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STABILITY THEORY METHODS IN PROBLEMS OF MATHEMATICAL MODELLING COMPLEX MECHANICAL SYSTEMS

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Abstract. Actual problems of dynamics and modelling complex dynamic systems of the singularly perturbed class are discussed in this research. Many applied questions of Mechanics, mechanical engineering are generating such problems. From mathematical point the object of the study in work is connected with systems for that mathematical models are presented by differential or algebraic- differential equations, with small parameter in different degrees under derivatives. Such mathematical models are describing the many dynamic processes of concrete physic-technical nature. Among them:
problems of modelling and analysis in theory of mechanical systems with big friction, in non-holonomic systems theory, in gyroscopic systems theory, in theory of the stabilizations and orientation systems, robotic systems,…
The main purpose: the determination of the conditions, under which the solving problems of the qualitative analysis and synthesis can be reduced to study of the shortened models of the more low order. With development of the approach, founded on stability theory methods of A.M.Lyapunov, on N.G.Chetayev ideas, on statements of P.A.Kuzmin, V.V.Rumyantsev, the examined qualitative problems are interpreted as stability problems under parametric non-regular perturbations.
The correctness (in accepted here sense) conditions of constructed shortened models is formulated, with expansion of the traditional statements of classical stability theory, with considering of non-regular parametric perturbations. Besides the solving original problem is obtained by analytical or numerically-analytical methods.

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1 INTRODUCTION

The research develops generalized approach, based on A.M.Lyapunov's methods [1], that make it possible to obtain the regular manners for mechanics problems of large-scale systems, including modelling problems and qualitative analysis [1-26]. The basic proposition, accepted here, is ascending to well-known N.G.Chetayev stability postulate [2,3], and to the L.K.Kuzmina singularity postulate [12] establishing the methodological connection between the modelling problems and Lyapunov's theory, deepened by P.A.Kuzmin concept of the stability with parametric perturbations [8]. Principal questions are solved here: the constructing of acceptable shortened mechanical-mathematical models by strong mathematical way; the rigorous manners of model simplification; the qualitative equivalence between original and approximate model. Universal conception, the elaboration to the mechanics problem lead to the reduction principle in the general qualitative analysis for the systems of singular class. The different mechanical systems are considered from unified point of view (as singularly perturbed ones); and in established approach set models (simplified models) are interpreted as asymptotic s-models. The problems of acceptability, of legitimacy are solved by methods of Lyapunov's theory [4,13]. In accordance with our main postulates, modeling the singular perturbations in original problem statement, we obtain the regular asymptotic manner that permits to receive effective algorithm for the division of the motions on the different-frequency components and for the construction of the idealized model. Here we can take as shortened model the approximate system, describing s-slow motions (slow ones of s- level). Moreover, establishing the hierarchy of the variables and introducing the different small and big parameters, we can receive the strict sequence of simplified approximate systems and corresponding simplified (set) models (asymptotic models):

$$\mathrm{IO} \rightarrow \mathrm{SPM} \rightarrow \mathrm{SPS} \rightarrow (\mathrm{SS}_0,\ \mathrm{SS}_1,...) \rightarrow (\mathrm{SM}_0,\ \mathrm{SM}_1,...) \qquad (1)$$

Here - IO – initial object; SPM – singularly perturbed model; SPS – singularly perturbed system; SS_i – simplified systems of i - level; SM_i – simplified model of i - level.

Using the methods of stability theory (A.M.Lyapunov, 1956; N.G.Chetayev, 1957; I.S.Gradstein, 1953) we can determine the conditions of acceptability (in corresponding sense) of approximate models; find the domains of parameters values allowing to reduce to the simplified systems; build the estimates. For the singular systems of some type, that are interesting for the applications (in the dynamics problems of stabilization systems; of gyrosystems; etc.), we get the different simplified (set) models. This asymptotic approach allows to give the substantiation of known models (D.R.Merkin, 1956) and to construct the new approximate models, with the conditions of their correctness (on (1)) for problems of s-stability, s-proximity, s- optimality,...

