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NON-SMOOTH AND INTERMITTENT MODEL OF CUTTING PROCESS

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Abstract. Harmful chatter vibrations in milling are generated mainly by regenerative mechanism. This mechanism connected together with process discontinuity cause troubles during numerical calculations, especially for two degree of freedom model (2dof). Therefore, here dynamics of 2dof system with two variants of smoothing function are analysed with the help of numerical methods using bifurcation diagrams and Poincare sections. Finally, some practical conclusions for cutting process are drawn from this study.

1 INTRODUCTION

Nowadays, cutting process is still one of the most popular manufacturing method. Also milling process, as a kind of cutting operation, is very often applied in industry practice. An increased industrial competition has driven the need for manufacturers to reduce costs and improve dimensional accuracy. The efficiency of a machining operation is determined by the metal removal rates, cycle time, machine down time and tool wear. A primary factor that limits process efficiency in machining is a phenomenon called chatter. Chatter is a dynamic instability that can limit material removal rates, cause a poor surface finish and potentially damage the tool and/or the workpiece. From the historical point of view, machine tool chatter goes back almost 100 years, when Taylor, as the first, described this phenomenon [1]. After the extensive work of Tlusty et al. [2], Tobias [3] and Kudinov [4, 5], the so-called regenerative effect has become the most commonly accepted explanation for machine tool chatter. However, much latter another chatter mechanism has been developed by Grabec [1]. This mechanism, called frictional chatter can cause interesting phenomena like deterministic chaos [1, 6-8]. The regenerative effect is related to the wavy workpiece surface generated by the previous cutting tooth passage. While, the frictional mechanism bases on dry friction between the tool and the workpiece.

In the literature, the most often continuous orthogonal cutting operations are analysed, e.g. turning. In case of turning, the governing equation is relatively simple because the tool has one cutting tooth (blade) which still is in contact with a workpiece providing that depth of cut is positive (when system vibrations are small) [6−10]. In case of milling, the direction and value of the cutting force is changing due to rotation of the multi-blade tool, and the cutting is interrupted as each tooth enters and leaves the workpiece. Consequently, the resulting equation of motion is a non-smooth and interrupted delay differential equation [11−14]. That causes troubles during numerical calculations, especially for two degree of freedom model (2dof). Therefore, here dynamics of the milling model is analysed with the help of numerical methods using bifurcation diagrams and Poincare sections. Finally, some practical conclusions for cutting process are drawn from this study.

2 NON-SMOOTH MODEL OF CUTTING

Models of milling process are non-smooth by nature because a cutting tool has several cutting blades, which are in contact with a workpiece during some time intervals of cutting. For the rest of time, tool blades are away from the workpiece. This causes discontinuities, which make difficulties in numerical simulations and analytical solutions, as well. Therefore, modelling of milling process is very important and complicated from technical point of view.

Generally, during milling material is removed from a workpiece by a cutting tool, which rotates with angular speed Ω (in rad/s, but $\Omega=\pi n/30$ if n [rpm]). A schematic representation of up-milling process is shown in Figure 1, as a two degree of freedom (2dof) system.

