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TRANSIENT THERMOELASTIC ANALYSIS OF A FUNCTIONALLY GRADED HOLLOW SPHERE WITH PIECEWISE POWER LAW

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1 INTRODUCTION

Functionally graded materials (FGMs) are nonhomogeneous material systems that two or more different material ingredients change continuously and gradually, and are used as constituents of the beam, strip, plate and shell types. FGMs subjected to several thermal loading consist of metals and ceramics as their constituents, and are remarkable heat-resistant materials for relaxation of thermal stress. Thermal stresses in various materials may reach their peaks in a transient state rather than in a steady state. Therefore, we focus on the transient problem of a FGM. Since it is difficult to obtain the exact solution of such a problem due to the nonlinearity of the governing equations, some approximate approach is required. One of the approaches is to express the material profiles by some specific functions of location. As the transient thermoelastic problems, two-dimensional solutions of the single-layered functionally graded cylindrical panels [1] and hollow cylinders [2] were obtained by this approach. This approach, however, has a drawback in the sense that the specific functions in the sigle layer lack the applicability to arbitrary profiles. Another approach is to apply the theory of laminated composites. The transient thermal stress problems for several models were analyzed by this approach. This approach has the drawback of discontinuity in the material profiles at interfaces.

In order to overcome such drawbacks, both approaches were integrated. Ootao et al. analyzed the transient one-dimensional thermoelastic problems in the FGM hollow cylinder [3] and FGM hollow circular disk [4] by the piecewise-power model. In the model, material properties are expressed by the power functions that are defined in each virtual layer and are continuous at all the interfaces. The model is advantageous because it can describe arbitrary profiles of material properties. Ootao and Ishihara extended the analysis by the model to the asymmetric transient thermoelastic problems in the FGM hollow cylinder [5].

To the authors' knowledge, however, the exact analysis for the transient two-dimensional thermoelastic problem of functionally graded hollow sphere by the model has not been reported.

From the viewpoint of above mentioned, we analyze the transient thermoelastic analysis for a functionally graded hollow sphere with piecewise power law due to axisymmetrical surface heating to guarantee arbitrary nonhomogeneity of material properties.

2 ANALYSIS

The functionally graded hollow sphere consists of many layers whose material properties are expressed by piecewise power law of position. The thermal and thermoelastic constants of each layer are expressed as power functions of the radial coordinate, and their values continue on the interfaces. The hollow sphere's inner and outer radii are defined r_a and r_b, respectively. Moreover, r_i is the outer radius of ith layer. Throughout this article, indices i (=1,2,…, N) are associated with the ith layer from the inner side of a functionally graded hollow sphere.

2.1 Heat conduction problem

We assume that the functionally graded hollow sphere is initially at zero temperature and is heated from the inner and outer surfaces by surrounding media with relative heat transfer coefficients (heat transfer coefficient/thermal conductivity) h_a and h_b . We denote the temperatures of the surrounding media by the functions $T_a f_a(\theta)$ and $T_b f_b(\theta)$. Then the temperature distribution shows a two-dimensional distribution in $r-\theta$ plane, and the transient heat conduction equation for the ith layer is taken in the following form:

$$c_i(r)\rho_i(r)\frac{\partial T_i}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\lambda_{ti}(r)r^2\frac{\partial T_i}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\lambda_{ti}(r)\sin\theta\frac{\partial T_i}{\partial\theta}\right);\, i=1,2,\cdots,N \tag{1}$$

The thermal conductivity λ_{ti} and the heat capacity per unit volume $c_i\rho_i$ in each layer are assumed to take the following forms:

$$\lambda_{ti}(r) = \lambda_{ti}^0 (r/r_{i-1})^{m_i}\,,\ c_i(r)\rho_i(r) = c_i^0\rho_i^0 (r/r_{i-1})^{k_i} \tag{2}$$

where

$$m_i = \frac{\ln(\overline{\lambda}_{t,i+1}^0/\overline{\lambda}_{ti}^0)}{\ln(\overline{r}_i/\overline{r}_{i-1})}\,,\ k_i = \frac{\ln(\overline{c}_{i+1}^0\overline{\rho}_{i+1}^0/\overline{c}_i^0\overline{\rho}_i^0)}{\ln(\overline{r}_i/\overline{r}_{i-1})} \quad ;i=1,\cdots,N \tag{3}$$

