

FINITE ELEMENT TECHNOLOGY FOR STEEL-ELASTOMER-SANDWICHES

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Abstract. Orthotropic bridges are widely used in today's street infrastructure, especially when large spans have to be bridged. They consist of steel deckplates with attached – typically longitudinal – steel stiffeners and transverse cross-beams. In order to alleviate the stress at the welding joints of the stiffeners and the deckplate a polyurethane and steel sandwich design is investigated. For the purpose of making accurate predictions for this construction, the orthotropic plate is modelled using solid-cube as well as recently developed solid-shell and solid-beam element formulations. These are 8-node reduced integration elements with a variable amount of integration points in one (solid-shell) or two axes (solid-beam) which feature assumed natural strain (ANS) and enhanced assumed strain (EAS) concepts to tackle different kinds of locking – one example being volumetric locking which happens with near-incompressible materials such as polyurethane. The implemented adaptive hourglass stabilization scheme avoids zero-energy hourglass deformation modes. A numerical investigation was carried out and the numerical displacement predictions were within 5% of the experimental results. The predictions of the developed solid elements were generally better than that of the Abaqus standard elements, even for very fine meshes.

1 INTRODUCTION

Unfortunately many existing orthotropic bridges do not concur with today's best practices. Typically the recommended minimum thickness of the steel and asphalt layer is not met and the largest allowable stiffener separation is exceeded. These shortcomings usually manifest in cracks and failure of the welded connection between the deckplate and the stiffeners.

The described premature failure of orthotropic bridges creates a demand for refurbishment and improvement of the existing bridges. One possible solution is the sandwich plate system (SPS). This system consists of two steel plates with a polyurethane core in between. Extensive measurements of specimen with various geometries were carried out in a previous research project [1]. The chosen geometry was afterwards applied to a German motorway under severe traffic.

Previously the numerical prediction of the plates was done using a “quasi-3-D”-model in which 4-node shell elements were linked with trusses forming a cantilever beam. The distance of the shells represented the thickness of the entire plate, i.e. of all 3 layers. The stiffness of the trusses and shells were chosen such that they approximate the shear and bending stiffness of the composite. The described approach led to predictions which were in good agreement with the experimental results.

The drawback of this method is that the mechanical properties of the composite have to be known, the material parameters of the constituents and the geometry alone are not sufficient. This is a consequence of each truss geometrically representing multiple layers. It is unclear whether introducing shells at each layer interface and trusses between all the neighbouring shells could alleviate this restriction. Even if the truss stiffness could be derived directly from material parameters and geometry rather than from a parameter fit, other problems still remain. For instance, implementing full 3-D material models is difficult since trusses are 1-D elements and shells are 2-D elements. Other problems arise in the context of large deformations.

The solid-cube, solid-shell and solid-beam elements allow the separate modelling of each layer with its physical thickness and material properties, thus avoiding the “smearing” of mechanical and structural parameters, see Figure 1.