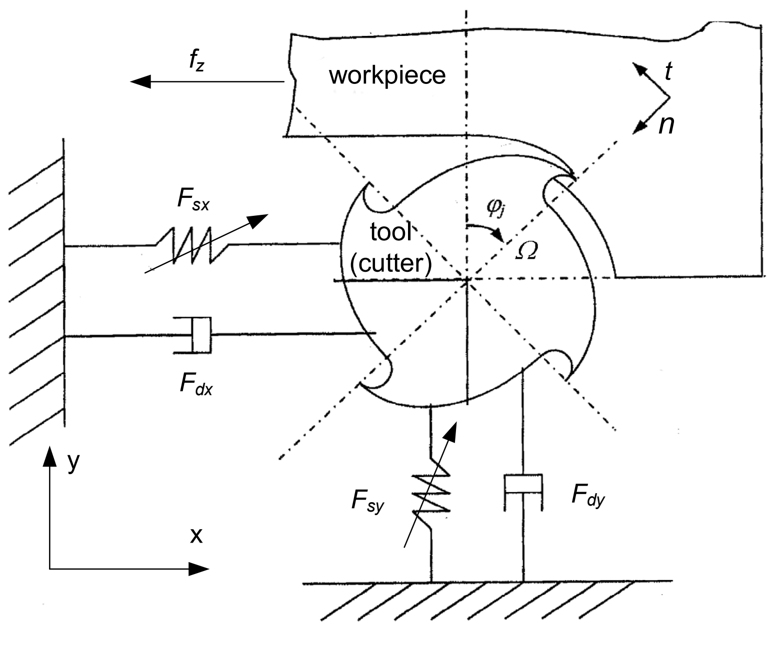

Figure 1: 2 dof model of milling

The cutting tool is represented as a rigid body suspended on visco-elastic systems both in x and y directions. Properties of the suspension are defined by a viscous damping coefficients c_x, c_y and nonlinear stiffness defined with the help of k_x, k_y, γ_x, γ_y. The cutting force F_j acting on j-th tooth ($j=1,2$ .., z) is decomposed on the tangential F_{tj} and normal F_{nj} components. Definition of the tangential and the normal forces is presented below:

$$\begin{aligned} F_{tj} &= K_t a_p h_j^{\kappa} \\ F_{nj} &= K_n a_p h_j^{\kappa}, \end{aligned} \tag{1}$$

where, z means the number of tool teeth, a_p is the axial depth of cut, h_j is a chip thickness. K_t and K_n are specific cutting forces which depend on the cutting material properties. Typical relationship between K_t and K_n for classical materials is $K_n \approx 0.3K_t$. The coefficient κ also depends on the material, and is usually estimated from 0.75 to 1.

The chip thickness $h_j(t)$ is a function of the feed rate f, the present tool vibrations ($x(t)$) and vibrations of the previous tooth (x(t-τ)). Theoretically, the chip thickness h_j can be positive or negative, but only positive value has a practital meaning. Therefore, the actual chip thickness h_j is defined with the help of Heaviside step function H() as follows:

$$\begin{aligned} h_j^* &= [(f_z + x(t) - x(t-\tau))\sin\varphi_j + \left(y(t) - y(t-\tau)\right)\cos\varphi_j] \\ h_j &= H(h_j^*)h_j^* \end{aligned} \tag{2}$$

where, $\tau=60/zn$ is the tooth passing period, φ_j is an angular tool position which is defined as follows:

$$\varphi_j = \Omega t + (j-1)\upsilon; \quad j = 1,2...,z \tag{3}$$

where, ν is an angle between subsequent teeth:

$$\upsilon = \frac{2\pi}{z} \tag{4}$$

We have to remember that j-th tool blade cuts material only when the position angle of a tool is between the entry (φ_s) and exit (φ_e) angle, therefore the step function g_j , is used to define whether the tool is in cut or not:

$$g_j\left(\varphi_j\right) = \begin{Bmatrix} 1, & \varphi_s \le \varphi_j \le \varphi_e \\ 0 & elsewhere \end{Bmatrix} \tag{5}$$

Projecting the forces on x and y direction, summing them for all cutting tooth and taking into account equation (5) the cutting forces are as follows:

$$\begin{aligned} F_x &= \sum_1^n (-F_{nj}\sin\varphi_j - F_{tj}\cos\varphi_j)g_j \\ F_y &= \sum_1^n (-F_{nj}\cos\varphi_j + F_{tj}\sin\varphi_j)g_j \end{aligned} \tag{6}$$

The entry *(φ_s)* and exit *(φ_e)* angles depend on the axial depth of cut (a_e) and the tool diameter (d) as follows:

$$\varphi_s = 0, \quad \cos\varphi_e = \frac{d - 2a_e}{d}. \tag{7}$$

In a case of full immersion milling, according to Figure 1 the equation (7) takes the form:

$$\varphi_s = 0, \quad \varphi_e = \pi / 2 \tag{8}$$

The conditions described in Eq. (5) are realized by non-smooth Heaviside function:

$$g_j = H(\sin\varphi_j) \cdot H(\cos\varphi_j - \cos\varphi_e) \tag{9}$$

but, for ease of numerical computations the discontinuous Heaviside function H() is replaced by its smooth approximation using the sigmoid function given by:

$$H(x) = \frac{1}{1 + e^{-\sigma x}}, \tag{10}$$

where, σ is a smoothing parameter which can have a key meaning in simulations.