In Eq. (3), $\overline{r}_0$ and $\overline{r}_N$ are

$$\overline{r}_0 = \overline{r}_a\,,\ \overline{r}_N = 1 \tag{4}$$

Substituting the Eq. (2) into the Eq. (1), the transient heat conduction equations in dimensionless form are

$$\frac{\partial\overline{T}_i}{\partial\tau} = \overline{\kappa}_i^0\overline{r}_{i-1}^{k_i-m_i}\{(m_i+2)\overline{r}^{m_i-k_i-1}\frac{\partial\overline{T}_i}{\partial\overline{r}} + \overline{r}^{m_i-k_i}\frac{\partial^2\overline{T}_i}{\partial\overline{r}^2} + \overline{r}^{m_i-k_i-2}\frac{\partial}{\partial\mu}[(1-\mu^2)\frac{\partial\overline{T}_i}{\partial\mu}]\} \tag{5}$$

where

$$\mu = \cos\theta \tag{6}$$

The initial and thermal boundary conditions in dimensionless form are

$$\tau = 0; \quad \overline{T}_i = 0 \quad ; \quad i = 1,2,\cdots,N \tag{7}$$

$$\bar{r} = \bar{r}_a ; \; \frac{\partial \overline{T}_1}{\partial \bar{r}} - H_a \overline{T}_1 = -H_a \overline{T}_a f_a(\mu) \tag{8}$$

$$\bar{r} = \bar{r}_i ; \; \overline{T}_i = \overline{T}_{i+1} \quad ; \; i = 1,2,\cdots,N-1 \tag{9}$$

$$\bar{r} = \bar{r}_i ; \bar{\lambda}_{ri} \frac{\partial \overline{T}_i}{\partial \bar{r}} = \bar{\lambda}_{r,i+1} \frac{\partial \overline{T}_{i+1}}{\partial \bar{r}} \; ; \; i = 1,2,\cdots,N-1 \tag{10}$$

$$\bar{r} = 1; \; \frac{\partial \overline{T}_N}{\partial \bar{r}} + H_b \overline{T}_N = H_b \overline{T}_b f_b(\mu) \tag{11}$$

In Eqs. (4), (5), (7)-(11), we introduced the following dimensionless values:

$$(\overline{T}_i, \overline{T}_a, \overline{T}_b) = (T_i, T_a, T_b)/T_0 , \; (\bar{r}, \bar{r}_i, \bar{r}_a) = (r, r_i, r_a)/r_b , \; \tau = \lambda_{t0} t/(c_0 \rho_0 {r_b}^2) ,$$
$$\bar{\kappa}_i^0 = \lambda_{ti}^0 /(c_i^0 \rho_i^0) , \; (\bar{\lambda}_{ti}, \bar{\lambda}_{ti}^0) = (\lambda_{ti}, \lambda_{ti}^0)/\lambda_{t0} , \; (H_a, H_b) = (h_a, h_b) r_b \tag{12}$$

where T_i is the temperature change; t is time; and T_0 , λ_{t0} and $c_0\rho_0$ are typical values of temperature, thermal conductivity, and heat capacity per unit volume, respectively. We expand the temperature functions $f_a(\mu)$ and $f_b(\mu)$ into the following series forms

$$\begin{Bmatrix} f_a(\mu) \\ f_b(\mu) \end{Bmatrix} = \sum_{n=0}^{\infty} \begin{Bmatrix} a_n \\ b_n \end{Bmatrix} P_n(\mu) , \; \begin{Bmatrix} a_n \\ b_n \end{Bmatrix} = \frac{2n+1}{2} \int_{-1}^{1} \begin{Bmatrix} f_a(\mu) \\ f_b(\mu) \end{Bmatrix} P_n(\mu)\, d\mu \tag{13}$$

