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ON LINEARIZATION OF NONLINEAR DYNAMIC SYSTEMS DESCRIBED BY STATE-DEPENDENT-PARAMETER (SDP) DISCRETE-TIME MODEL

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Abstract. This work develops and applies the exact linearization of nonlinear systems described by State Dependent Parameter (SDP) models. This allows the implementation of conventional Proportional-Integral-Plus (PIP) control using its linear strategy. A simple demonstrator consists of a DC motor, drives a belt as an automated line, and equipped with a single board Reconfigurable Input-Output (sbRIO-9631) card, within a Field Programmable Gate Array (FPGA) and with a real time processor for control is used. A discrete-time SDP model, for the nonlinear system, is first constructed, in which its parameters are system state dependent. Then, an exact linearization is applied in order to return to the linear model, which is subsequently used to design and apply the PIP control algorithm based on linear system design strategies. Preliminary experimental results demonstrate that the approach yields an acceptable control performance for the proposed demonstrator.

1 INTRODUCTION

Exact linearization by feedback for nonlinear continuous-time systems has been studied by many authors, e.g. [1,2]. Such authors discuss the conditions required for exact linearization; and use of a local coordinate transformation in the state space, in order to transform the nonlinear system into a linear, controllable system. Guay [1] suggests orbital feedback linearization or state dependent time scaling for nonlinear systems for which their linearizability conditions are not met.

The analogy between discrete-time and continuous-time systems has motivated other authors to study linearization by feedback for discrete-time systems; e.g. [3-5]. Exact linearization by feedback for SDP-TF models had previously been hinted out by McCabe et al. [5], although not explicitly utilized and fully developed.

However, partial and exact linearization of nonlinear SDP systems by local coordinate transformation has been first developed and implemented successfully by Shaban and Taylor [6], the simulation of deadbeat response using pole placement approach could not be achieved for systems having more than two samples delay.

Recently, the necessary and sufficient conditions for exact linearization of such SDP systems has been fully defined and developed by Shaban [7], such that any nonlinear SDP model can now be linearized and its typical deadbeat simulation is straightforwardly achieved. The author provided several simulation examples to validate the linearization methodology; however on-line implementation has not been investigated yet.

The current paper develops and examines the exact linearization by local coordinate transformation of nonlinear SDP system, the linear PIP control algorithm is then developed to control a simple conveyor demonstrator driven by DC motor, which equipped with sbRIO-9631 within FPGA board for controller implementation. The system is considered as fast mechatronics system, since its sampling rate is 50 Samples/s.

The implementation suggests that for such SDP nonlinear model structures, the exact linearization approach is straightforward and can be utilized to develop a fixed gain controller. The linearized SDP-PIP control applied to the system achieves acceptable tracking performance for the selected set points. Nevertheless, the linearized SDP-PIP control shows excellent tracking performance, the on-line implementation still confront some difficulties due to the iterations required to solve the mapping equation for the control action.

2 NONLINEAR SDP-PIP CONTROL

State Dependent Parameter, Proportional Integral Plus (SDP-PIP) control, used for a wide class of nonlinear systems, is a promised research area being investigated to improve linear Proportional Integral Plus (PIP) control. However, SDP model has been introduced earlier by Young [8], the practical development of this model is of more recent origin [9]. The general form of nonlinear discrete-time SDP model can take the form:

$$y_k = \frac{b_{1,k+1}\mathrm{z}^{-1} + \cdots + b_{m,k+m}\mathrm{z}^{-m}}{1 + a_{1,k+1}\mathrm{z}^{-1} + \cdots + a_{n,k+n}\mathrm{z}^{-n}} u_k = \frac{B_k(\mathrm{z}^{-1})}{A_k(\mathrm{z}^{-1})} u_k \tag{1}$$

where u_k and y_k are the system input and output respectively. However, the model parameters $a_{i,k+i}$ $\forall 1 \le i \le n$ and $b_{j,k+1}$ $\forall 1 \le j \le m$ are functions of the system states. Also the operator z^{-1} is the backward shift operator. The corresponding Non-Minimal State Space (NMSS) form for the SDP model (1), so-called SDP-NMSS, can be represented as:

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{F}_{k+1}\,\mathbf{x}_k + \mathbf{g}_{k+1}\,u_k + \mathbf{d}\,r_{k+1} \\ y_k &= \mathbf{h}\,\mathbf{x}_k \end{aligned} \tag{2}$$

