11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

NEW METHODOLOGIES IN RELIABILITY-BASED DESIGN OPTIMIZATION FOR AEROSPACE STRUCTURES

M. N. Coccon[1,2], M. Menegozzo[1,3]*, and U. Galvanetto[1,4]

[1]Department of Industrial Engineering
University of Padova, Via Marzolo 9, 35131 Padova, Italy
[2]marco.coccon@studenti.unipd.it
[3]marco.menegozzo@studenti.unipd.it
[4] ugo.galvanetto@unipd.it; URL: http://www.dii.unipd.it/category/ruoli/personale-docente?key=6668BBE4D2254EFC31BAC8842F19BB3E

Key Words: *Reliability-based Design Optimization, Aeroelasticity, Wing design*

Abstract. The purpose of this paper is the design of a high aspect ratio wing, in which the main issue is to address the necessity of minimizing the wing mass and of avoiding aeroelastic problems, while maintaining a prearranged level of reliability. The impossibility to give a deterministic and exact solution to the problem is due to the intrinsic level of randomness both in the material properties and in the external loads. In particular, uncertainties in the aerodynamic forces are mainly caused by random gusts, which are accounted for by the present analysis following a model of the wing dynamics proposed by Cesnik and Brown. The wing design is then performed via a Reliability-Based Design Optimization employing a Non Dominated Sorting Genetic Algorithm. The objectives of the current analysis concern the structural weight as well as the location of the elastic axis, whose variation is determined over a pre-established set of control points in the spanwise direction. Lightness and aeroelastic stability are then achieved while satisfying the probabilistic constraints (e.g., on the maximum allowable torsion angle). The proposed analysis is still a work in progress, and only the results relative to the optimization of a single wing section are presented.

1. INTRODUCTION

During the last decades, there has been an ever increasing incentive for aerospace vehicles to have better performances, higher reliability and decreasing cost and failure risk. A diversified quantity of uncertainties arises from the aerospace vehicle system itself, as well as from the environmental and operational conditions it is involved in. Such uncertainties may cause system performance to change or fluctuate, thus constituting an inherent source of risk. Reliability-Based Design Optimization (RBDO) provides an answer to this kind of issues [1]. RBDO can be used to enhance the reliability of a physical system, and to maximize its performances in full compliance with the desired feasibility constraints. The present paper

describes a reliability-based approach to the design of a high Aspect Ratio wing. The structure is modeled based on the work of Cesnik and Brown [2,3], in which the wing is analyzed as a one-dimensional beam undergoing three-dimensional bending and twisting deformations, while the contribution of extensional and shear deformations is neglected. The aerodynamic forces are introduced in this model following the original theory of Peters *et al.* [4,5]. Such a formulation allows highlighting the forces that are due to random gusts, which entail the most of the uncertainties in aerodynamic forces. Then, the optimization scheme introduced by Huo *et al.*[6] is reviewed, where the influence of the elastic axis location is investigated in the context of the aeroelastic stability of a composite wing. Noticing that the divergence velocity increases for decreasing values of the distance between the aerodynamic centre and the elastic axis, such a distance was considered in [6], together with the wing mass, to define the problem objective function. This work partially influenced the analysis proposed in the present paper, in which the optimization process is extended in order to account for uncertainties in both the material properties and in the aerodynamic loads.
Here, a Performance Measure Approach [7,8] employing inverse First Order Reliability Method (iFORM) is used to evaluate the probabilistic constraints on the maximum allowable strain and stress over the wing span. Differently from sampling-based methods, such as Monte Carlo, the proposed algorithm provides a good compromise between precision requirements and computational costs. Although the development of this optimization procedure is still underway, some preliminary results are presented, limited to the design of a single airfoil. The goal of this simulation was to minimize its cross-sectional area, together with the distance between the aerodynamic centre and the shear centre. The thicknesses of the shear webs and their mutual distance along the chord were chosen as design variables. As for the constraint functions, the minimum allowable values for bending stiffness and twisting stiffness were specified, and the target reliability index was set to one. Prior to RBDO-based simulations, deterministic optimization of the airfoil was carried out as well. Subsequent analyses via Monte Carlo Simulation (MCS) showed that only the probabilistically optimized model fully satisfies the reliability requirements. Simulations brought to an optimal airfoil geometry, which meets the problem requirements and constraints as well. The authors' intention is to extend this kind of analysis to the whole wing. This will be done by simultaneously performing RBDO to different airfoils along the wingspan. To do so, a Non Dominated Sorting Genetic Algorithm (NSGA) [9,10] will be used, in order to reach an acceptable level of numerical efficiency.