where $P_n(\mu)$ is the Legendre function of order n . Introducing the Laplace transformation with respect to the variable τ and the method of separation of variables, the solution of Eq. (5) can be obtained so as to satisfy conditions (7)-(11). This solution is shown as follows:

$$\overline{T}_i(\bar{r}, \mu, \tau) = \frac{1}{F_0}[\overline{A}'_{i0} + \overline{B}'_{i0} \bar{r}^{-(m_i+1)}] + \sum_{n=1}^{\infty} \frac{1}{F_n} (\overline{A}'_{in} \bar{r}^{\xi_{i1}} + \overline{B}'_{in} \bar{r}^{\xi_{i2}}) P_n(\mu)$$
$$+ \sum_{n=0}^{\infty} \sum_{j=1}^{\infty} \frac{2\bar{r}^{-(m_i+1)/2}}{\mu_{1j} \Delta'_n(\mu_{1j})} \; \exp[-\frac{\mu_{1j}^2}{4} \cdot \frac{\bar{\kappa}_1^0}{\bar{r}_a^{m_1-k_1}} (2 - m_1 + k_1)^2 \tau]$$
$$\times [\overline{A}_{iq} J_{\gamma_i}(\Omega_i \mu_{1j} \bar{r}^{1-\frac{m_i-k_i}{2}}) + \overline{B}_{iq} Y_{\gamma_i}(\Omega_i \mu_{1j} \bar{r}^{1-\frac{m_i-k_i}{2}})] \; P_n(\mu) \tag{14}$$

In Eq. (14), ς_{i1}, ς_{i2}, γ_i, Ω_i and $\Delta'_n(\mu_{1j})$ are

$$\varsigma_{i1},\varsigma_{i2} = \frac{-(m_i+1)\pm\sqrt{(m_i+1)^2+4n(n+1)}}{2}\,,\ \gamma_i = \frac{\sqrt{(m_i+1)^2+4n(n+1)}}{|2-m_i+k_i|}\,,$$

$$\Omega_i = \sqrt{\frac{\bar{\kappa}_1^0}{\bar{\kappa}_i^0}\cdot\frac{\bar{r}_{i-1}^{m_i-k_i}}{\bar{r}_a^{m_1-k_1}}\left(\frac{2-m_1+k_1}{2-m_i+k_i}\right)^2}\,,\ \Delta'_n(\mu_{1j}) = \left.\frac{d\Delta_n}{d\mu_1}\right|_{\mu_1=\mu_{1j}} \tag{15}$$

and μ_{1j} the *j*th positive root of the following transcendental equation:

$$\Delta_n(\mu_1) = 0 \tag{16}$$

2.2 Thermoelastic problem

We now analyze the transient thermal stress of a functionally graded hollow sphere. The displacement-strain relations are expressed in dimensionless form as follows:

$$\bar{\varepsilon}_{rri} = \bar{u}_{ri},_{\bar{r}}\,,\ \bar{\varepsilon}_{\theta\theta i} = \bar{r}^{-1}(\bar{u}_{\theta i},_{\theta}+\bar{u}_{ri})\,,\ \bar{\varepsilon}_{\phi\phi i} = \bar{r}^{-1}(\bar{u}_{ri}+\bar{u}_{\theta i}\cot\theta)\,,$$

$$\bar{\varepsilon}_{r\theta i} = [\bar{r}^{-1}(\bar{u}_{ri},_{\theta}-\bar{u}_{\theta i})+\bar{u}_{\theta i},_{\bar{r}}\,]/2\,,\ \bar{\varepsilon}_{r\phi i} = \bar{\varepsilon}_{\theta\phi i} = 0 \quad ; i=1,\cdots,N \tag{17}$$

where a comma denotes partial differentiation with respect to the variable that follows. The stress-strain relation in dimensionless form is given by the following relation:

$$\begin{Bmatrix}\bar{\sigma}_{rri}\\ \bar{\sigma}_{\theta\theta i}\\ \bar{\sigma}_{\phi\phi i}\\ \bar{\sigma}_{r\theta i}\end{Bmatrix} = \frac{\bar{E}_i}{(1+\nu_i)(1-2\nu_i)}\begin{bmatrix}1-\nu_i & \nu_i & \nu_i & 0\\ \nu_i & 1-\nu_i & \nu_i & 0\\ \nu_i & \nu_i & 1-\nu_i & 0\\ 0 & 0 & 0 & 1-2\nu_i\end{bmatrix}\begin{Bmatrix}\bar{\varepsilon}_{rri}\\ \bar{\varepsilon}_{\theta\theta i}\\ \bar{\varepsilon}_{\phi\phi i}\\ \bar{\varepsilon}_{r\theta i}\end{Bmatrix} - \frac{\bar{\alpha}_i\bar{E}_i\bar{T}_i}{1-2\nu_i}\begin{Bmatrix}1\\1\\1\\0\end{Bmatrix} \tag{18}$$

The Young's modulus E_i, the coefficient of linear thermal expansion α_i and Poisson's ratio ν_i are assumed to take the following forms:

$$\bar{E}_i(\bar{r}) = \bar{E}_i^0(\bar{r}/\bar{r}_{i-1})^{l_i}\,,\bar{\alpha}_i(\bar{r}) = \bar{\alpha}_i^0(\bar{r}/\bar{r}_{i-1})^{b_i}\,,\nu_i = const\,(\nu_i \neq \nu_{i+1}) \tag{19}$$

where

$$l_i = \frac{\ln(\bar{E}_{i+1}^0/\bar{E}_i^0)}{\ln(\bar{r}_i/\bar{r}_{i-1})}\,,\ b_i = \frac{\ln(\bar{\alpha}_{i+1}^0/\bar{\alpha}_i^0)}{\ln(\bar{r}_i/\bar{r}_{i-1})} \quad ; i=1,\cdots,N \tag{20}$$

Substituting Eqs. (17)-(19) into the equilibrium equations, the displacement equations of equilibrium are written as

$$\bar{u}_{ri},_{\bar{r}\bar{r}}+(l_i+2)\bar{r}^{-1}\bar{u}_{ri},_{\bar{r}}+2(\frac{\nu_i l_i}{1-\nu_i}-1)\bar{u}_{ri}\bar{r}^{-2}+\frac{1-2\nu_i}{2(1-\nu_i)}\bar{r}^{-2}[(1-\mu^2)\bar{u}_{ri},_{\mu\mu}-2\mu\bar{u}_{ri},_{\mu}]$$

$$+\frac{1}{2(1-\nu_i)}\bar{r}^{-1}\frac{\partial}{\partial\bar{r}}(-\bar{u}_{\theta i},_{\mu}\sin\theta+\bar{u}_{\theta i}\cot\theta)$$

$$+[\frac{\nu_i l_i}{1-\nu_i}-\frac{3-4\nu_i}{2(1-\nu_i)}]\bar{r}^{-2}(-\bar{u}_{\theta i},_{\mu}\sin\theta+\bar{u}_{\theta i}\cot\theta)=\frac{(1+\nu_i)\bar{\alpha}_i^0}{(1-\nu_i)\bar{r}_{i-1}^{b_i}}[(l_i+b_i)\bar{r}^{b_i-1}\bar{T}_i+\bar{r}^{b_i}\bar{T}_i,_{\bar{r}} \tag{21}$$

$$\bar{u}_{\theta i},_{\bar{r}\bar{r}}+(l_i+2)\bar{r}^{-1}\bar{u}_{\theta i},_{\bar{r}}+\frac{1-\nu_i}{1-2\nu_i}2\bar{r}^{-2}[(1-\mu^2)\bar{u}_{\theta i},_{\mu\mu}-2\mu\bar{u}_{\theta i},_{\mu}]$$

$$-[l_i+\frac{1}{1-\mu^2}\cdot\frac{2(1-\nu_i)}{1-2\nu_i}]\bar{r}^{-2}\bar{u}_{\theta i}-\frac{(1-\mu^2)^{1/2}}{1-2\nu_i}\bar{r}^{-1}\bar{u}_{ri},_{\bar{r}\mu}$$