The $n+m$ dimensional non-minimal state space vector $\mathbf{x}_k$ consists of the present and past sampled values of the output and input variables as follows:

$$\mathbf{x}_k = \begin{bmatrix} y_k & y_{k-1} & \cdots & y_{k-(n-1)} & u_{k-1} & \cdots & u_{k-(m-1)} & z_k \end{bmatrix}^T \tag{3}$$

Here, $z_k = z_{k-1} + (r_k - y_k)$ is the integral-of-error state, introduced to ensure inherent type 1 performance, giving that r_k is the reference level. The full definition of the state transition matrix $\mathbf{F}_{k+1}$, input vector $\mathbf{g}_{k+1}$, command input vector $\mathbf{d}$, and output vector $\mathbf{h}$ is existed in [7].

The control law associated with the SDP-TF model (1) takes the usual State Variable Feedback (SVF) form:

$$u_k = -\mathbf{v}_k\,\mathbf{x}_k \tag{4}$$

where the SVF gain vector is $\mathbf{v}_k = [f_{o,k} \quad f_{1,k} \quad \cdots \quad f_{n-1,k} \quad g_{1,k} \quad \cdots \quad g_{m-1,k} \quad -k_{I,k}]$. The control action, u_k, represented in (4) can be implemented as shown in Figure 1, where $F_k(\mathrm{z}^{-1})$ and $G_k(\mathrm{z}^{-1})$ are defined as:

$$
\begin{aligned}
F_k(\mathrm{z}^{-1}) &= f_{o,k} + f_{1,k}\mathrm{z}^{-1} + \cdots + f_{n-1,k}\mathrm{z}^{-(n-1)} \\
G_k(\mathrm{z}^{-1}) &= g_{1,k}\mathrm{z}^{-1} + g_{2,k}\mathrm{z}^{-2} + \cdots + g_{m-1,k}\mathrm{z}^{-(m-1)}
\end{aligned}
\tag{5}
$$

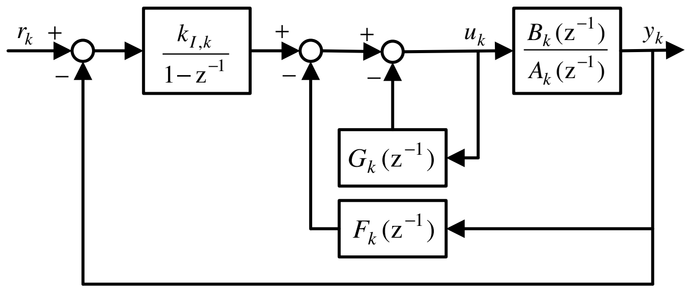

Figure 1: The implementation of SDP-PIP control action (4) on SDP-TF (1)

Equation (5) gives the elements of the SDP-SVF gain vector, for which its elements are variable (state dependent) and their values are updated at each sampling instant.

Linear Quadratic (LQ) control design can be implemented by optimizing the following LQ cost function:

$$
J = \sum_{i=0}^{\infty} \left\{ \mathbf{x}_i^T \mathbf{Q}\,\mathbf{x}_i + R\,u_i^2 \right\} \tag{6}
$$

Here, the optimum SDP-SVF gain vector is obtained such that it satisfies predetermined conditions or weighting criteria. The procedure to solve this minimization problem is by means of the Algebraic Riccati Equation (ARE), see equation (7), derived from the standard LQ cost function (6). In this case, the SDP-SVF gain vector $\mathbf{v}_k$ is obtained recursively at every sample k based on the SDP-NMSS system matrices defined at that sampling instant as:

$$
\begin{aligned}
\mathbf{v}_k &= \left[\mathbf{g}_k^T\,\mathbf{P}^{(i+1)}\,\mathbf{g}_k + R\right]^{-1} \mathbf{g}_k^T\,\mathbf{P}^{(i+1)}\,\mathbf{F}_k \\
\mathbf{P}^{(i)} &= \mathbf{F}_k^T\,\mathbf{P}^{(i+1)}\left[\mathbf{F}_k - \mathbf{g}_k\,\mathbf{v}_k\right] + \mathbf{Q}
\end{aligned}
\tag{7}
$$