2 INITIAL TENETS

Contemplating the applications to the mechanics problems, we shall take as initial mathematical model the Lagrange's equations (or their extension: Lagrange's-Maxwell's equations, Gaponov's ones,…)

IM $$\frac{d}{dt}\frac{\P T}{\P q^{\bullet}} - \frac{\P T}{\P q} = Q \quad , \frac{dq}{dt} = q^{\bullet} \qquad (2)$$

We call this initial model (IM).

Here q is k-dimensional vector, k is number of the freedom degrees of the original system.

In accordance with the using manner, supposing that (2) is the model of the singularly perturbed object, and it is SPM, we introduce in (2) the small parameter $m > 0$. Further with the using of the corresponding transformation of the variables

$$(q, q^{\bullet}) \to y \qquad (3)$$

we shall lead the initial model (2) to the standard form of the singular equations

IS $$M(m)\frac{dy}{dt} = Y(t, m, y) \qquad (4)$$

We shall consider such systems, for which in these equations of type (4)

$$M(m) = \left\| M_{ij}(m) \right\|, M_{ij}(m) = m^{a_i} E$$

a_i - nonnegative numbers, $0 \leq a_i \leq r$; E are identity matrices.

We call (4) initial system (IS), and it is SPS.

Taking into consideration in (4) only the members containing m in power no greater than s ($0 \leq s < r$) we shall obtain the shortened systems (approximate ones) of type

SS_s $$M_s(m)\frac{dy}{dt} = Y_s(t, m, y) \qquad (5)$$

Here s is the number, chosen or given in advance, $0 \leq s \leq r$-1. Note, in general case SS_s is also SPS.

We shall call (5) the simplified system of s-level (s-system, SS_s). For the singularly perturbed systems, that are considered here, the order of system (5) is lower than order of initial system (4). In applications to mechanics these shortened systems lead to the simplified models (as the approximate models of s-level). For the constructing of asymptotic model of s-level we must return in (5) from new variables (y) to the old variables (q, $q^{\bullet}$). Then obtain approximate model of s-level (s-model): $SS_s \to SM_s$. We can obtain the sequence of approximate models (as set models) in mechanics, that is corresponding the sequence of these s-systems (with $s=0,1,2, \ldots, r-1$).

We have the problem (important both for the theory and for applications): in which cases and under which conditions it is possible to reduce system (4) to the s-system (5) and respectively $IM \to SM_s$?

Such problems for the equations with small parameter under derivatives were considered by many authors (N.G.Chetayev, 1957; I.S.Gradstein, 1953; N.N.Krasovsky, 1961; B.S.Razumikhin, 1963; L.K.Kuzmina, 1982; A.N.Tikhonov, 1952;...). Here for solving this problem, following the Chetayev's method, we must introduce the deviations (as new variables):

$$k = y - y^s$$

Here $y = y(t, m)$ is the solution of the initial system (4); $y^s = y^s(t, m)$ is the solution of the approximate system (5). Using the Lyapunov's methods we can investigate the differential equations for these variables

$$\tilde{M}(m)\frac{dk}{dt} = K(t, m, k) \tag{6}$$

and we can determine the conditions with which we have

$$|| k || < e \quad \text{(for all } t > t_0\text{)},$$

if $0 £ m £ m_*$.

Here $e > 0$ is any small number, given in advance. Further these methods permit us to get the estimations for m [3]. Respectively, returning to the state variables, we obtain the conditions for validity of constructed approximate model (*s*-model) and the domains estimations of permissible parameters values.

3 SOME RESULTS

The mechanical systems with the big parameters.
a) System with the non-rigid elements.