$$-(1-\mu^2)^{1/2}[l_i+\frac{4(1-\nu_i)}{1-2\nu_i}\bar{r}^{-2}\bar{u}_{ri},_{\mu}=-\frac{2(1+\nu_i)\bar{\alpha}_i^0}{(1-2\nu_i)\bar{r}_{i-1}^{b_i}}\bar{r}^{b_i-1}(1-\mu^2)^{1/2}\bar{T}_i,_{\mu} \tag{22}$$

In Eqs. (17)-(22), the following dimensionless values are introduced:

$$\bar{\sigma}_{kli}=\frac{\sigma_{kli}}{\alpha_0 E_0 T_0},\ \bar{\varepsilon}_{kli}=\frac{\varepsilon_{kli}}{\alpha_0 T_0},\ (\bar{\alpha}_i,\bar{\alpha}_i^0)=\frac{(\alpha_i,\alpha_i^0)}{\alpha_0},$$

$$(\bar{E}_i,\bar{E}_i^0)=\frac{(E_i,E_i^0)}{E_0},\ (\bar{u}_{ri},\bar{u}_{\theta i})=\frac{(u_{ri},u_{\theta i})}{\alpha_0 T_0 r_b} \tag{23}$$

where σ_{kli} are the stress components, ε_{kli} are the strain tensor, $(u_{ri},u_{\theta i})$ are the displacement components and α_0 and E_0 are the typical values of the coefficient of linear thermal expansion and Young's modulus, respectively. If the inner and outer surfaces are traction free, and the interfaces of the each layer are perfectly bonded, then the boundary conditions of inner and outer surfaces and the conditions of continuity on the interfaces can be represented as follows:

$$\bar{r}=\bar{r}_a;\ \bar{\sigma}_{rr1}=0,\ \bar{\sigma}_{r\theta 1}=0,$$

$$\bar{r}=\bar{r}_i;\ \bar{\sigma}_{rri}=\bar{\sigma}_{rr,i+1},\ \bar{\sigma}_{r\theta i}=\bar{\sigma}_{r\theta,i+1},\ \bar{u}_{ri}=\bar{u}_{r,i+1},\ \bar{u}_{\theta i}=\bar{u}_{\theta,i+1}\ ;i=1,2,\cdots,N-1,$$

$$\bar{r}=1;\ \bar{\sigma}_{rrN}=0,\bar{\sigma}_{r\theta N}=0 \tag{24}$$

We assume the solutions of Eqs. (21) and (22) in the following form.

$$\bar{u}_{ri} = \sum_{n=0}^{\infty}[U_{rcni}(\bar{r}) + U_{rpni}(\bar{r})]P_n(\mu)\,,$$

$$\bar{u}_{\theta i} = \sum_{n=1}^{\infty}[U_{\theta cni}(\bar{r}) + U_{\theta pni}(\bar{r})](1-\mu^2)^{1/2}\frac{d}{d\mu}P_n(\mu) \tag{25}$$

In Eq. (25), the first term on the right side gives the homogeneous solution and the second term of right side gives the particular solution. We now consider the homogeneous solution, and show U_{rcqi} and $U_{\theta cqi}$ as follows:

$$(U_{rcqi}, U_{\theta cqi}) = (U^0_{rcqi}, U^0_{\theta cqi})\bar{r}^{\lambda_i} \tag{26}$$

Substituting Eq. (26) and the first term on the right side of Eq. (25) into the homogeneous equations Eqs. (21) and (22), the condition that nontrivial solutions of $(U^0_{rcni}, U^0_{\theta cni})$ for $n \geq 2$ exist leads to the following equation:

$$\lambda_i^4 + 2(l_i+1)\lambda_i^3 + [l_i(\frac{1+\nu_i}{1-\nu_i} + l_i) - 1 - 2n(n+1)]\lambda_i^2$$

$$+ (l_i+1)[\frac{3\nu_i - 1}{1-\nu_i}l_i - 2 - 2n(n+1)]\lambda_i$$

$$+ 2l_i + \frac{l_i^2\nu_i}{1-\nu_i}[n(n+1)-2] - n(n+1)(l_i+2) + n^2(n+1)^2 = 0 \tag{27}$$

From Eq. (27), there might be four real roots, two real roots and one pair of conjugate complex roots, or two pairs of conjugate complex roots.