According to Figure 1, the equation of motion, for the two degree of freedom milling model can be written as follows:

$$\begin{aligned} m_x\ddot{x} + F_{dx} + F_{sx} &= F_x \\ m_y\ddot{y} + F_{dy} + F_{sy} &= F_y \end{aligned}, \tag{11}$$

where, m_x and m_y is mass of the tool. The stiffness (F_s) and damping force (F_d) in x and y directions are defined:

$$\begin{aligned} &F_{dx} = c_x\dot{x}; \quad F_{dy} = c_y\dot{y} \\ &F_{sx} = k_x x + \gamma x^3; \quad F_{sy} = k_y y + \gamma y^3 \end{aligned} \tag{12}$$

c means the viscous damping coefficient, k and γ are linear and nonlinear stiffness coefficients respectively. Finally, the equation of motion takes the form:

$$\begin{aligned} \ddot{x} + 2\varsigma_x\omega_x\dot{x} + \omega_x{}^2 x + \frac{\gamma_x}{m_x}x^3 &= \frac{1}{m_x}F_x \\ \ddot{y} + 2\varsigma_y\omega_y\dot{y} + \omega_y{}^2 y + \frac{\gamma_y}{m_y}y^3 &= \frac{1}{m_y}F_y \end{aligned} \tag{13}$$

where, damping coefficient are defined

$$\varsigma_y = c_y / (2m_y\omega_y), \quad \varsigma_x = c_x / (2m_x\omega_x) \tag{14}$$

In this study symmetry between x and y directions is assumed. In consequence the visco – elastic coefficients, taken to numerical calculations, indexed as x and y have the same value.

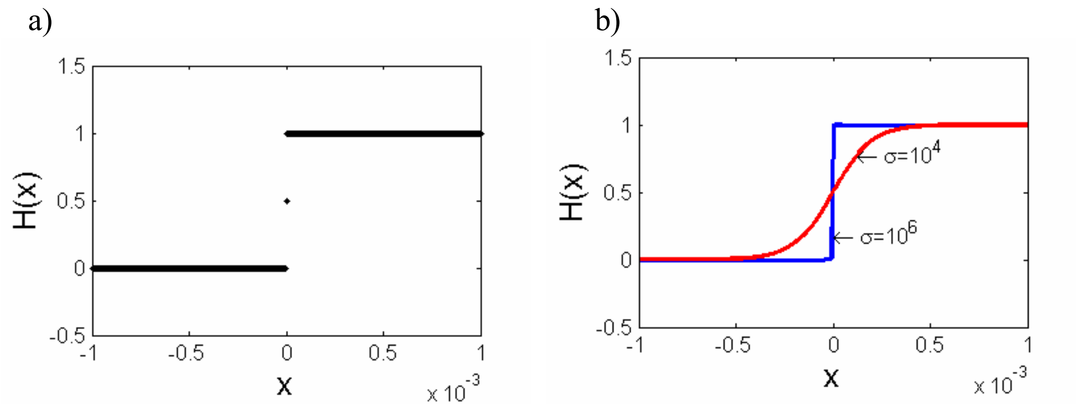

Figure 2: Heaviside function (a) and its smotthed approximation (b)

The substitution (10) does not have any significant effect on the system dynamics providing that σ is big enough because the original Heaviside function Figure 2a is identical like the smooth one presented by blue line in Figure 2b. The results are obtained with the smoother version almost 10 times faster using the Matlab-Simulink inbuilt command *ode45* with a specified the absolute and the relative tolerances of 10^{-6} for numerical integration. On the other hand, too small σ (Figure 2a, red line) can get worst precision of the numerical results. This problem is analysed in the next section. In our computation we assume $\sigma = 10^6$ but the results are compared also with σ=10^2.

3 CUTTING DYNAMCS

On the basis of nonlinear and discontinuous two degree of freedom model presented in the Figure 1, chatter vibrations are analyzed numerically in Matlab-Simulink. Parameters for numerical analysis are taken from [15] as follows: $f_z=0.01\cdot 10^{-3}\,m$, $\omega_{nx}=\omega_{ny}=920rad/s$, $mx=my=2.573kg$, $\xi_x=\xi_y=0.0032$, $\gamma_x=\gamma_y=2\cdot 10^{12}\,N/m^3$, z is from 1 to 4, $K_t=5.5\cdot 10^{8}\,N/m^2$, $K_n=2\cdot 10^{8}\,N/m^2$, $n\in\langle 500,20000\rangle\,rpm$, $d=12\cdot 10^{-3}\,m$, $a_e=4\cdot 10^{-3}\,m$, $a_p\in\langle 0,2\rangle\cdot 10^{-3}\,m$, $\kappa=1$ and initial conditions $x(t=0)=0.00001m$, $\dot{x}(t=0)=0m/s$, $y(t=0)=0.00001m$, $\dot{y}(t=0)=0m/s$.