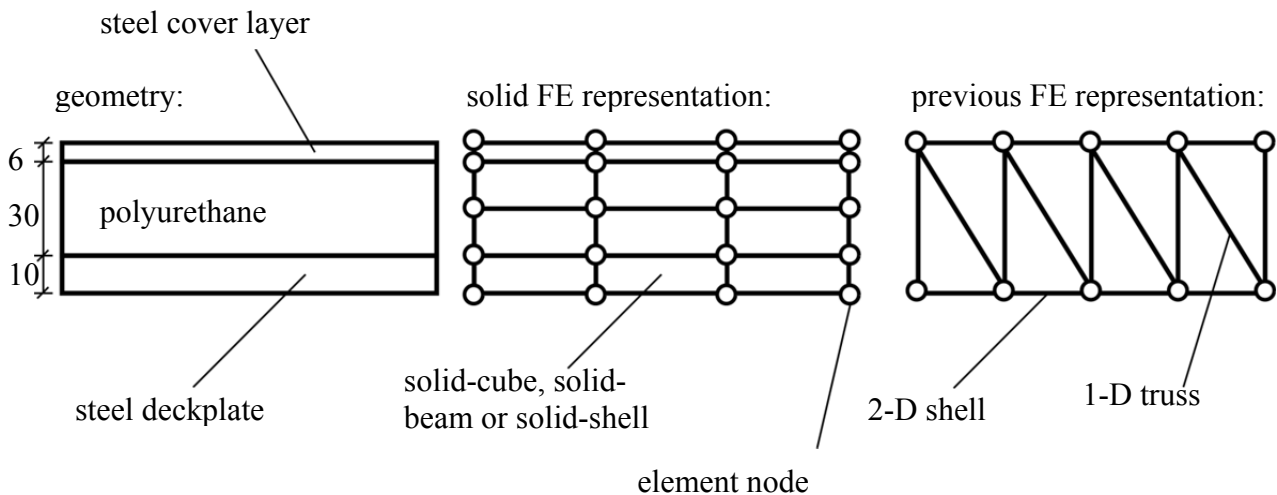


Figure 1: Comparison of the plate geometry and its FE representations.

2 Q1STX ELEMENT

Very comprehensible and detailed overviews of the underlying theory for the solid-shell and solid-beam element formulations are provided in [2] and [3]. A brief summary of the main aspects is provided in the following.

Solid cube, solid-shell and solid-beam elements have in common that they are 8 node elements with linear isoparametric ansatz functions and reduced integration. The solid-shell element has a user-defined number of integration points along the shell axis as depicted in Figure 2. Similarly the solid-beam element has its integration points in the plane perpendicular to the beam axis. The solid-cube element is then the trivial case of solid-shell and solid-beam element where the number of integration points per axis is equal to one.

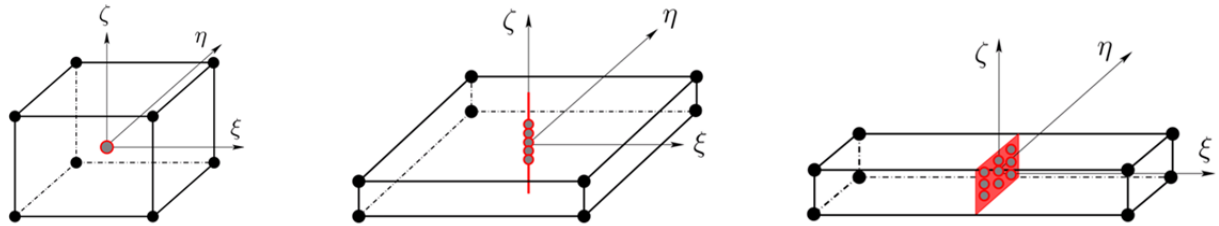


Figure 2: From left to right: solid-cube (Q1STc), solid-shell (Q1STs) and solid-beam (Q1STb). Black points represent nodes and grey points represent integration points.

These three formulations are combined in the Q1STx element. The „x“ in Q1STx indicates that the element can be used as a solid-cube, solid-shell or solid-beam element, denoted by Q1STc, Q1STs and Q1STb respectively.

As previously mentioned there are several locking phenomena which need to be treated if the element formulation is to give accurate results.

In order to address curvature thickness locking and transverse shear locking the assumed natural strain method (ANS) is used. Curvature thickness locking stems from incorrect thickness discretization which results in an overestimation of transverse normal strains. Transverse shear locking is a result of the linear ansatz function's inability to resolve element curvature. The proper cross-section of a bended element can only be resolved at the ends of the element. Instead of proper curvature, the bending is therefore represented by a trapezoidal deformation which leads to unphysical transverse shear strains.

However, ANS cannot cure all locking phenomena. Volumetric locking – as it happens with nearly incompressible materials such as the polyurethane core investigated in this paper – is tackled by using the enhanced assumed strain (EAS) concept. Volumetric locking leads to too stiff structural responses which manifest in an overestimation of stresses and an underestimation of strains.

EAS can also take care of Poisson thickness locking and enable a trapezoidal deformation mode of the beam plane caused by in plane strains in bending situations.

Since the Q1STx is a reduced integration element, hourglassing is generally a problem. The zero-energy modes stemming from hourglass deformation modes are addressed by an adaptive hourglass stabilization scheme.

The aforementioned features make the Q1STx very suitable for sandwich constructions which are usually characterized by multiple stacked layers of varying but generally small thickness

compared to other dimensions. Since the Q1STx is proper volume element, even complex geometries as the one investigated in this study can be realized since a coupling of shells beams and cubes is as straightforward as with any other volume element.