where $\mathbf{Q} = \mathrm{diag}\left[q_{y_1} \quad \ldots.. \quad q_{y_n} \quad q_{u_1} \quad \ldots.. \quad q_{u_m} \quad q_e\right]$ is a square matrix with dimension $n+m$, for which $q_{y_i}\ \forall\, i = 1,\ 2,\ \ldots,\ n$ are the output weighting parameters, $q_{u_j}\ \forall\, j = 1,\ 2,\ \ldots,\ m$ are the lagged input weighting parameters, and q_e provides the weighting of the integral of error state variable z_k, and R is an additional scalar weight of input. Also $\mathbf{P}$ is symmetrical-positive definite matrix and its initial value, $\mathbf{P}^{(i+1)} = \mathbf{Q}$; finally, $\mathbf{v}_k$ is the control gain vector.

A prerequisite of global controllability is that the NMSS system (2) is piecewise controllable at each sample k. The SDP-NMSS controllability conditions are developed at [6]. It is worth to note that, due to the time variation of the SDP-TF parameters, the controllability conditions are not necessarily guaranteed at each sample. Therefore, singularities may arise during the implementation of the SDP-PIP control [7]. Exact linearization of SDP-TF illumines these singularities such that they can be overcome [6,7].

3 EXACT LINEARIZATION

The conventional sample delay term, δ, used in linear discrete-time TF models, may give an apparent value in case of nonlinear discrete-time SDP-TF models. This is due to the inappropriate structure of these SDP models. Therefore, the term relative degree, ρ, has been first declared to nonlinear SDP-TF to determine its actual samples delay [6,7]. This consequently allows the proper constitution for SDP-TF models [7]. The conditions for determining the relative degree, ρ, can be introduced as

$$\begin{aligned} &\frac{\partial \Delta^i y_k}{\partial \tilde{\mathbf{x}}_k}\tilde{\mathbf{g}}_{k+1} = 0 \qquad \forall \quad i = 0, ..., \rho-2 \\ &\frac{\partial \Delta^{\rho-1} y_k}{\partial \tilde{\mathbf{x}}_k}\tilde{\mathbf{g}}_{k+1} \neq 0 \end{aligned} \tag{8}$$

for which $\tilde{\mathbf{g}}_{k+1}$ is the regulator input vector [7], and the regulator state space vector $\tilde{\mathbf{x}}_k$ is similar to $\mathbf{x}_k$ in (3) without the integral of error state, z_k.

Relative degree ρ, defined in (8) for SDP models, is exactly equal to the number of samples required for the input to start affect the system output. In case of linear discrete-time TF models, the relative degree of the system exactly equals to its sample delay, i.e. $\rho = \delta$. However, in nonlinear SDP-TF models, its several possible structures may lead to relative degree ρ, less than the apparent sample delay δ, i.e. $\rho < \delta$. This improper structure of SDP-TF models may lead to difficulties in the design of SDP-SVF control, as well as this will confront difficulties in case of linearizing the SDP-SVF control gains.

Determining the relative degree, ρ, of the nonlinear SDP model, the steps of the exact linearization by local coordinate transformation can be summarized in the following steps [7]:

1- Define the new linearized input state space vector $\overline{\mathbf{X}}_k$, which its elements are defined as:

$$U_{k-q} = \Delta^{\rho-q} y_k - \sum_{i=2}^{n} \Delta^{\rho-(q+1)} y_{k-(i-1)} + \sum_{j=q}^{\rho-2} \Delta^{j-q} U_{k-(j+1)} \quad \forall q = 1, ..., \rho-1 \tag{9}$$

In case of nonlinear systems with unity relative degree, i.e. $\rho = 1$, there is no use for transformation (11), since there is no input state space vector.

2- The new linearized model can be constructed using transformation (11), at $q = \rho-1$, i.e.

$$U_{k-(\rho-1)} = \Delta y_k - \sum_{i=2}^{n} y_{k-(i-1)} \tag{10}$$

The linearized model, in incremental form, is then

$$\Delta y_k = \sum_{i=2}^{n} Y_{k-(i-1)} + U_{k-(\rho-1)}$$

The difference state Δy_k is defined in forward difference as $\Delta y_k = y_{k+1} - y_k$. Therefore, the expansion of model (10), with step-behind leads to:

$$y_k = y_{k-1} + y_{k-2} + \ldots + y_{k-n} + U_{k-\rho} \tag{11}$$

This is a linear system with unity parameters at all input and output terms, for which the corresponding linear TF model is:

$$y_k = \frac{z^{-\rho}}{1 - z^{-1} - z^{-2} - \ldots - - z^{-n}} U_k \tag{12}$$

3- Closed-loop TF or NMSS form can be constructed in the usual manner for the linearized system (12). Either pole placement or LQ control design can be utilised to find the time-invariant SVF gain vector, $\mathbf{v}$. The linearized control input, U_k, can be then obtained using the control law, $U_k = -\mathbf{v}\,\mathbf{X}_k$. Considering that the linearized state space vector is $\mathbf{X}_k = \left[y_k \quad \ldots \quad y_{k-(n-1)} \;\vdots\; \overline{\mathbf{X}}_k \;\vdots\; z_k \right]^T$.

4- The mapping between the transitional linearized input U_k and the actual system input u_k can be obtained from the $(\rho-1)^{\text{th}}$ difference of the transformation in (10), i.e.

$$\Delta^{\rho-1} U_{k-(\rho-1)} = \Delta^{\rho} y_k - \sum_{i=2}^{n} \Delta^{\rho-1} y_{k-(i-1)} \tag{13}$$

4 MECHATRONICS DEMONSTRATOR

The proposed mechatronics demonstrator is composed of a simple conveyor system driven by DC motor with its Pulse Width Modulation (PWM) driver, digital encoder for signal feedback, and Field Programmable Gate Array (FPGA) board equipped with single board Reconfigurable IO (sbRIO-9631) for control processing, see Figure 2.

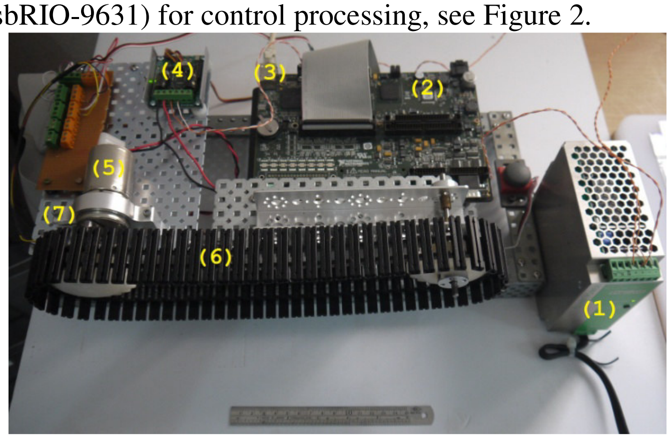

Figure 2: The Mechatronic system used for evaluating the implemented controllers, showing the DC motor (5) with its PWM board (4) and encoder (7). The sbRIO-9631 (2) and Ethernet port (3), within the FPGA board, are also shown. The whole system is activated by a power supply (1) to drive a simple conveyor system (6)

Here, the sbRIO-9631 is characterized by 266 MHz real-time processor with 1 M gate Xilinx Spartan FPGA, 110 IO lines. It also has 64 MB of DRAM for embedded operations and 128 MB of nonvolatile memory for program storing and data logging. The device is featured by a built-in 10/100 Mbits/s Ethernet port to conduct signal communication via network.

Traditionally, the programming of FPGA board requires experienced digital software designers. However, nowadays, National Instruments (NI) Corporation has simplified this task by using visual programming system design by means of LabVIEW FPGA module. The LabVIEW FPGA module defines the FPGA logic, which provides timing, triggering, processing and custom IO measurements. FPGA logic provides analog loop rates exceed 100KS/s, and digital loop rates up to 1 MS/s. It is also possible to evaluate multiple rungs of Boolean logic using single-cycle loop at rate of 40 MHz.

The high dynamics of the mechatronics demonstrator requires high sampling rate. Several experimental trials suggest the use of time step of 20 ms, this rate is sufficient to describe the inherent dynamics for the system.

The description of nonlinear dynamic systems using SDP models is first depicted in Young [8]. The approach exploits both recursive Kalman Filtering and Fixed Interval Smoothing (FIS) methods, within an iterative backfitting algorithm that involves special reordering of the data. All these statistical tools and associated algorithms have been assembled as CAPTAIN toolbox within the Matlab® environment (www.es.lancs.ac.uk/cres/captain) [10].