The numerical simulations were made for four variants, for different number of tool blades z. The calculation time of all simulation was 20 seconds. To determine stability lobes diagrams maximum of response in the x direction are used. This maximum was searched in the time interval from 19 to 20 seconds. Obtained diagrams (Figure 3) are presented in the form of color maps, where the different colors correspond to the respective levels of maximum of coordinate *x*. Figure 3 shows areas, where vibrations during cutting process are safe (blue colour) or unsafe (low or high levels of vibration, respectively). Increasing the number of tool blades the dangerous area of vibration is shifted in the direction lower speeds (n). However, a larger number of blades can increase amplitude vibration for smaller depth of cut a_p. That means the unstable lobes are lower situated in the diagrams.

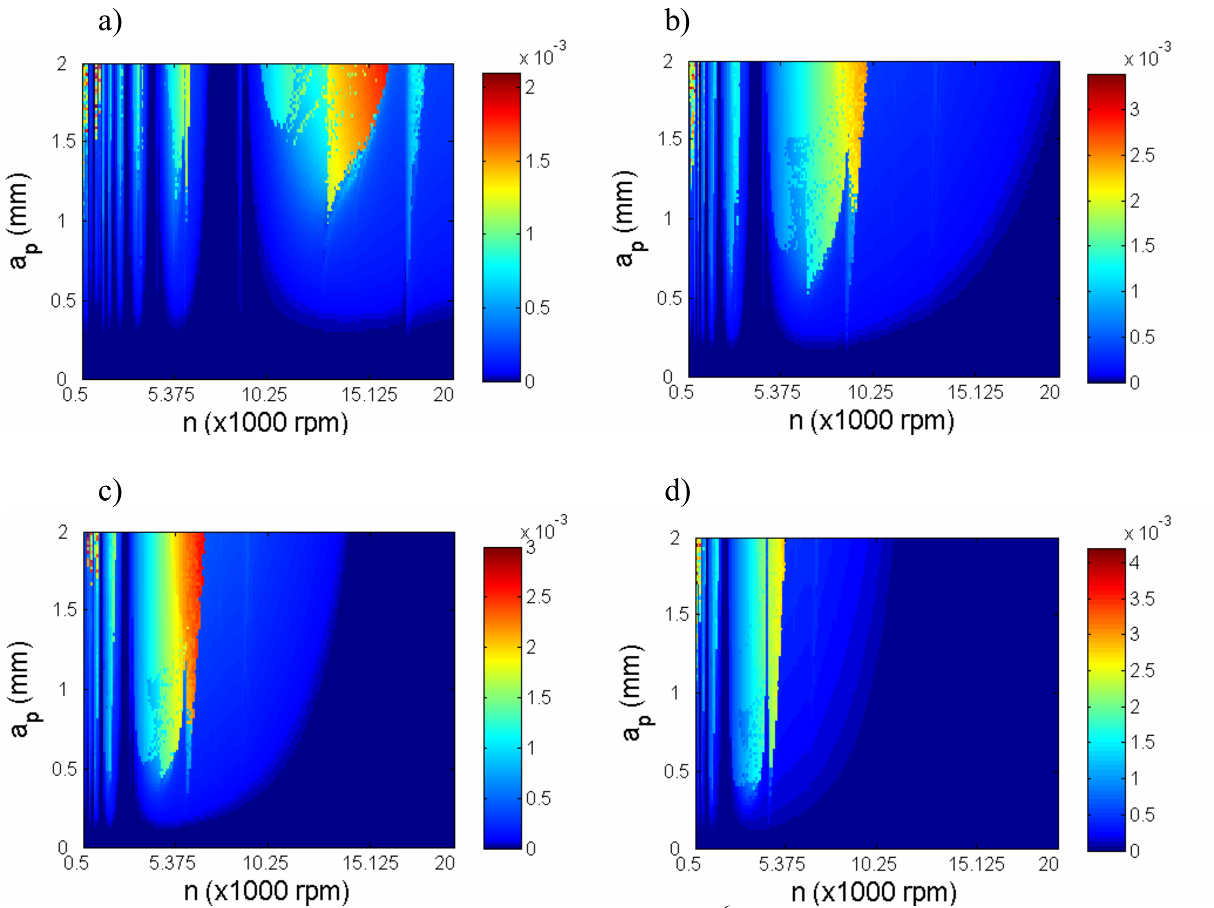

Figure 3: Stability lobes diagrams of coordinate x for σ=10^6 and z=1 (a), z=2 (b), z=3 (c), z=4 (d)