One problem which arises in these complex geometries, however, is the assignment of the proper element type to each individual element. For irregular geometries cube, shell and beam elements are typically not occurring in separate regions but are rather scattered over the entire domain. Especially for irregular geometries a generally valid criterion to determine when an element should be a cube, shell or beam is not easily obtained. For instance the ratio of the shortest to the longest edge of the element – which many FEA tools provide the user with – is not necessarily very conclusive in this question. Although this criterion is useful for elements of approximately cuboid shape, it is misleading for elements of e.g. trapezoidal shape, which can have a high ratio of shortest to longest edge without being reasonable candidates for the use of a shell element formulation. This makes it challenging to assign the element type manually even if there was an inclination to do so. What is more, the aspect ratio changes with each step of mesh refinement which necessitates the repetition of the chosen procedure every time the mesh is changed.

For this reason the Q1STx code was modified to enable automatic element type selection based on the aspect ratio of the element in the reference configuration. In the current version the Q1STx is called with a slenderness parameter and the number of quadrature points per space dimension. This leaves the formulation open at the time the input file is created. In order to determine the aspect ratio ρ , the eigenvalues λ_i of the element Jacobian at the element center \mathbf{J}_0 multiplied with its transpose, are evaluated.

$$\mathbf{J}_0 = \left. \frac{\partial \mathbf{X}}{\partial \xi} \right|_{\xi=0} \quad (1)$$

$$\det(\mathbf{J}_0^T \mathbf{J}_0 - \lambda \mathbf{I}) = 0 \quad (2)$$

Here \mathbf{X} denotes the element reference coordinates and ξ denotes the natural element coordinates. The square roots of the eigenvalues $\sqrt{\lambda_i}$ are equal to the singular values of a singular value decomposition of \mathbf{J}_0 [4].

$$\mathbf{J}_0 = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^* \quad (3)$$

$$\mathbf{J}_0^T \mathbf{J}_0 = (\mathbf{V} \mathbf{\Sigma}^T \mathbf{U}^*) \cdot (\mathbf{U} \mathbf{\Sigma} \mathbf{V}^*) = \mathbf{V} \mathbf{\Sigma}^T \mathbf{\Sigma} \mathbf{V}^* \quad (4)$$

Here \mathbf{U} and \mathbf{V} are unitary matrices and \mathbf{U}^* and \mathbf{V}^* are the associated conjugate transpose matrices. $\mathbf{\Sigma}$ is the matrix containing the singular values. The singular values can be interpreted geometrically as stretches.

This means that $\sqrt{\lambda_i}$ is a measure of how much the i -th element axis is stretched when it is mapped to the natural coordinate system axis. Since the natural coordinate axes always have a constant length of 2, the ratio of the eigenvalue square roots corresponds to the ratio of the element axes in the reference configuration.

The element type is determined by comparing singular values of the Jacobian at the element center. If two singular values are at least ρ times greater than the third singular value Q1STs is selected, that is the quadrature point space dimension is set to one. If one singular value is at least ρ times greater than the other two singular values, Q1STb is selected, that is

Table 1: Overview of the mesh parameters.

	coarse mesh	medium mesh	fine mesh	truss model
number of elements	5340	22476	365988	31648
number of nodes	8334	34131	451458	30768 + unreported number of interim nodes
number of nodes on which external forces are acting	21	27	225	unknown

The medium mesh on which this study focuses is depicted in some detail in Figure 4.