2. DYNAMIC ANALYSIS OF HIGH ASPECT RATIO WINGS

Hereinafter, the formulation proposed by Cesnik and Brown [2,3] for high aspect ratio flexible wings is briefly reviewed. Let us introduce the local coordinate system, w, attached to the deformed beam reference line as shown in Fig.1. Such a frame is initially aligned with the inertial frame, B, that is fixed to the fuselage (it is assumed that the fuselage does not rotate, nor accelerate).

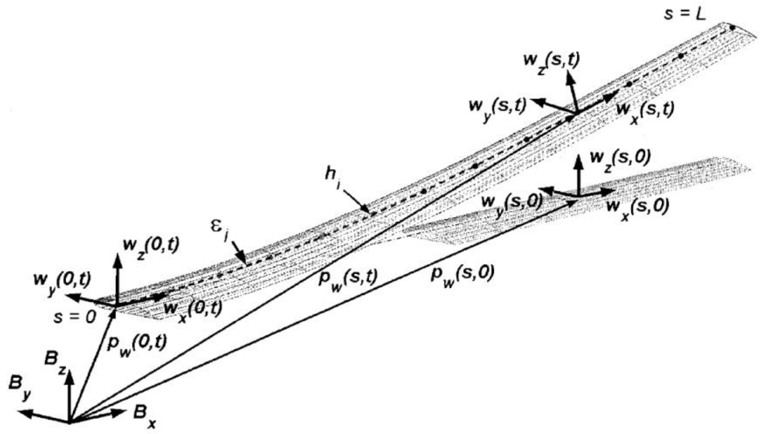

Fig. 1. Wing coordinate systems [3]

In the proposed model, the wing is analyzed as a one-dimensional beam undergoing three-dimensional bending and twisting deformations. Furthermore, the contribution of extensional and shear deformations to overall wing deformation is neglected. Under this assumption, the shape of the wing is completely defined by the distribution of the curvature along the beam coordinate, s

$$\frac{\partial h(s,t)}{\partial s} = A(k_x, k_y, k_z)h(s,t) = \begin{bmatrix} 0 & I & 0 & 0 \\ 0 & 0 & k_z I & -k_y I \\ 0 & -k_z I & 0 & k_x I \\ 0 & k_y I & -k_x I & 0 \end{bmatrix} h(s,t) \tag{1}$$

where $h = (p, w_x, w_y, w_z)^T$, being p the position vector of a point on the beam reference line, which is expressed in the inertial frame as well as the three component vectors w_x, w_y and w_z. This equation is solved by discretizing the reference line into n_k equally spaced coordinates, which define a set of n_k-1 three-node elements as illustrated in Fig. 2.

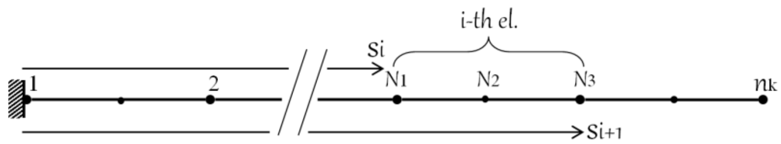

Fig. 2. Discretization of the beam reference line.