The numerical models which use the original Heavisade functions and approximate functions with big values of σ give practically the same results. Next calculations show the influence of values of parameter σ on obtained results. For comparison the bifurcation analisys is applied with different values of σ. The tested system is not excitated by periodic external forces. In this case traditional bifurcation analysis (strobostopic method based on the frequency of external force) can not be used. In this paper bifurcation diagrams were created for points *x(t)*, which are taken provided that $\dot{x}(t) = 0m/s$. This method allows for the implementation of quantitative and qualitative analysis. From the diagram the maksimum and minimum levels of vibration and type of motion (no motion or periodic, quasi-periodic, chaotic motion) are possible to determined. Figure 4 presents comparison of the bifurcation diagrams versus speed n for two different values of parametr σ: small value 10^2 and big value 10^6. Both cases of system response differ in quality a bit especially about n=13000 rpm. Figures 5 and 6 present the phase portraits and the time series for the one selected speed *n=17783rpm*. In this case, both solutions are quasi-periodic motion, but a change of the parameter σ reveals two significantly different solutions.

The bifurcation analysis is also repeated versus depth of cut a_p for various σ (Figure 7). In this cae we can see explicitly different solutions depending on σ (see Figure 8 and 9 where the phase portraits and the time series are presented).

Generally, in selected examples (Figure 6 and 9) when decreaseing value of σ the level of vibration in the x direction is reduced. The use of small values of parametr σ deteriorates the accuracy of the numerical calculations.

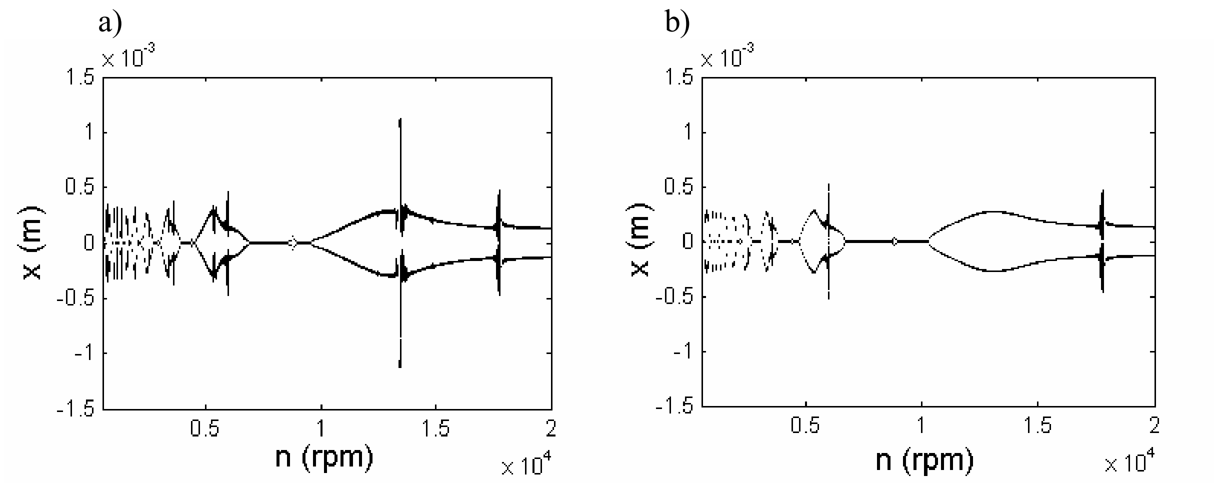

Figure 4: Bifurcation diagrams versus speed n for a_p*=1mm, z=1,* $\sigma=10^6$ (a), $\sigma=10^2$ (b)