Case 1: real roots for λ_i

Given J_{iR} real roots for λ_i, $U_{rcni}(\bar{r})$ and $U_{\theta cni}(\bar{r})$ are given by the following expressions:

$$U_{rcni}(\bar{r}) = \sum_{J=1}^{J_{iR}} F^{(i)}_{nJ}\bar{r}^{\lambda_{iJ}}\,,\; U_{\theta cni}(\bar{r}) = \sum_{J=1}^{J_{iR}} M^{(i)}_{nJ}(\lambda_{iJ})F^{(i)}_{nJ}\bar{r}^{\lambda_{iJ}} \tag{28}$$

where

$$M^{(i)}_{nJ}(\lambda_{iJ}) = -\frac{\lambda_{iJ}^2 + (l_i+1)\lambda_{iJ} + \dfrac{2\nu_i l_i}{1-\nu_i} - n(n+1)\dfrac{1-2\nu_i}{2(1-\nu_i)} - 2}{\dfrac{n(n+1)}{2(1-\nu_i)}(\lambda_{iJ} + 2\nu_i l_i - 3 + 4\nu_i)} \tag{29}$$

In Eq. (28), $F^{(i)}_{nJ}$ are unknown constants.

Case 2: complex roots for λ_i

If the complex root for λ_i is expressed by $\lambda_{iJ} = \alpha_{iJ} \pm j\beta_{iJ}$, and given J_{iI} pairs of complex roots for λ_i, $U_{rcni}(\bar{r})$ and $U_{\theta cni}(\bar{r})$ are given by the following expressions:

$$U_{rcni}(\bar{r}) = \sum_{J=1}^{J_{iI}} \left[C_{1J}^{(i)} \bar{r}^{\alpha_{iJ}} \cos(\beta_{iJ} \ln \bar{r}) + C_{2J}^{(i)} \bar{r}^{\alpha_{iJ}} \sin(\beta_{iJ} \ln \bar{r}) \right],$$

$$U_{\theta cni}(\bar{r}) = \sum_{J=1}^{J_{iI}} \left\{ C_{1J}^{(i)} \bar{r}^{\alpha_{iJ}} [\Gamma_{iJ} \cos(\beta_{iJ} \ln \bar{r}) - \Omega_{iJ} \sin(\beta_{iJ} \ln \bar{r})] \right.$$
$$\left. + C_{2J}^{(i)} \bar{r}^{\alpha_{iJ}} \left[\Omega_{iJ} \cos(\beta_{iJ} \ln \bar{r}) + \Gamma_{iJ} \sin(\beta_{iJ} \ln \bar{r}) \right] \right\} \tag{30}$$

where

$$\Gamma_{iJ} = R_e[M_{nJ}^{(i)}\big|_{\lambda_{iJ} = \alpha_{iJ} + j\beta_{iJ}}],\ \Omega_{iJ} = I_m[M_{nJ}^{(i)}\big|_{\lambda_{iJ} = \alpha_{iJ} + j\beta_{iJ}}] \tag{31}$$

In Eq. (31), j, $R_e[\ ]$ and $I_m[\ ]$ are imaginary unit $j = \sqrt{-1}$, real part and imaginary part, respectively. Furthermore, in Eq. (30) , $C_{1J}^{(i)}$ and $C_{2J}^{(i)}$ are unknown constants. The homogeneous solutions for $n = 0,1$ are omitted here.

In order to obtain the particular solution, we use the series expansions of the Bessel functions in Eq. (14). The detail of the particular solutions is omitted here for the sake of brevity. Then, the stress components can be evaluated by substituting Eq. (25) into Eq. (17), and later into Eq.(18). The unknown constants in the homogeneous solution are determined so as to satisfy the condition (24).