The curvature is assumed to vary linearly in the element, so that only the curvatures at the nodes need to be considered to solve Eq (1). In this way, the wing kinematics is simply defined by $3n_k$ generalized coordinates, i.e., the $3n_k$ elements of curvature vector, $\tilde{k}$, which is given by

$$\tilde{k} = (k_{x,1}, k_{y,1}, k_{z,1}, \dots, k_{x,n_k}, k_{y,n_k}, k_{z,n_k}) \tag{2}$$

This vector can be used to define the equation of motion, which is provided by the total virtual work done on the wing, i.e. the sum of the internal virtual work (i.e., work done by inertial forces, internal damping and elastic stresses) and the external virtual work (i.e., due to gravity loads and aerodynamic forces)

$$\delta W = \delta \tilde{k}^T \left(-M \frac{\partial^2 \tilde{k}}{\partial t^2} - C \frac{\partial \tilde{k}}{\partial t} - K\tilde{k} + F \right) \tag{3}$$

Since the virtual displacement $\delta\tilde{k}$ is arbitrary, the equation of motion is obtained from Eq (3) by setting the total virtual work equal to zero. In the following, only the aerodynamic loads are calculated in order to explain the methodology proposed by Cesnik and Brown [2,3].

3. AERODYNAMIC LOADS

In this Section, the aerodynamic model used by Cesnik and Brown [2,3] is presented, which is based on the original formulation of Refs. [4,5]. The proposed theory has been developed for a thin deformable airfoil undergoing large motion in a subsonic flow, with small deformations about that motion.

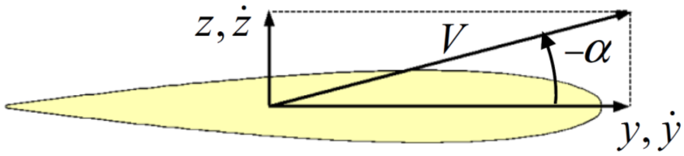

Fig. 3. Airfoil motion definition [2]

The lift, moment and drag forces are nonlinear functions of the state vector $\psi = (y, z, \alpha, \dot{y}, \dot{z}, \dot{\alpha}, \lambda_0)$ and its derivative, $\dot{\psi}$, where (y, z, α) is the instantaneous motion of the airfoil as illustrated in Fig. 3, and λ_0 is the induced flow due to the free vorticity. For example, the lift is given by

$$L = 2\pi\rho b \left\{ \dot{y} \left[\left(\frac{1}{2} b - d \right) \dot{\alpha} - \dot{z} - \lambda_0 \right] - \frac{1}{2} b\ddot{z} - \frac{1}{2} bd\ddot{\alpha} \right\} \tag{4}$$

where b is the semichord, and d is the distance of the midchord from the beam reference axis. The expressions of the moment, M, and drag, D, are similar to that of Eq. (4), while λ_0 is obtained through the finite-state inflow theory [4], i.e. by superimposing N states $(\lambda_1, \lambda_2, \dots, \lambda_N)$, where the λ_i's, $i = 1, \dots, N$ are defined by a set of differential equations

$$\dot{\lambda} = E_1\lambda + E_2\ddot{z} + E_3\ddot{\alpha} + E_4\dot{\alpha} \tag{5}$$

The airload vector, $F = (L, M, D)^T$ is then linearized about time t_a and, following the finite element procedure described in the previous Section, the virtual work done by F can be written in terms of the curvature vector, $\tilde{k}$, that is

$$\delta W = \int_{s=0}^{L} (L\delta z + D\delta y + M\delta\alpha)ds = \delta\tilde{k}^T B\tilde{F} \tag{6}$$

where $\tilde{F}$ is the vector of the nodal aerodynamic forces. This vector is further linearized about the instantaneous state vector, $\varphi_a = \left(\tilde{k}_a, \dot{\tilde{k}}_a\right)^T$ as

$$\varphi \cong \varphi_a + \Delta\varphi \quad \Longrightarrow \quad \tilde{F} \cong \tilde{F}_a + \left[\tilde{F}_{\ddot{\tilde{k}}} \ \ \tilde{F}_{\Delta\dot{\tilde{k}}} \ \ \tilde{F}_{\Delta\tilde{k}}\right] \begin{Bmatrix} \ddot{\tilde{k}} \\ \Delta\dot{\tilde{k}} \\ \Delta\tilde{k} \end{Bmatrix} + [\tilde{F}_\lambda \ \ \tilde{F}_w] \begin{Bmatrix} \lambda \\ w \end{Bmatrix} \tag{7}$$

in which the terms in square brackets are Jacobian matrixes; and *w* is the gust input that can be seen as an airfoil velocity term in Eq. (4). Such a term represents the source of uncertainty in the reliability-based design optimization (RBDO) of the wing structure, as described in the next Sections.