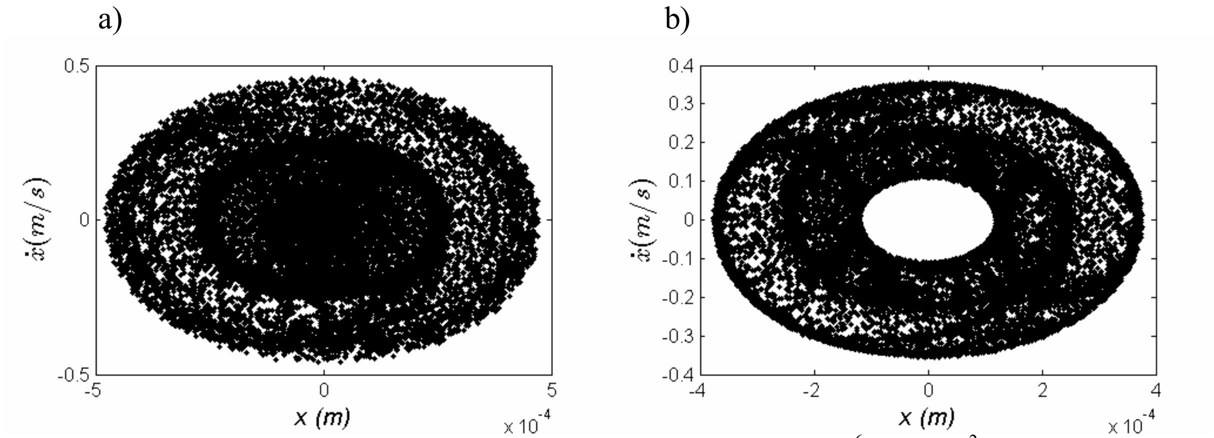

Figure 5: Phase space for *n=17783rpm,* a_p*=1mm, z=1,* $\sigma=10^6$ (a), $\sigma=10^2$ (b)

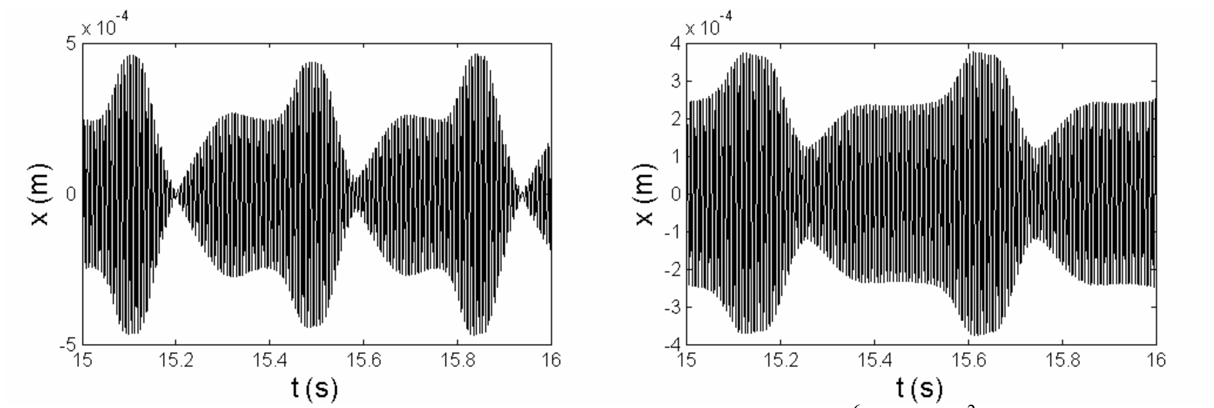

Figure 6: Time series for *n=17783rpm,* a_p*=1mm, z=1,* $\sigma=10^6$ (a), $\sigma=10^2$ (b)

a) b)

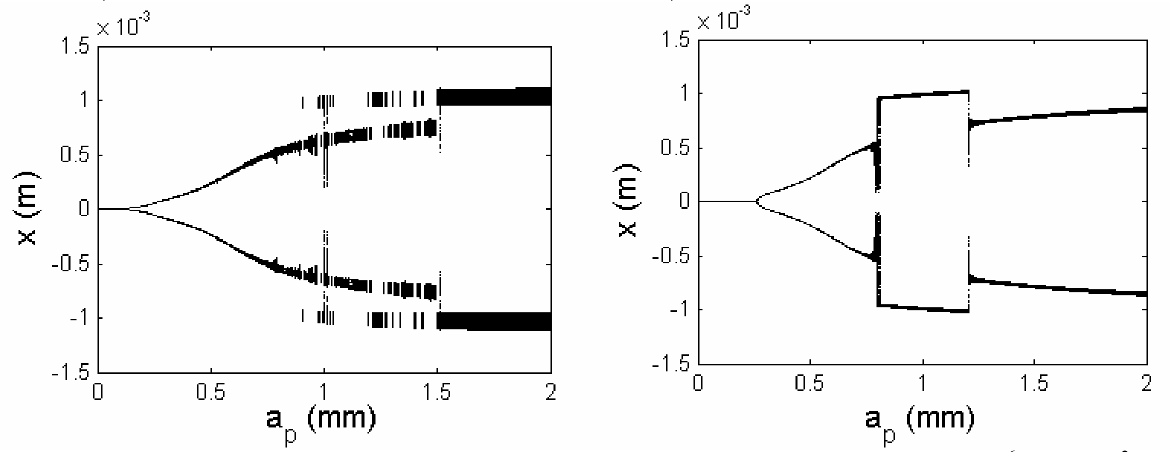

Figure 7: Bifurcation diagrams versus the axial depth of cut a_p for *n =6000rpm, z=2,* $\sigma=10^6$ (a), $\sigma=10^2$ (b)

a) b)