3 NUMERICAL RESULTS

We consider the functionally graded materials composed of titanium alloy (Ti-6Al-4V) and zirconium oxide (ZrO_2). The materials of the inner and outer surfaces are titanium alloy 100% and zirconium oxide 100%, respectively. We assume that the hollow sphere is partially heated from the outer surface by surrounding media. The material properties g_i of the interface between ith layer and (i+1)th layer are assumed as follows:

$$g_i = g_a + (g_b - g_a) f_i,\ 0 \le f_i \le 1\ ; i = 1,2,\cdots, N-1 \tag{32}$$

where g_a is the material property of the inner surface, and g_b is the material property of the outer surface. The numerical parameters of heat conduction, shape and f_i are presented as follows:

$$H_b = 1.0,\ H_a = H_b \bar{\lambda}_{tb} / \bar{\lambda}_{ta},\ \bar{T}_a = 0,\ \bar{T}_b = 1.0,\ \bar{r}_a = 0.7,$$

$$f_b(\mu) = \begin{cases} (\cos^2 \theta_b - \mu^2)/(\cos^2 \theta_b - 1) : 0 \le \mu \le \cos\theta_b \\ 0 : \cos\theta_b \le \mu \end{cases},\ \theta_b = 60^\circ \tag{33}$$

$$N = 2,\ \bar{r}_1 = 0.85,\ f_1 = 0.1, 0.5, 0.9 \tag{34}$$

The numerical results for f_1=0.5 are shown in Figs. 1-4. Fig. 1 shows the variation of temperature change on the heated surface ($\bar{r}=1$). From Fig. 1, the temperature rise can clearly be seen in the heated region ($0^\circ \le \theta \le 60^\circ$) . And the temperature rises as the time proceeds and is greatest in a steady state. Figs. 2 and 3 show the variations of thermal stress $\bar{\sigma}_{\theta\theta}$. The variation on the heated surface ($\bar{r}=1$) is shown in Fig. 2, the variation on the inner surface ($\bar{r}=\bar{r}_a$) is shown in Fig. 3. From Fig. 2, the thermal stress $\bar{\sigma}_{\theta\theta}$ shows compressive stress on the outer surface, and the maximum compressive stress occurs in a transient state. From Fig. 3, the tensile stress occurs on the inner surface. Fig. 4 shows the variation of thermal stress $\bar{\sigma}_{rr}$ in the radial direction on the middle cross section ($\theta=0$). From Fig. 4, the maximum tensile stress occurs in a transient state.

In order to examine the influence of the material property distribution for two-layered FGM model, the distribution of the thermal stress $\bar{\sigma}_{\theta\theta}$ for $f_1=0.1, 0.5, 0.9$ is shown in Fig. 5. From Fig. 5, the large tensile stress occurs inner part of the hollow sphere and large compressive stress occurs on the inner and outer surfaces. As shown in Fig. 5, the maximum tensile stress and the maximum compressive stress decrease when the parameter f_1 decreases.

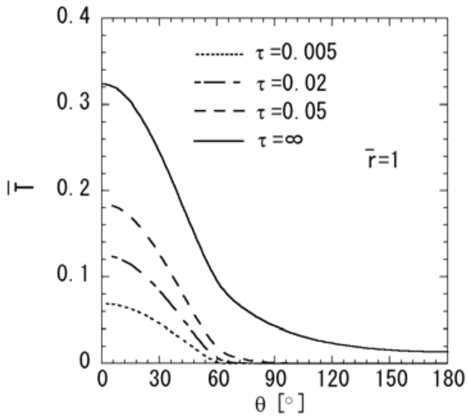

Figure 1:Variation of temperature change on the heated surface ($f_1=0.5$, $\bar{r}=1.0$)