4. OPTIMIZATION DESIGN BASED ON ELASTIC AXIS

A Multidisciplinary Design Optimization (MDO) problem has been recently proposed in [6] to investigate the composite wing elastic axis and its influence on the aeroelasticity application problem. The elastic centre is a point on a section of the beam, where a shear force can be applied without inducing any torsion, and the elastic axis is the line of all elastic centres. In general, the mechanical behavior of an aerofoil is mainly characterized by the position of three points: elastic centre, centre of mass and aerodynamic centre. Hereinafter, it will be shown that the elastic axis position plays a crucial role in the aeroelastic stability of an airfoil. Let *e* be defined as the distance of the elastic axis to the aerodynamic center, and α the airfoil attack angle, which is composed of initial attack angle, α_0, and torsion angle, θ. The aerodynamic moment on the elastic axis, M_e, can be written as

$$M_e = qScC_{MAC} + qS\left[\frac{\partial C_L}{\partial\alpha}(\alpha_0 + \theta)\right]e \tag{8}$$

where q is the dynamic pressure; *S* is the airfoil area, *c* is the cross section chord length; and C_{MAC} and $C_L$are respectively the moment and lift coefficients. If a torsional spring is used to simulate the torsional elasticity of the airfoil, i.e. $M_e = k_\theta\theta$, the equilibrium equation of aerodynamic moment and spring elastic moment can be written as

$$\theta = \frac{(qS)/(k_\theta)(e(\partial C_L/\partial\alpha)\alpha_0 + cC_{MAC})}{1 - \big((qeS)/k_\theta\big)(\partial C_L/\partial\alpha)} \tag{9}$$

The torsion angle becomes infinite when the denominator of Eq. (9) equals zero, indicating that the wing is in a divergent state (instability). The corresponding divergence pressure, q_D, is given by

$$q_D = \frac{k_\theta}{eS(\partial C_L/\partial\alpha)} \tag{10}$$

from which it is seen that divergence instability is more likely to occur for high values of e. Hence, an optimization design based on elastic axis was proposed in [6]. In this study, the wing mass and the distance from the elastic axis to the aerodynamic center define the following objective function

$$f = \frac{W_1 M}{M_{max}} + \frac{W_2 e}{c} \tag{11}$$

where M is the wing mass after optimization, M_{max} is the maximum wing mass in the design variable span, and W_1 and W_2 are weight coefficients. The constraints of the problem concern the maximum torsion angle, maximum strain and flutter frequency.

The optimization process described above can be extended to a reliability-based design optimization analysis, where the objective function in Eq. (11) is minimized while considering the uncertainties introduced by the randomness on the material properties as well as on the aerodynamic loads (random gusts). In the following Section, a reliability-based methodology is proposed for the optimization of the high-aspect-ratio wing model of Sections 2-3, and some preliminary results are presented.

5. RELIABILITY-BASED DESIGN OPTIMIZATION: SOME PRELIMINARY RESULTS

The proposed analysis is focused on the geometrical characteristics of the airfoils along the wing span, such as the thicknesses of the shear webs and their position along the chord. Indicating with n_A the number of design variables for each airfoil, the total number of design variables in the present RBDO problem is $n = n_A \cdot n_E$, where n_E is the number of beam elements in the spanwise direction ($n_E = n_k$ -1, see Fig. 2). In order to obtain an accurate representation of the wing dynamics by Eq. (3), a fine spacing coordinate set is required for the beam reference line. As a result, the total number of design variables, n, is high, so that a Non Dominated Sorting Genetic Algorithm [10] is used to achieve numerical efficiency. This searching scheme is run with an initial set of design of experiments (DOE), and the probabilistic constraints are accounted for by a Performance Measure Approach (PMA) [7,8]. At the end of the analysis, several designs are found in the Pareto front, from which the user can choose the optimal solution. Work is still underway and a brief overview on the PMA is presented here, followed by some preliminary results limited to a single airfoil section.