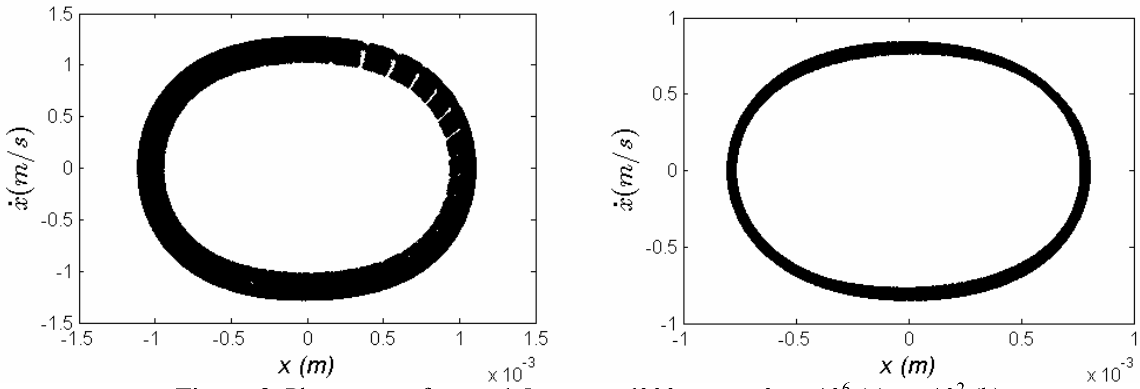

Figure 8: Phase space for *a_p=1.5mm, n =6000rpm, z=2,* $\sigma=10^6$ (a), $\sigma=10^2$ (b)

a) b)

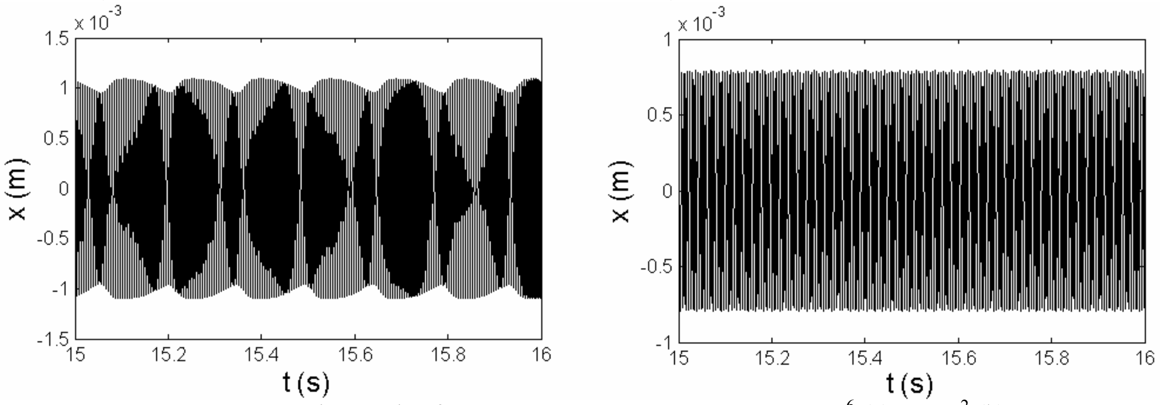

Figure 9: Time series for *n=6000rpm, a_p=1.5mm, z=2,* $\sigma=10^6$ (a), $\sigma=10^2$ (b)