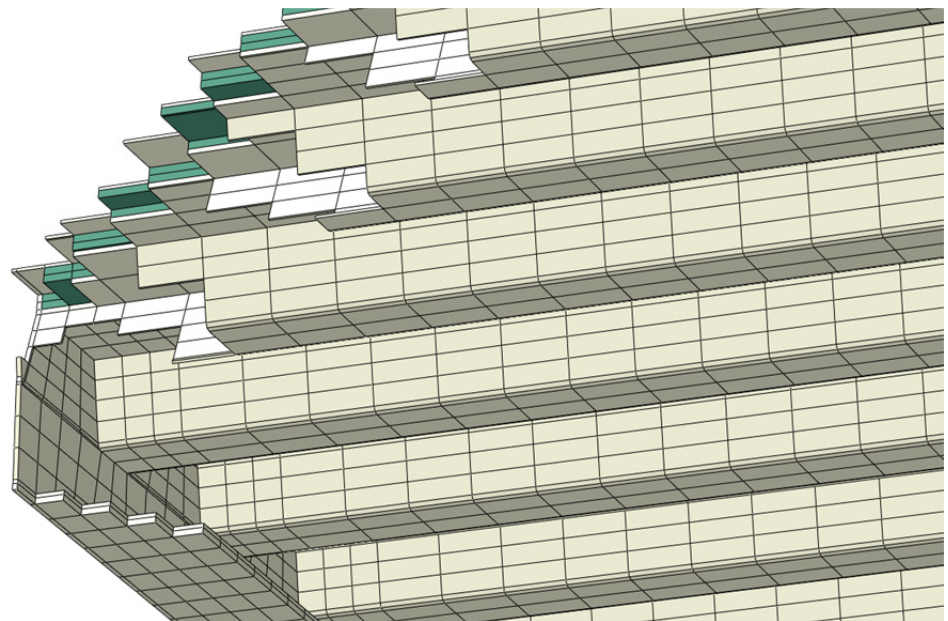


Figure 4 Detail of the medium mesh. The dark layer represents the core layer.

The element assignment provided by the proposed algorithm is shown in Figure 5. QDPSD refers to the QuaDratrate Point Space Dimension (0 cube, 1 shell, 2 beam).

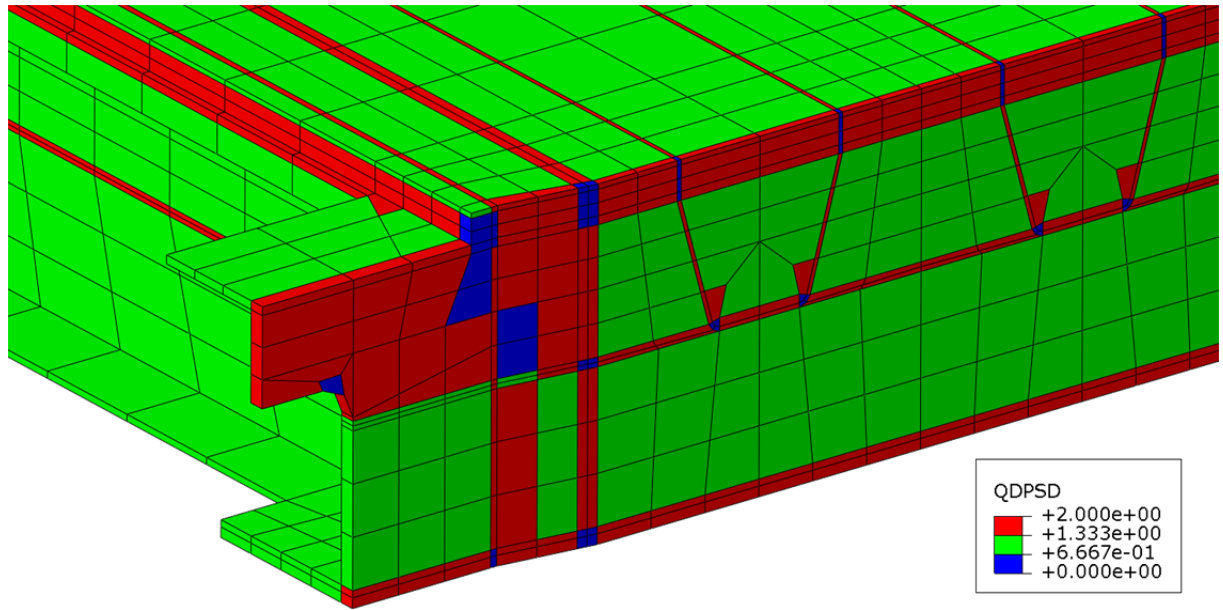


Figure 5 Element type assignment according to automatic element selection algorithm with $\rho = 3$ for the medium mesh.

The element assignment agrees very well with intuition.