Let us consider a problem of a strict mathematical substantiation of a transition to the approximate model, used in mechanics of the system with non-rigid elements (to absolutely rigid system) in critical case. As an example of such mechanical system we shall consider the gyrostabilization system. Here there is a critical case of zero-roots. We shall solve a stability problem of the steady motion for such system, supposing that the elements of the system are not absolutely rigid (we neglect the mass of non-rigid elements). Assuming (2) as the mathematical model of initial object, we shall accept the differential equations of perturbed motion in a form

$$\frac{d}{dt} a q_M^{\bullet} + (b + g) q_M^{\bullet} + c q_M = Q_M'' , \qquad \frac{dq}{dt} = q_M^{\bullet} \tag{7}$$

Here all notations are as in our work [12,14] and $q_M = || q_1, ..., q_4 ||^T$. In accordance with used method we must introduce in (7) a small parameter. We suppose that the elements of this system have a sufficiently high rigidity and we assume $c_{44} = c_{44}^* / m^2$, $b_{44} = b_{44}^* / m$, $m > 0$ is a small parameter.

Further, using the constructed transformation of type

$$z = \|a_1, a_2\|^T q_M^{\bullet} + \|b_1^0 + g_1^0, b_2^0 + g_2^0\|^T q_M , \quad k_1 = \|a_1, a_2, a_3\|^T q_M^{\bullet} , \quad k_2 = a_4 q_M^{\bullet} , \quad q_j = q_j \ (j = 1,4)$$

where a_i , b_i , g_i are submatrices of matrices *a, b, g,* we shall lead (7) to singularly perturbed form (4) of type

$$\frac{dz}{dt} = Z(t,m,z,x), \quad M(m)\frac{dx}{dt} = P(m)x + X(t,m,z,x) \tag{8}$$

$$x = \|x_1, x_2, x_3\|^T, \quad x_1 = \|k_1, q_1\|^T, \quad x_2 = k_2, \quad x_3 = q_4$$

$$a_1 = 0, \quad a_2 = 2, \quad a_3 = 0, \quad P_{2i}(m) = mP'_{2i}(m) \quad (i = 1,2)$$

Accepting the shortened system of *0*-level for approximate one to (8) and returning to old variables, we receive the simplified approximate model

$$\frac{d}{dt} a^* \dot{q}_M + (b^* + g^*)\dot{q}_M + c^* q_M = Q^*{}_M, \quad \frac{dq}{dt} = \dot{q}_M \tag{9}$$

q is vector of generalized coordinates of absolutely rigid system. We shall call it a "limit" model. By methods of N.G.Chetayev (1957) we receive a statement

Theorem 1. If $\left|b_{ij}^0 + g_{ij}^0\right|_{i=1,2}^{j=2,3} \neq 0, \quad \left|c_{ij}\right| \neq 0$ and all roots (except m zero roots) of characteristic equation of simplified system (9) have negative real parts, then with sufficiently small values of m (sufficiently high rigidity of the system elements):- from the zero solution stability of simplified system (9) the zero solution stability of the full system is succeeded; for preassigned numbers $x > 0$, $h > 0$, $g > 0$ (where x and g can be taken as small as one wishes);and - there is such a value of m_*, that in a perturbed motion when $0 < m < m_*$ for every $t \geq t_0 + g$

$$\left\|q_i - q_i^*\right\| < x, \quad \left\|\dot{q}_i - \dot{q}_i^*\right\| < x \quad (i = 1,2,3,4)$$

if $q_{i0} = q_{j0}^*, \quad \dot{q}_{i0} = \dot{q}_{i0}^* \quad (j = 1,2,3), \quad \left\|q_{40}\right\| < h, \quad \left\|\dot{q}_{40}\right\| < h$. Here by the index "*" a solution of simplified system (9) was marked ($q_4^* \equiv 0, \quad \dot{q}_4^* \equiv 0$); and without that index it is a solution of the full system (7). These results, complementing already known [22] justify for the systems, considered here, admissibility of simplified limit model (as approximate model of *0*-level) and determine the conditions, under which the considered transition is correct (in a meaning, adopted here).