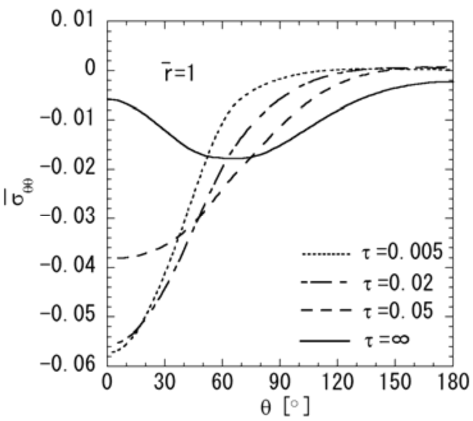

Figure 2:Variation of thermal stress $\bar{\sigma}_{\theta\theta}$ on the heated surface ($f_1=0.5$, $\bar{r}=1.0$)

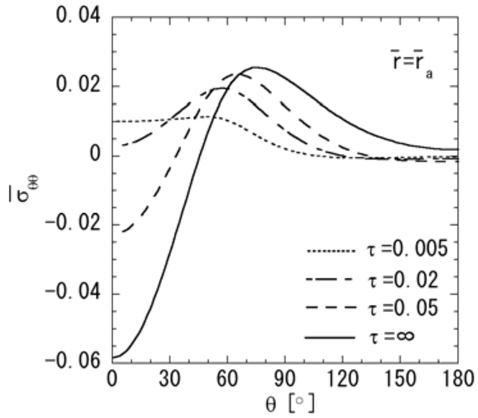

Figure 3:Variation of thermal stress $\overline{\sigma}_{\theta\theta}$ on the inner surface ($f_1 = 0.5$, $\bar{r} = 1.0$)

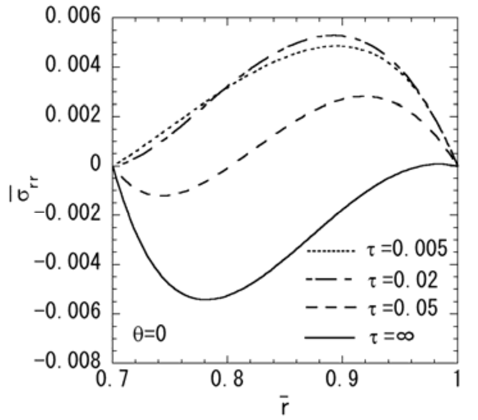

Figure 4:Variation of thermal stress $\overline{\sigma}_{rr}$ in the radial direction ($f_1 = 0.5$, $\theta = 0$)

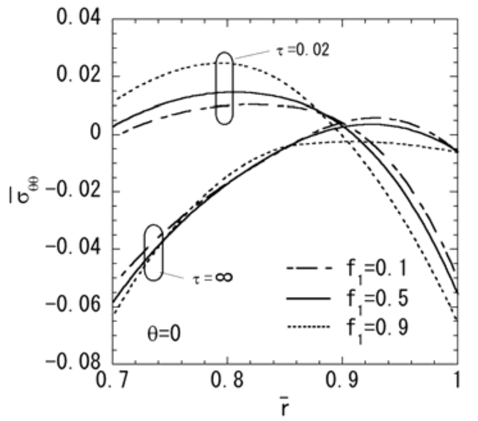

Figure 5:Variation of thermal stress $\overline{\sigma}_{\theta\theta}$ in the radial direction ($f_1 = 0.1, 0.5, 0.9$, $\theta = 0$)

4 CONCLUSIONS

In the present article, we analyzed the transient thermoelastic problem involving a functionally graded hollow sphere with piecewise power law due to axisymmetrical heating from its surfaces. The thermal and thermoelastic constants of each layer are expressed as power functions of the radial coordinate in the radial direction, and their values continue on the interfaces. We obtained the exact solution for the transient two-dimensional temperature and transient thermoelastic response of a functionally hollow sphere with piecewise power law.

As an illustration, we carried out numerical calculations for the functionally graded materials composed of titanium alloy (Ti-6Al-4V) and zirconium oxide (ZrO_2) and examined the behaviors in the transient state for the temperature change, the thermal stress displacements. Furthermore, the influence of the functional grading on the thermal stresses is investigated.

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