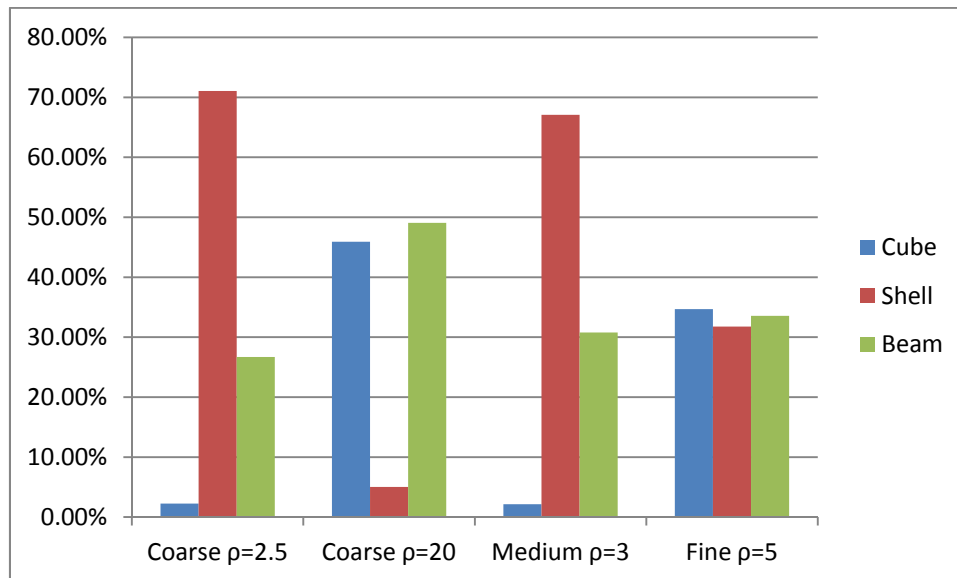


Figure 6: Element type assignments for the different mesh cases and slenderness ratios in percent.

In this study the mesh refinement was done in such a way that the element ratio would not change too much at a given reasonable choice of ρ . This can for instance be observed when the element type distribution of the coarse mesh for $\rho = 2.5$ and the element type distribution of the medium mesh for $\rho = 3$ is compared, see Figure 6. The percentage of cubes increases

as ρ is increased. ρ values between 2.5 and 5 were found to provide the most agreeable results.

4 RESULTS AND VALIDATION

A variety of quantities were measured experimentally. The numerical investigation focuses on the deflection of the centerline of the orthotropic plate since this is the quantity for which the most experimental data is available.

The results of the coarse mesh for several element types are shown in Figure 7. All used element types underestimate the occurring displacement. The relative error of the Q1STx elements is considerably lower than that of the Abaqus standard elements C3D8 and C3D8R, see Figure 8.

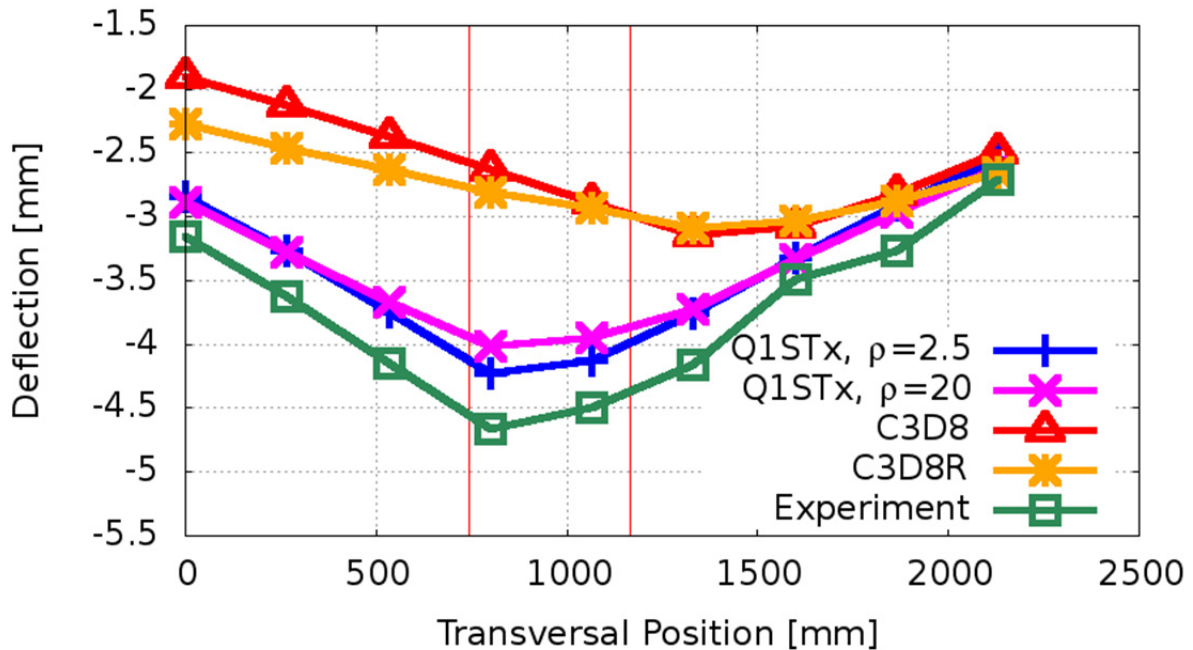


Figure 7: Deflection at the centerline, coarse mesh. $P=100$ kN. The vertical red lines mark the area to which the load was applied.