The first stage of the analysis requires the identification of typical discrete-time linear TF models. Therefore, open-loop experiment is conducted by means of LabVIEW FPGA module, and random input signal has been applied, with time step 0.02 s, to the mechatronics system. The collected input/output data are then collected and stored for modelling process. CAPTAIN toolbox reveals that a first order with four samples delay gives an appropriate model structure across a wide range of input operating conditions, i.e.

$$y_k = -a_{1,k}\, y_{k-1} + b_{4,k}\, u_{k-4} \tag{14}$$

The SDP analysis of this mechatronics system suggests that the time-variant parameters, $a_{1,k}$ and $b_{4,k}$, are both functions of the lagged input variable u_{k-4}, for which the optimum state dependent parameters are defined by

$$\begin{aligned} a_{1,k} &= -0.2085\times10^{-6}\, u_{k-4}^2 - 0.7224 \\ b_{4,k} &= -0.003425\times10^{-6}\, u_{k-4}^2 + 0.005869 \end{aligned} \tag{15}$$

4 LINEARIZED SDP-PIP CONTROL FOR THE DEMONSTRATOR

The nonlinear SDP model (14) shows that the regressors, y_{k-1} and u_{k-4}, and their time variant parameters (15), $a_{1,k}$ and $b_{4,k}$, are lagged by equal steps. Therefore, the given structure for the SDP model guarantees that the relative degree equals the samples delay, $\rho = \delta = 4$. It is now possible to define the new linearized input state vectors, according to equation (9), as:

$$\begin{aligned} U_{k-1} &= \Delta^3 y_k + U_{k-2} + \Delta U_{k-3} \\ U_{k-2} &= \Delta^2 y_k + U_{k-3} \\ U_{k-3} &= \Delta y_k \end{aligned} \tag{16}$$

Recalling that $\rho = 4$ and $n = 1$, then the linearization of the SDP model (14) takes the form of (10), $U_{k-3} = \Delta y_k$. Giving that $\Delta y_k = y_{k+1} - y_k$, then the linearized model takes the form:

$$y_k = y_{k-1} + U_{k-4} \tag{17}$$

As expected, the linearized system has unity parameters at all input and output terms. The corresponding linearized TF model is then:

$$y_k = \frac{z^{-4}}{1 - z^{-1}} U_k \tag{18}$$

The linear NMSS form can now be constructed, and LQ control design can be used to find the linearized time-invariant SVF gain vector, $\mathbf{v}$. The control law, $U_k = -\mathbf{v}\,\mathbf{X}_k$, can be applied, given that the new linearized state vector is $\mathbf{X}_k = [y_k \quad U_{k-1} \quad U_{k-2} \quad U_{k-3} \quad z_k]^T$.

According to this system, the mapping equation (13), which maps between the transitional linearized input U_k and the actual system input u_k, takes the form:

$$\Delta^3 U_{k-3} = \Delta^4 y_k \tag{19}$$

The 4th difference of the output y_k can be obtained using model (14) as follows:

$$\begin{aligned}\Delta^4 y_k &= \Delta^3 y_{k+1} - \Delta^3 y_k \\ &= -\left(\alpha_1 u_k^2 + \alpha_2 + 1\right)\left(y_k + 3\Delta y_k + 3\Delta^2 y_k + \Delta^3 y_k\right) \\ &\quad -\left(\Delta y_k + 3\Delta^2 y_k + 3\Delta^3 y_k\right) + \left(\beta_1 u_k^2 + \beta_2\right) u_k\end{aligned}$$

By substituting into equation (19), the following 3rd order mapping equation is obtained:

$$c_1 u_k^3 + c_2 u_k^2 + c_3 u_k + c_4 = 0 \tag{20}$$

for which

$$\begin{aligned}c_1 &= \beta_1 \\ c_2 &= -\alpha_1\left(y_k + 3\Delta y_k + 3\Delta^2 y_k + \Delta^3 y_k\right) \\ c_3 &= \beta_2 \\ c_4 &= -(\alpha_2 + 1)\left(y_k + 3\Delta y_k + 3\Delta^2 y_k + \Delta^3 y_k\right) \\ &\quad -\left(\Delta y_k + 3\Delta^2 y_k + 3\Delta^3 y_k\right) - \Delta^3 U_{k-3}\end{aligned}$$

Iterative Newton-Raphson method can be used in (20) to get the nearby solution for u_k, given an appropriate initial value.