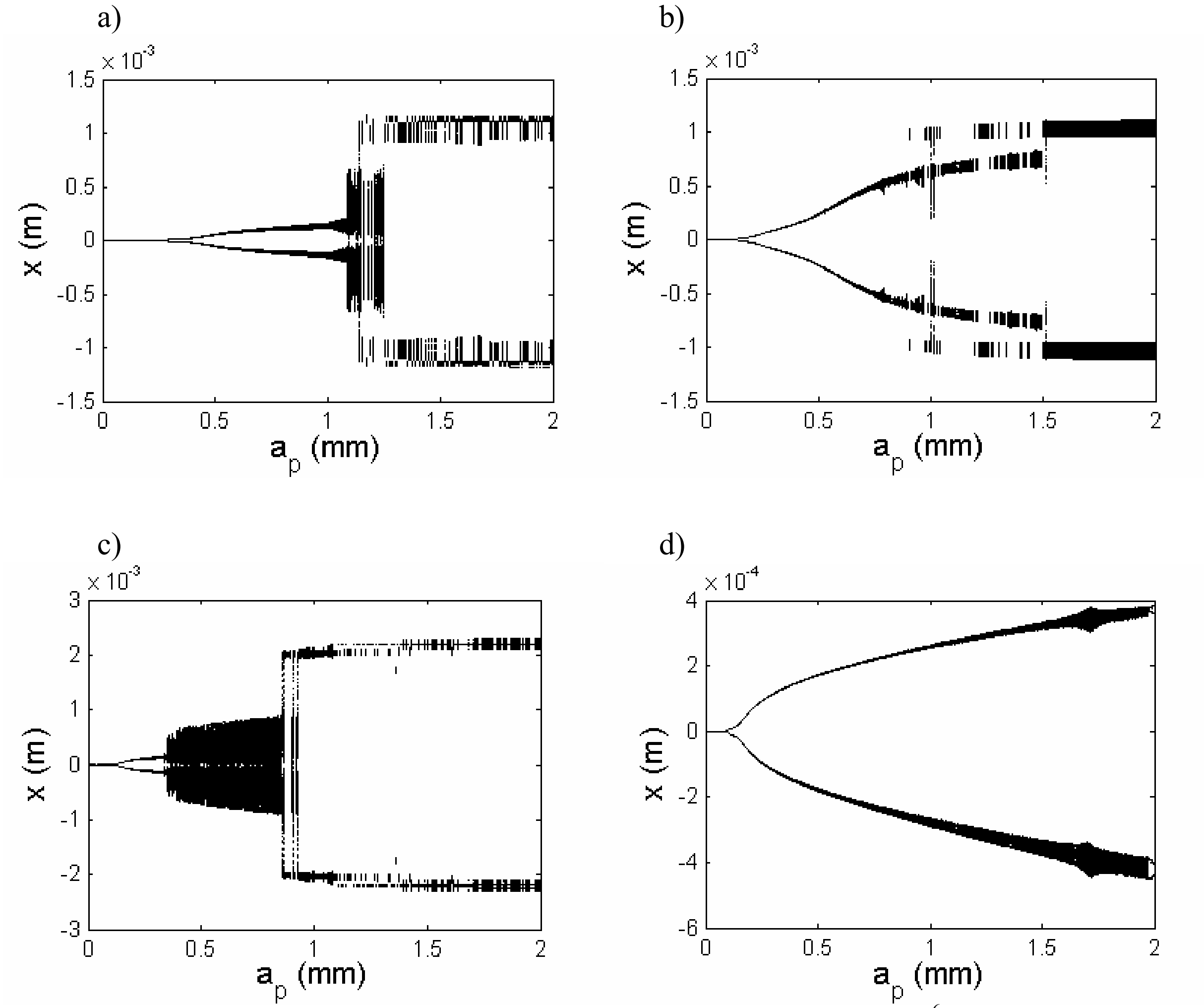

Figure 10: Bifurcation diagrams versus the axial depth of cut a_p for n *=6000rpm,* σ=10^6 , z=1(a), z=2(b), z=3(c), z=4(d)

Figure 10 presents comparison of the bifurcation diagrams for different number of tool blades *z*. These diagrams made for the speed *n=6000rpm* show that from practical point of view, the tool with four blades is the best because the vibrations are smaller than tools with *z=1, 2* or *3* for all depths of cut.

The whole analysis is consequently presented only in *x* direction because there is not observable differences between *x* and *y* directions.

4 CONCLUSIONS

In the paper a strongly nonlinear and discontinuous two degree of freedom model of cutting process is presented. Obtained results of numerical calculations show the influence of the number of tool blades on location of the zone with chatter vibrations. Stability lobes diagrams present the regions where the tool with one, two, three or four blades is preferred to apply. For example, for speed *n=6000rpm* should be applied tool with four blades.

The discontinuous Heaviside function is approximated by smooth function with parameter σ. The values of this parameter influence the accuracy of the numerical simulations, therefore it should be estimated carefully.

This model will be developed in the nearest future in order to explain frictional effect which is mentioned in this paper.

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