Two different values for the slenderness ratio were investigated. The accuracy of the element with the lower slenderness ratio was better in this case. The reason for this is that at $\rho = 20$ approximately 45% of the elements are cubes and only 5% are shells, see Figure 6. Nonetheless, the Q1STx still outperforms the standard elements distinctly, see Figure 8. This is possibly due to the extensive treatment of locking in the Q1STx formulation.

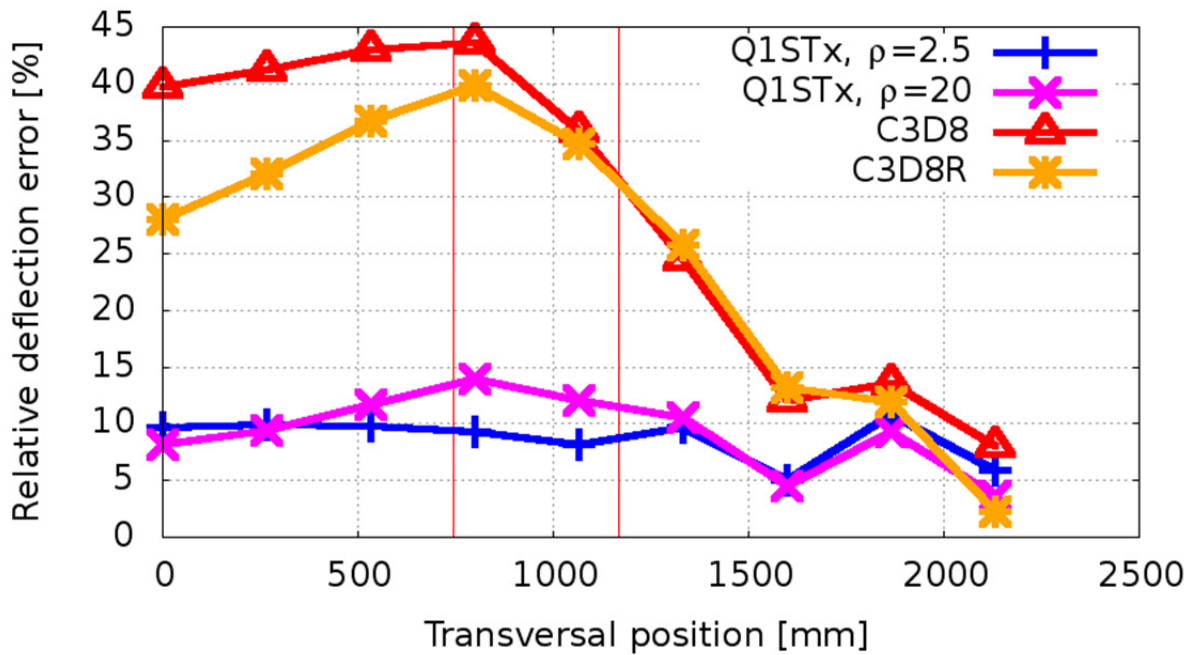


Figure 8: Relative error of the predicted deflection along the centerline compared with the experimental result, coarse mesh. $P=100$ kN. The vertical red lines mark the area to which the load was applied.

The accuracy of the prediction for the medium mesh is very good for the Q1STx and the truss model. In nominal terms both the truss model and the Q1STx perform very well, see Figure 9. The C3D8R, which underestimated the deflections for the coarse mesh, overestimates the deflections for the medium mesh.



Figure 9: Deflection at the centerline, medium mesh. $P=100$ kN. The vertical red lines mark the area to which the load was applied.

The relative error of the C3D8R elements has significantly decreased but is still not quite satisfactory. The Q1STx and the truss model are also close in terms of relative error, see Figure 10.