Regarding LQ control design, the linear NMSS form for the linearized model (17) takes the form of (2), for which

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -1 & 1 \end{bmatrix}, \quad \mathbf{g} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{d} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{h} = [1 \quad 0 \quad 0 \quad 0 \quad 0]$$

The straightforward implementation of the ARE in (7) provides the time-invariant control gain vector, $\mathbf{v}$. Several experimental trials suggest the use of weightings, $\mathbf{Q} = \text{diag}[1 \quad 1 \quad 1 \quad 1 \quad 1]$, and $R = 5$. The associated SVF gain vector is:

$$\mathbf{v} = [1.271 \quad 0.825 \quad 1.048 \quad 1.271 \quad -0.223] \tag{21}$$

The closed-loop response using SVF gain vector (21) is shown in Figure 3.

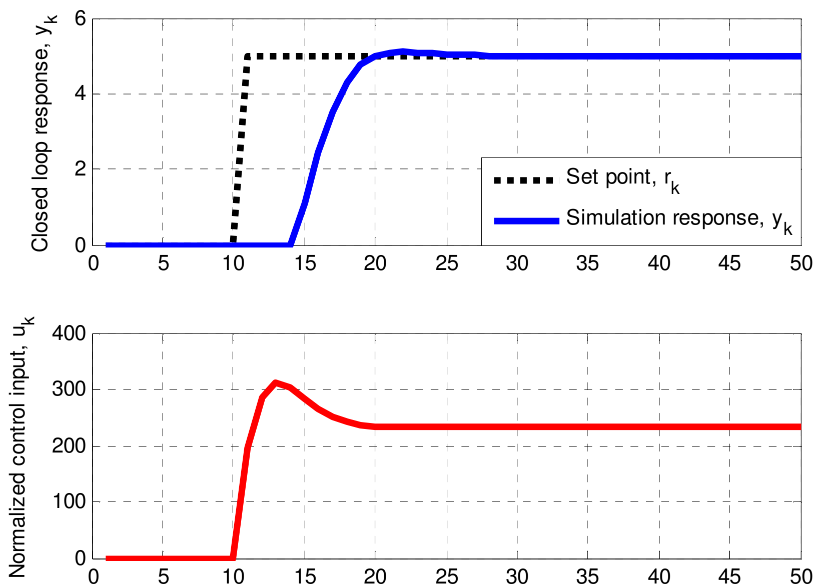

Figure 3: The closed-loop response of the SDP model (21) using exact linearization, for a set point selected at 5, using LQ method

5 IMPLEMENTATION

The exact linearized controller is implemented and examined at two different levels of set points, see Figure 4. As depicted in this figure, the controller provides zero steady state error, however the control input suffers from severe noise because of the low sensitivity of the feedback signal coming from the encoder. It is worth to note here that exact linearization, till the moment, does not prove robustness towards such output disturbances [7].

6 CONCLUSION

This work investigates the applicability of the novel exact linearization method, applied to relatively fast mechatronics demonstrator, for which the time step is selected to be 0.02 s. The paper considers the construction of the nonlinear SDP-TF, followed by full development of exact linearization approach to achieve the time-invariant gain vector for controlling the nonlinear system. The necessary and sufficient conditions for exact linearization methodology are discussed so that the achieved controllable linear system is equivalent to the given nonlinear system. However, the feasible SDP-TF model can be controlled using the basic nonlinear SDP-PIP approach, the present paper applies an exact linearization to study its applicability and reliability for such nonlinear systems.

Regarding the simulation study, the exact linearization of the SDP model (14) provides satisfactory closed-loop response, by using suboptimal LQ method, using weightings $\mathbf{Q} = \mathrm{diag}[1 \quad 1 \quad 1 \quad 1 \quad 1]$, and $R = 5$. The application of the linearized SDP control requires iterative or some numerical methods to implement.

The on-line implementation of the exact linearization of SDP-PIP controller demonstrates satisfactory system response at two different levels of set points. The noise shown in the system response may arise because of the built-in disturbances come from the encoder. Also, the severe noise existed in the control input may be happened due to the high sensitivity of the linearized controller to these output disturbances.

Finally, the paper shades the light on the applicability of the exact linearization of SDP-PIP control, in order to achieve time-invariant gains. However, the approach provides successful application; the proposed approach still needs further investigations regarding implementing the control law and solving the mapping equation, as well as study its performance in case of output and input disturbances. Moreover, the numerical method used here for solving the mapping equation, for the linearized control action, needs more exploration.