A general RBDO problem implementing PMA is given by

$$\begin{cases} find & d \\ min & f(d;\mu_X) \\ s.t. & g_i^{P_{f,i}}(d;X) \geq 0\,, \quad i = 1,2,\dots l \\ & g_j^D(d) \geq 0\,, \qquad j = 1,2,\dots m \\ & d_k^L \leq d_k \leq d_k^U\,, \quad k = 1,2,\dots n \end{cases} \tag{12}$$

where f is the objective function which has to be minimized; μ_X is the mean vector of the random variables X; d is the vector of the design variables, whose lower and upper bounds are d^L and d^U, respectively; g_j^D is the j-th deterministic constraint; and $g_i^{P_{f,i}}$ is the $P_{f,i}$-th percentile value of the i-th performance function, g_i, i.e.

$$P\left[g_i(d,X) \leq g_i^{P_{f,i}}\right] = P_{f,i} \tag{13}$$

in which $P_{f,i}$ is the corresponding target probability of failure. The percentile $g_i^{P_{f,i}}$ is determined by inverse reliability analysis, and inverse First-Order Reliability Method (iFORM) [9] can be used

$$\begin{cases} find & U \\ min & g_i(d,U) \\ s.t. & \|U\| = \beta_i \end{cases} \tag{14}$$

where U corresponds to X mapped into the corresponding space of uncorrelated standard normal variables; and β_i is the reliability index related to the i-th performance function, which is defined as the opposite of the inverse cumulative density function of the standard normal distribution evaluated in $P_{f,i}$, i.e. $\beta_i = -\Phi^{-1}(P_{f,i})$. The solution of Eq. (14), U^*, is such that, once transformed back to the space of the original random variables, i.e. X^*, it results $g_i^{R_i}(d,X) \cong g_i(d,X^*)$. The optimization problem in Eq. (12) will be solved through the NSGA searching scheme for the analysis of the entire wing. Hereinafter, only the results relative to a single airfoil are presented and further developments are planned.

Let us consider the airfoil shown in Fig. 4, which is modeled as a thin walled structure with a single cell closed section, consisting of two horizontal skins and two vertical shear webs.

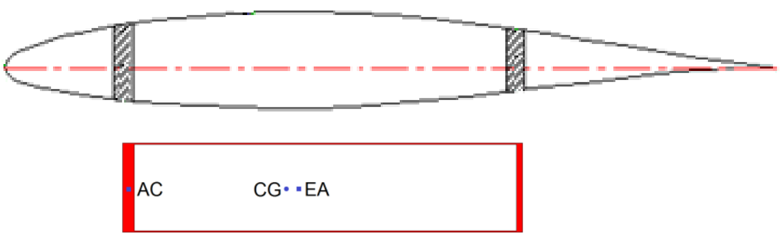

Fig. 4: Structural simplified model of the wing cross section

For the sake of simplicity, the material properties are assumed to be constant over the section. Furthermore, the aerodynamic center, AC, is supposed to lie on the vertical axis of the left shear web and aligned with the center of gravity, CG, and the elastic axis, EA. The objectives to minimize are the wing cross sectional area (representative of its mass) and the distance between the AC and EA, here indicated with A and e, respectively. With reference to the previous Section, the objective function f is expressed by

$$f = 0.5\left(\frac{A}{A_0}\right) + 0.5\left(\frac{e}{e_0}\right) \tag{15}$$

where A_0 and e_0 are the values of A and e prior to optimization. In this way, the value of f is initially equal to unity and it decreases during the optimization process. Three design variables are considered, i.e. the thicknesses of the two shear webs, t_2 and t_3, and their distance, l, while the thickness of the skin, t_1, has a constant value of 2 mm. The initial configuration is reported in Table 1.

