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DYNAMICS OF AN ELASTIC WEB IN ROLL-TO-ROLL SYSTEMS USING FINITE ELEMENT METHOD

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Abstract. Roll-to-Roll systems handling web material such as papers, polymers, textiles or metals are very common in the industry. Web handling systems are recently used to produce new technologies such as semi-conductors, thin solar panels, printed electronics, etc. One of the main objectives in web handling machinery is to reach an expected web speed and web tension. Models that are usually used for the tuning of machineries are only one-dimensional and they do not take into account cross-machine components of the elastic web dynamics and therefore several three-dimensional mechanical phenomena initiated by these lateral components of the web dynamics. In order to optimize roll-to-roll systems, it is necessary to develop new models. This paper focuses on finite element modeling of roll-to-roll systems in order to study the elastic web dynamics. The main objective is to simulate web wrinkles by adjusting mechanical parameters, boundary and initial conditions. These parameters and conditions are given by physical laws and experimental data measured on real industrial production lines. The major part of the study deals with roll-to-roll systems having a misaligned roller in order to validate the finite element model and to study the influence of parameters with the software Recurdyn [1].

1 INTRODUCTION

Web processing systems are largely used in the manufacturing of various products in electronics, construction materials, pharmaceuticals, renewable energy, packaging and everyday consumer goods. A web is a flexible material such as foil, paper, metal, textile and plastic film which moves over rollers through different processing stages. Commonly, stages are coating, plating, laminating and printing. Webs are stored and transported as rolls. It is a largely used production system because of its capability of maximum productivity (high speed continuous process) with minimum waste which means low production costs.

One of the main objectives in web handling machineries is to reach an expected web speed while maintaining the web tension within an acceptable range around the web tension reference in the entire processing line. But even if these parameters are properly controlled, it does not mean that it will have no dysfunctions. Indeed models that are usually used for the tuning of machineries in web speed or tension are only one-dimensional [2]. They do not take into account cross-machine components of the elastic web dynamics and therefore the models can not reproduce several three-dimensional mechanical phenomena of the web dynamics in roll-to-roll production lines.

In order to address problems and to optimize roll-to-roll systems, it is necessary to develop new models. Industrial web handling production lines are complex system: large scale, multibody and the contact web/roller is difficult to analyse (Flexible-rigid contact). One way to replicate this complexity is to use a finite element modeling [3] model coupled with multibody dynamics with the help of an analytical theory which has shown good results [4] [5] in wrinkling analysis.

In roll-to-roll systems, troughs and wrinkles of the web are the most common and the most expensive defects. In addition, these are difficult to detect and it is even more difficult to understand their origins. It is usually an accumulation of several origins. It can come from of a misaligned roller, a centering roller, a lateral guide, temperature and moisture, localized friction, non-uniform tension, high tension, etc. The first wrinkle models were developed by Gehlbach and Good [4] [6] [7] which are prediction models for isotropic webs (using beam and thin plate buckling [8]). After this, Good and Beisel [9] create a model of buckling of anisotropic web. By using the classical buckling theory [6], Hashimoto [5] focuses more on a paper-web wrinkle prediction model based on experimental observations of an-isotropic webs. The theory is validated by experiments on paper-web with different Young modulus. The web velocity is taken into account in the friction law only.

The work of Jacques [10] shows good results on the prediction of wrinkles for metal sheet and also help drastically the understanding of such wrinkles. Apart from this work, published finite element models of troughs and wrinkles do not take into account the contact between the web and rollers.

This paper focuses on finite element modeling of roll-to-roll systems in order to study elastic web dynamics. The main objective is to simulate web wrinkles by adjusting mechanical parameters, boundary and initial conditions. These parameters and conditions are given by physical laws and experimental data measured on real industrial production lines. The major part of the study deals with roll-to-roll systems having a misaligned roller in order to validate the finite element model and to study the influence of parameters.

2 ONE DIMENSIONAL MODEL

Simple web traction can produce troughs if the tension is sufficient. In fact, when the web is deformed longitudinally due to its elastic properties, the web has the tendency to retract laterally on its center (Poisson's ratio) and therefor produces compressive stresses. If these stresses exceed the bending stress, the stresses lead to out of plane web deformations (troughs) which cannot be reproduced by 1D models.

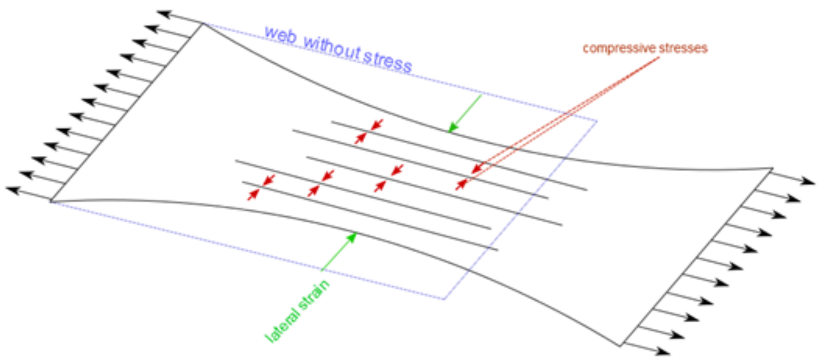

Figure 1: Troughs due to web tension

Table 1: List of Symbols

m	the half wave number in the x direction	E_x	MD Young modulus
n	the half wave number in the y direction	E_z	CD Young modulus
a	web span length between the 2 roller	ϵ_x	MD strain
b	the width of the web	ϵ_z	CD strain
t_f	thickness of the web	γ_{xz}	shear strain
A_{mn}	the maximum amplitude of out-of-plane deformation	υ_x	MD Poisson's ratio
L	web width	υ_z	CD Poisson's ratio
t_f	web thickness	N_x, N_z	Normal forces

The non-linear model of a web transport system is built from the equations describing the velocity of each roller and the web tension behaviour between two consecutive rollers [11, 2].