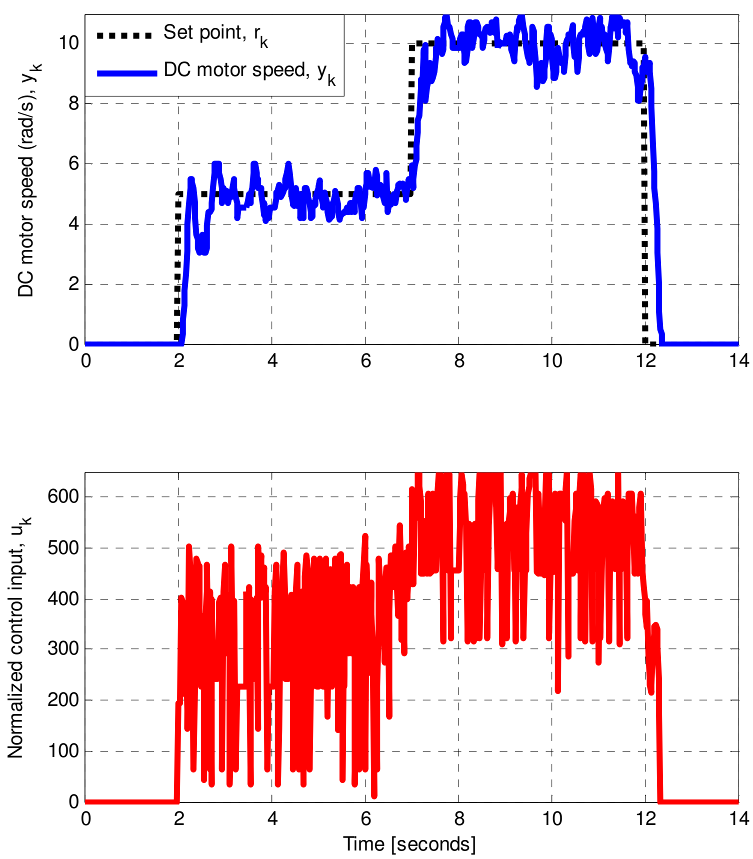

Figure 4: The implementation of the PIP control based on the exact linearization method, at two different levels of set points

REFERENCES

[1] M. Guay. An algorithm for orbital feedback linearization of single-input control affine systems. Systems and Control Letters, 38: 271-281, 1999.

[2] H.G. Lee., A. Arapostathis, and S.I. Marcus. Linearization of discrete-time systems, Int. J. Control, 45(5): 1803-1822, 1987.

[3] M. Sergio Savares, and O. Guido Guardabassi. Approximate I/O feedback linearization of discrete-time non-linear systems via virtual input direct design. Automatica, 34(6): 715-722, 1988.

[4] C. Califano, S. Monaco, and D. Normand-Cyrot. On the problem of feedback linearization. Systems and Control Letters, 36: 61-67, 1999.

[5] A.P. McCabe, P.C. Young, A. Chotai, and C.J. Taylor. Proportional-Integral-Plus (PIP) control of non-linear systems. Systems Science, 26: 25-46. Publisher: OR RAN, Warszawa, 2000.

[6] E.M. Shaban, and C. J. Taylor, Proportional-Integral-Plus control of a class of nonlinear systems using exact and partial linearization by feedback, ICGST International Journal on Automatic Control and System Engineering (ACSE), Volume (6), Special Issue, pages 55 – 70, March 2006.

[7] E.M. Shaban. Deadbeat response of nonlinear systems described by discrete-time state dependent parameter using exact linearization by local coordinate transformation. Journal of American Science, Vol. 8, No. 10, 2012.

[8] P.C. Young. A general theory of modeling for badly defined dynamic systems. In G.C. Vansteenkiste (ed.), Modeling, Identification and Control in Environmental Systems, North Holland: Amsterdam, 103-135, 1978.

[9] P.C. Young. Stochastic, dynamic modelling and signal processing: time variable and state dependent parameter estimation. Appears in Nonlinear and nonstationary signal processing, W.J. Fitzgerald et al. (Ed.), Cambridge University Press, 74-114, 2000.

[10] D.J. Pedregal, C.J. Taylor, and P.C. Young. System Identification, Time Series Analysis and Forecasting. The Captain Toolbox. Handbook v2.0, February 2007.