Table 1. Initial configuration of the wing cross section

Parameters	Value [mm]	Bounds [mm]
t_2	*8*	*5-15*
t_3	*15*	*10-25*
l	*500*	*350-600*

In this example, uncertainties are due to lack of knowledge on the material properties. In particular, the Young modulus, E, and the shear modulus, G, are assumed normally distributed with mean values μ_E = *71700 MPa*, μ_G = *26900 MPa* (Al 7075-T6) and coefficients of variation equal to *10%*. It should be noted that the values of A and e are not influenced by E and G, so that the objective function only depends on the design variables, i.e. $f = f(t_2, t_3, l)$. However, the random variables E and G are introduced in the constraints, which establish the minimum required values for the section bending and twisting stiffnesses, i.e. $K_{B,min}$ = *120 N/mm* and $K_{T,min}$ = 1.6×10^6 *N/mm*. The corresponding performance functions are defined by the difference

$$g_i(t_2, t_3, l; E, G) = K_{i,min} - K_i(t_2, t_3, l; E, G)\,, \quad i = \{B, T\} \tag{16}$$

where B stands for bending and T for twisting. Following the PMA approach described above, the constraint equations are based on the $P_{f,i}$-th percentile, $g_i^{P_{f,i}}$, of the performance function g_i. A reliability index $\beta = 1$ is assumed for both the bending and twisting constraints, so that the target probabilities of failure are equal to $P_f = \Phi(-\beta_i)$ =*15.87%*. The mathematical model of the present reliability-based optimization problem is finally given as

$$\begin{cases} find & t_2, t_3, l \\ min & f(t_2, t_3, l) \\ s.t. & g_i^{P_f}(t_2, t_3, l; E, G) \geq 0\,, \quad i = \{B, T\} \\ & t_2^L \leq t_2 \leq t_2^U \\ & t_3^L \leq t_3 \leq t_3^U \\ & l^L \leq l \leq l^U \end{cases} \tag{17}$$

In Table 2, the results of the solution of the problem represented by Eq. (17) are compared to those provided by the equivalent deterministic problem, which is obtained from the proposed RBDO by replacing the constraints $g_i^{P_f}(t_2, t_3, l; E, G) \geq 0$ with deterministic performance functions, i.e. $g_i(t_2, t_3, l; \mu_E, \mu_G) \geq 0$.

Table 2. Comparison between deterministic optimum design and proposed RBDO.

Parameters	Initial configuration	Deterministic optimization	Proposed RBDO
f	*1.000*	*0.7632*	*0.9272*
A *[mm²]*	*5450*	*4987*	*5440*
e *[mm]*	*217.0*	*132.6*	*185.8*
K_B *[N/mm]*	*141.2*	*120.0*	*141.0*
K_T *[N/mm]*	$1.91 \mathrm{x} 10^6$	$1.95 \mathrm{x} 10^6$	$1.84 \mathrm{x} 10^6$
t_2 *[mm]*	*8*	*5*	*5*
t_3 *[mm]*	*15*	*18.1*	*17.9*
l *[mm]*	*500*	*380.8*	*499.9*

From Table 2, it can be seen that the smallest value of the objective function is achieved by the deterministic optimization. Both the cross section area and the position of the elastic axis are smaller than those obtained by RBDO, and such a difference is mainly due to the choice of *l*, i.e. the distance between the shear webs. Despite the deterministic design looks like the most performant, it does not meet the reliability requirements on the minimum bending stiffness. This fact is proved by a series of crude Monte Carlo Sampling (MCS) analyses estimating the reliabilities of both the optimal solutions in Table 2. The results are summarized in Table 3, which shows the failure probabilities and the corresponding reliability indexes of the constraint functions for the bending and twisting stiffness. Only the RBDO design guaranties a failure probability $P_f < 15.87\%$ (or, equivalently, a reliability index $\beta > 1$) for both the constraints, while the probability of the deterministic design to have a bending stiffness less than *120 N/mm* is equal to *49.9%*.