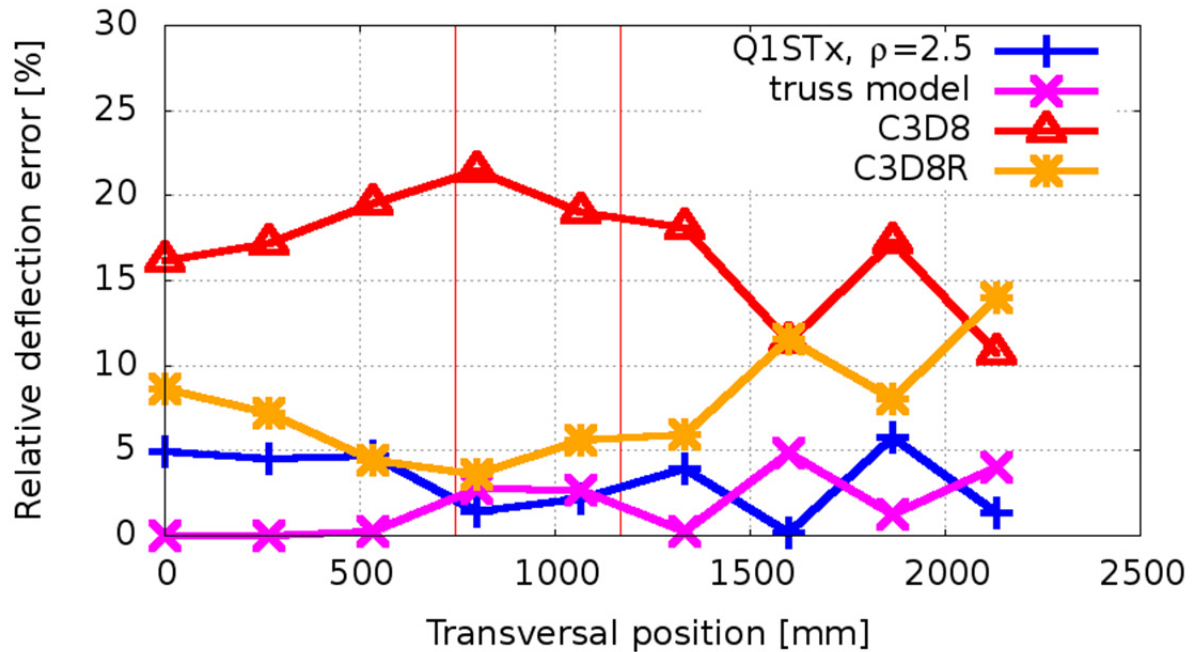


Figure 10: Relative error of the predicted deflection along the centerline compared with the experimental result, medium mesh. $P=100$ kN. The vertical red lines mark the area to which the load was applied.

For the fine mesh the Q1STx performs very well. The C3D8R also predicts the result with reasonable accuracy except for the region where the load is applied and the deformations are the highest, see Figure 11.

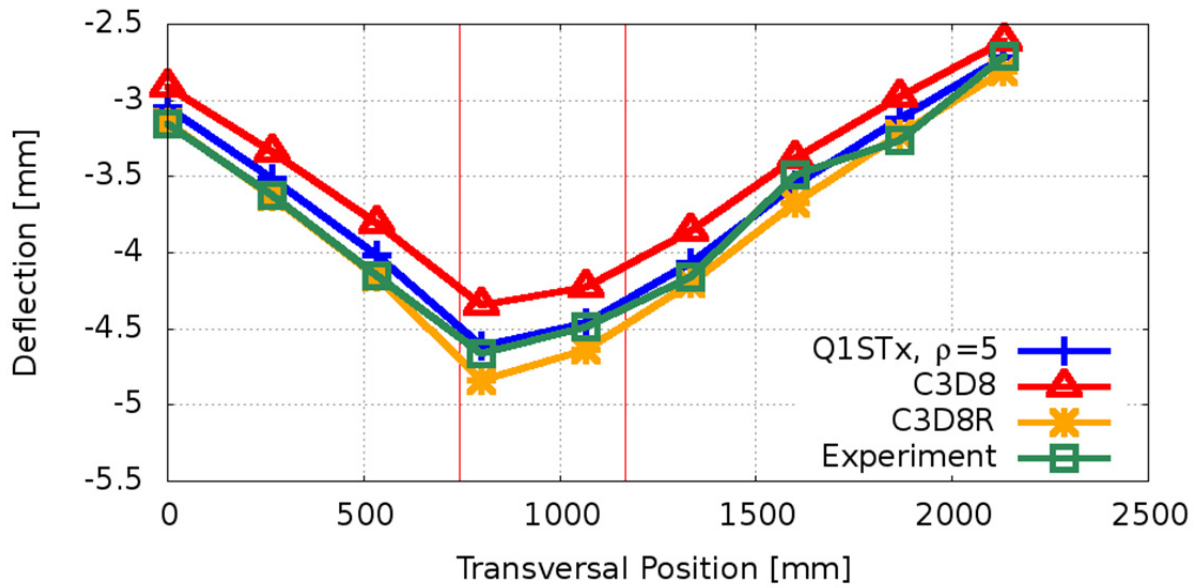


Figure 11: Deflection at the centerline, medium mesh. $P=100$ kN. The vertical red lines mark the area to which the load was applied.

