

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