Table 3. Reliabilities estimated by MCS simulation

Probabilistic constraints	Deterministic optimization		Proposed RBDO	
	P_f [%]	β	P_f [%]	β
$K_{B,min}$ - K_B	***49.9***	***-0.1181***	*6.8*	*1.4909*
$K_{T,min}$ - K_T	*3.6*	*1.8384*	*9.8*	*1.2930*

6. CONCLUSIONS

This paper presents an efficient approach for reliability-based design optimization of high aspect ratio wings. First, the structural model of the wing is introduced, which is analyzed as a one-dimensional beam undergoing three-dimensional bending and twisting deformations. The main steps that lead to the aeroelastic equation of motion are briefly overviewed. In this model, uncertainties are introduced due to randomness on the material properties as well as on the aerodynamic loads (caused by random gusts). Then, an elastic axis-based RBDO strategy is proposed to ensure lightness and aeroelastic stability while keeping prearranged levels of reliability on the constraints (e.g., maximum allowable stress and displacements).

Work is still underway, and some preliminary results are shown relatively to the optimization of a single airfoil. Here, the thicknesses of the shear webs and their position along the chord are determined so that the cross-sectional area and the distance of the elastic axis to the aerodynamic center are minimized. Probabilistic constraints establish the minimum required values for the section bending and twisting stiffnesses, and they are evaluated by a Performance Measure Approach employing inverse first-order reliability analysis. The results are compared with those provided by a deterministic optimization, revealing the ability of RBDO to minimize the objective function while meeting the reliability requirements.

Further developments are planned, in which the optimization process is simultaneously carried out on numerous cross sections along the wing span. To this end, a Non Dominated Sorting Genetic Algorithm will be used to achieve numerical efficiency.

REFERENCES

[1] Wen Yao, Xiaoqian Chen, Wencai Luo, Michel van Tooren, Jian Guo, Review of uncertainty- based multidisciplinary design optimization methods for aerospace vehicles, Progress in Aerospace Sciences 47 (2011) 450–479.

[2] C. E. S. Cesnik, E. L. Brown, “Modeling of high aspect ratio active flexible wings for roll control”, 43rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Con 22-25 April 2002, Denver, Colorado

[3] E. L. Brown, “Integrated strain actuation in aircraft with highly flexible composite wings”, Ph.D. thesis, Massachusetts Institute of Technology, June 2003.

[4] Peters, D. A. and Johnson, M. J., "Finite-State Airloads for Deformable Airfoils on Fixed and Rotating Wings", In *Symposium on Aeroelasticity and Fluid/Structure Interaction, Proceedings of the Winter Annual Meeting*. ASME, November 6 -11, 1994.

[5] Peters, D. A., Karunamoorthy, S., and Cao, W. M., "Finite State Induced Flow Models; Part I: Two-Dimensional Thin Airfoil", *Journal of Aircraft*, Vol. 32, No. 2, March ñ April 1995, pp. 313-322.

[6] S.H. Huo, F.S. Wang, Z. Yuan and Z.F. Yue, “Composite wing elastic axis for aeroelasticity optimization design”, Aircraft Engineering and Aerospace Technology: An International Journal 85/1 (2013) 10–1

[7] Frangopol D. M., Maute K., Reliability-Based Optimization of Civil and Aerospace Structural Systems. Chapter 24 in: Engineering Design Reliability Handbook, Ed. Nikolaidis E., Ghiocel D.M., Singhal S., CRC Press LCC, 2005.

[8] Tu J, Choi K K, Young H P. “A new study on reliability-based design optimization”. Journal of Mechanical Design 1999; 121(4):557–64.

[9] Tu, J. and Choi, K.K., 1997, “A Performance Measure Approach in Reliability-Based Structural Optimization.” Technical Report R97-02, Center for Computer-Aided Design, The University of Iowa, Iowa City, IA.

[10] Srinivas, N., Deb, K. Muiltiobjective Optimization Using Nondominated Sorting in Genetic Algorithms, Evolutionary Computation, Vol. 2, No. 3, pp. 221-248, 1994.