The C3D8 accuracy is not as good, see Figure 12. The mesh may not be converged for that element type yet or locking may prevent convergence to the correct result altogether.

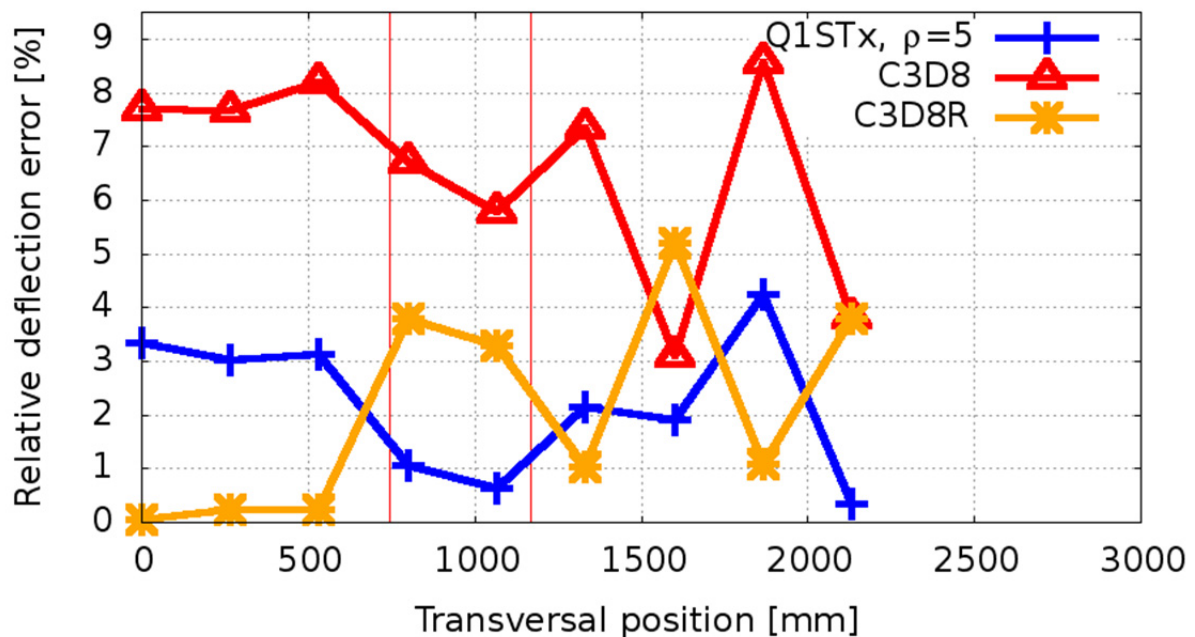


Figure 12: Relative error of the predicted deflection along the centerline compared with the experimental result, fine mesh. $P=100$ kN. (The vertical red lines mark the area to which the load was applied.)

5 CONCLUSION

With the proposed element formulation the centerline deflections of the orthotropic plate were predicted within 5% of the experimental result for a mesh consisting of 22476 elements.

Q1STx elements showed superior convergence behaviour compared with C3D8 and C3D8R elements. The Q1STx model was converged for 22476 elements. The C3D8R model was converged except for high deformation regions at 365988 elements. No satisfactory prediction could be made with C3D8 elements for any of the investigated meshes.

The accuracy of the previous numerical predictions made with a truss and shell cantilever-like model could be matched with a similar number of elements. The advantage of the new more general approach is that it needs only minimal adjustment for other more general sandwich problems.

Element type selection was performed automatically using an algorithm that determines the element type based on the aspect ratio of the element. The element assignments produced by this approach were consistent with intuition and led to the aforementioned good convergence and result accuracy. The user effort required was minimal since only the slenderness ratio and the number of integration points per space direction had to be specified. Slenderness ratios between 2.5 and 5 provided the most agreeable results in this example.

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