Remark. According to (1) we can introduce other simplified model (as set model) for (7). This is asymptotic model of *1*-level, which has s+(*n*-s)/*2* of freedom degrees (if in (8) take into consideration members containing m in power no greater than 1). This model is ***new one.*** System (8) belongs to the special critical case, when all eigenvalues of matrix P_{22}, corresponding to the fast x_2, are zero. These results are important for aerospace engineering [26].

b) System with the fast rotors.

Using our method, here we shall consider the gyrostabilization systems (electromechanical model). Assuming the gyroscopes rotors are fast we can obtain approximate models. Let the differential equations for such systems are represented [9,20,24] in the form

$$\frac{d}{dt} a q_M^{\bullet} + (b^0 + g^0) q_M^{\bullet} = Q_M' + Q_M'' \tag{10}$$

$$\frac{d}{dt} L q_E^{\bullet} + B^0 q_3^{\bullet} + R^0 q_E^{\bullet} = Q_E' + Q_E'', \quad \frac{dq_M}{dt} = q_M^{\bullet}$$

$$Q_M' = \left\| 0,\ A^0 q_E^{\bullet},\ -c^0 q_4 \right\|^T, \quad Q_E' = -\left\| w^0 q_1,\ W^0 q_E^{\bullet},\ 0 \right\|^T$$

For fast gyroscopes $g = g^* H$, $H = 1/m$, $m > 0$ is small dimensionless parameter.

Here the necessary transformation of the variables is constructed:

$t = m t$;

$$z = m^2 a_1 \frac{dq_M}{dt} + (m b_1^0 + g_1^0) q_M, \quad x_1 = a \frac{dq_M}{dt}, \quad x_2 = L q_E^{\bullet}, \quad x_3 = q_1, \quad x_4 = q_4$$

and the system (10) in new variables is SPS of type

$$\frac{dz}{dt} = Z(t, m, z, x), \quad M(m) \frac{dx}{dt} = P(m) x + X(t, m, z, x)$$

$$x = \left\| x_1, x_2, x_3 \right\|^T, \quad a_1 = 2, \quad a_2 = 1, \quad a_3 = 0 \tag{11}$$

In this case the shortened system of *1*-level leads to precessional model (approximate model SM_1)

$$(b^0 + g^0) \frac{dq_M}{dt} = Q_M' + \overline{Q}_M'' \tag{12}$$

$$\frac{d}{dt} L q_E^{\bullet} + B^0 q_3^{\bullet} + R^0 q_E^{\bullet} = Q_E' + Q_E''$$

The shortened system of *0*-level leads to limit model – approximate model SM_0 ; it's *new model* for gyro systems of considered class

$$g^0 q_M^{\bullet} = Q_M' + \tilde{Q}_M'', \quad R^0 q_E^{\bullet} = Q_E' + \tilde{Q}_E'' \tag{13}$$

The possibility of using approximate system more simple than known precessional model was pointed early by D.R.Merkin, 1956. This approach allows to obtain these models by *strict means* as approximate models of corresponding levels. These shortened models get obvious "physical" interpretation in application to mechanics. The conditions of the models correctness in critical case may be found by same method as in above using stability theory methods.

Theorem 2. If $\left|g^0\right| \neq 0,\ \ \left|g_{ij}^0\right|_{i=1,2}^{j=2,3} \neq 0$ and all roots (except *m* zero roots) of the characteristic equation for precessional system (12) have negative real parts and equation $\left|a^0 l + b^0 + g^0\right| = 0$ satisfies the Hurwitz conditions then with sufficiently big H-values:- from the zero–solution stability of precessional system (12) the zero–solution stability of full system (10) is followed;- and for preassigned numbers $x > 0,\ h > 0,\ g > 0$ (x and g can be taken as small as one wishes), there is such a H_*–value that with all $H > H_*$ in disturbed motion for all $t \geq t_0 + g$

$$\left\|\dot{q}_M - \dot{q}_M^*\right\| < x,\quad \left\|q_M - q_M^*\right\| < x,\quad \left\|\dot{q}_E - \dot{q}_E^*\right\| < x$$

if $\left\|\dot{q}_{M0} - \dot{q}_{M0}^*\right\| < x,\quad q_{M0} = q_{M0}^*,\quad \dot{q}_{E0} = \dot{q}_{E0}^*$, (by index "*" the solution of precessional system (12) marked).

This result gives the ***strict substantiation*** of a approximate model use, corresponding to known precessional model in critical case. With the transition to this model the stability properties are kept and there is proximity of solutions in an infinite time interval.

Theorem 3. If $\left|g^0\right| \neq 0,\ \ \left|g^0\right|_{i=1,2}^{j=2,3} \neq 0$ and all roots (except *m* zero roots) of the characteristic equation for limit system (13) are found in the left half-plane and equations $\left|Ll + R^0 + W^0\right| = 0$, $\left|a^0 l + b^0 + g^0\right| = 0$ satisfy the Hurwitz conditions then with sufficiently big H-values:- from zero-solution stability of the limit system (13) the zero-solution stability of the full system (10) follows;- and for preassigned positive numbers x, h, g (x and g can be taken as small as one wishes), there is such a H_{**}–value, that with all $H > H_{**}$ in perturbed motion for all $t \geq t_0 + g$

$$\left\|\dot{q}_M - \dot{q}_M^*\right\| < x,\quad \left\|q_M - q_M^*\right\| < x,\quad \left\|\dot{q}_E - \dot{q}_E^*\right\| < x$$

if $\left\|\dot{q}_{M0} - \dot{q}_{M0}^{**}\right\| < h,\quad \left\|\dot{q}_{E0} - \dot{q}_{E0}^{**}\right\| < h,\quad q_{M0} = q_{M0}^{**}$ (by "**" index the solution of limit system (13) was marked).

This result determines conditions, in which a transition to a *limit model* in analysis of electromechanical models with fast gyroscopes is permissible.

Remark. These results supplement and generalize already known in the perturbations theory and in gyroscopes theory. Shortened models are asymptotic approximate models corresponding of levels, which have (*u*+*n/2*) and (*n*+*u*)/*2* of freedom degrees. System (11) belongs to the specific critical case (in sense of A.M.Lyapunov), when all eigenvalues of matrix, corresponding to the fast variables x_1, are imaginary.

The mechanical systems with the small parameters.

Here we shall consider the electromechanical system of type (10) without writing and denoting it (14).

c) System with the small constants of time.

We shall suppose that the transitional processes in electric circuits of follow-up systems are quick-response. Therefore we assume that $L=L^* m_1$, $A=A^* m_1$, $B=B^* m_1$, $m_1>0$ is small parameter. We found (constructed) the corresponding of variables transformation:

$$t = m_1 t,\quad x_1 = a\frac{dq_M}{dt},\quad x_2 = L^* \dot{q}_E,\quad x_3 = q_1,\quad z = m_1 a_1 \frac{dq_M}{dt} + (b_1^0 + g_1^0) q_M$$

In these variables the system (14) has the form of type (11) where $a_1 = 1$, $a_2 = 2$, $a_3 = 0$. In accordance with the above adduced results we obtain here (for the electromechanical system with the quick-response, small-inertial electrical circuits) two types of simplified models (SM_1, SM_0):

$$\frac{d}{dt} a\dot{q}_M + (b^0 + g^0)\dot{q}_M = Q'_M + Q''_M \tag{15}$$

$$R\dot{q}_E = Q'_E + \overline{Q}''_E,\quad \frac{dq_M}{dt} = \dot{q}_M$$

and

$$(b+g)\dot{q}_M = Q'_M + \tilde{Q}''_M \tag{16}$$

$$R\dot{q}_E = Q'_E + \tilde{Q}''_E,\quad \frac{dq_M}{dt} = \dot{q}_M$$

The conditions of the admissibility of these approximate models we can obtain by same method based on ideas of N.G.Chetayev (with the estimate of m^* values).

Remark. The system (16) and the system (13) are in principle the different approximate models, with the different conditions of their acceptability: (13) is SM of *0*-level on m-parameter, (16) is SM of *0*-level on m_1-parameter. The domains of their validity are different.

d) System with the small controlling elements.

Here we suppose, that mass and the inertia moments of gyroscopes and their mounts are smaller in comparison with ones of controlled (stabilized) objects. In accordance with this we introduce in (14) m_2 –other small parameter; we construct the required transformation of variables and obtain the approximate models (including limit model on m_2 parameter). It is *new model*. Also *the conditions of acceptability are new,* different from known [21]. It is *important result.*

e) Also in framework of this approach it is discussed actual problems of modelling for mechanical systems theory with the friction, Newton's model of mass point dynamics, non-holonomic systems theory, robotic systems, …Do not showing here these applications, we would like to note: by analogy with considered problems, using the same methods, other singularly perturbed problems (as problems of modelling of complex multidisciplinary objects) and systems can be considered. The conditions of admissibility of approximate models may be obtained for the other qualitative problems in specific critical cases. By the methods, developed here, we can construct not only set models and conditions of correctness for approximate models, but also the evaluations of admissible values of a small parameter and permissible domains.

4 CONCLUSION

We would like to note.

These elaborated manners corroborated:
for all considered systems always there exists the necessary variables transformation, that allows to lead the initial mathematical model (2) to the standard form of (4); to obtain the shortened models on elaborated scheme (1)(as approximate asymptotic models, correct in dynamic problems); to get the acceptability conditions. These results are extending and supplementing the known ones for reduction principle and comparison method [10-19].
Developed asymptotic approach, based on stability postulate and singularity postulate (N.G.Chetayev, L.K.Kuzmina), gives effective algorithm to the solving problem of modelling in general. It allows to elaborate *regular* methods of *engineering level*, that are available for broad dissemination in engineering practice. Besides *the procedures* of constructing of working models, with the determination of their acceptability domains, *are formalized*.For this it is elaborated the regular algorithm of engineering level. Moreover we obtain *the possibility to investigate a original complex system by analytical or numerically-analytical methods.*
The worked out methods are very effective for examples from Mechanics. New elegant outcomes are obtained, that are interesting both for theory and for applications, both in general theory of singularly perturbed systems and in applied engineering problems. Also this approach is very perspective from gnosiological view point, for general Knowledge theory. With reference to systems dynamics with friction this developed approach allows to reveal interesting new models.

From this side, very *interesting gnosiological aspects are revealed* by this approach. Understanding of models idealization problem through the singularly perturbed problem is opening the way for discussion possibility of mechanics models, that are different from Newton's mechanics model (as example, Aristotle's mechanics model, AM, of 1-st order; hypothetical mechanics model (HM) of 3-rd order,…).AM is revealed as approximate asymptotic model for system with big friction…

Note, these meditations are no imagination play, but ones are very important for general Knowledge theory, for deep understanding of Mechanics… In particularly, last model (HM)

discussion allows to explain, that Newton's model (as approximate model for HM) may be non-acceptable for description of dynamics of original system, if there is critical case on Lyapunov for this approximate model (as example, in case of conservative system). In this case we can receive big divergence between theoretical results (corresponding the solution of approximate model) and test data (corresponding original system, HM).
Methodology of A.M.Lyapunov theory, these methods and ideas give constructive tool for developing of our Knowledge in whole. This approach makes it is possible the solving of the fundamental problem of modeling In Mechanics.

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