The speed V_i of a roller i (see Figure 2), which depends on the upstream web tension T_{i-1} and the downstream web tension T_i, is given by :

$$J_i \frac{dV_i}{dt} = (T_i - T_{i-1})\ R_i^2 + K_i\ R_i u_i - f_d\ V_i \tag{1}$$

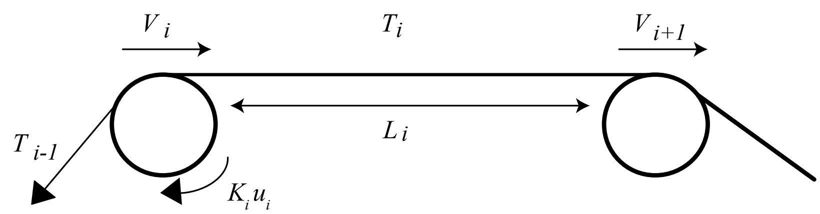

Figure 2: ANALYZED WEB SPAN

The strain ϵ_i of web span i, which depends on the upstream web strain ϵ_{i-1} and the speeds of the two consecutive rollers, is given by the differential equation [2]:

$$\frac{d}{dt}\left(\frac{a_i}{1+\epsilon_i}\right) = -\frac{V_{i+1}}{1+\epsilon_i} + \frac{V_i}{1+\epsilon_{i-1}} \tag{2}$$

The web tension is determined using the non-linear differential equation 2 and the Hooke's law.This equation can be linearized around a working point T_0, V_0. The linear equation becomes [2]:

$$a_i \frac{dt_i}{dt} = V_0(t_{i-1} - t_i) + (v_{i+1} - v_i)(ES + T_0) \tag{3}$$

This model shows good results for a longitudinal dynamics study and is used in order to find optimized robust controllers for roll-to-roll system. However it has some limitations. First, this model assumes that no slippage occurs between the web and the roller. In reality, slippage in industrial plants occurs. Moreover, slippage plays an important role in web wrinkles. Secondly, the lateral dynamics of the web and the out-of-plane behavior can not be simulated.

3 CLASSICAL BUCKLING THEORY

Gehlbach et al. [6] established the basic of the algebraic wrinkles model. The main goal is to find the limits in term of web tension and roller misalignment angle. When these limits are reached, wrinkles can appear.
Considering a rectangular plate element in the deformed configuration. The plate material is assumed to be isotropic, homogeneous and obeys Hooke's law. Plate thickness and applied forces are considered constant.The buckling theory of a such rectangular plate subjected to loads in both X and Y directions have been established by Timoshenko and Gere [8]. With the differential equation for the deflection surface w, resulting from the sum of moments equals zero, moment-displacement relations, the Hooke's law, Good and Beisel [9] have found an equation of the critical buckling stress for an isotropic web with the following dimension: a the web span length between two rollers, b the web width.

$$w = A_{mn} sin(\tfrac{m\pi x}{a}) sin(\tfrac{n\pi y}{a}) \tag{4}$$

respectively, the strain energy in buckling of the web and the work of forces are:

$$\Delta U = \frac{1}{2} \int \int \int_V (\sigma_x \epsilon_x + \sigma_z \epsilon_z + \tau_{xz} \gamma_{xz}) dx dy dz \quad (5)$$

$$\Delta T = \frac{1}{2} \int \int_A \{ N_x (\frac{\partial}{\partial x})^2 + N_z (\frac{\partial}{\partial z})^2 \} dx dz \quad (6)$$

From the thin plate theory, the strain energy in buckling of the web is equal to the work of forces [8]. From equations 4 to 6 (regarding Hashimoto [5]), the critical misalignment angle and the critical web tension that will create a wrinkle on the roller are:

$$\theta_{cr} = \frac{6a^2 \tau_{cr}}{E_x L^2} \quad (7)$$

$$T_{cr} = \frac{2t_f^2}{\mu L} \sqrt{\frac{E_x E_z}{3(1 - \upsilon_x \upsilon_z)}} \quad (8)$$

The stress necessary to overcome the friction force is:
$\sigma_f = \mu \frac{T}{2R} \frac{L}{tf}$

Figure 3 shows the area of wrinkling create dby the critical misalignment angle curve and the critical web tension curve. For a nominal friction of 0.5 wrinkles will appear if the web tension exceeds 600 N/m and if the misalignment angle of roller is higher than 0.02 degrees. If the friction increases, the critical misalignment angle curve is unchanged but the critical web tension needed to create wrinkles decreases. The used parameters shown in table 2

Table 2: Parameters table

Web thickness	0.67 mm
Web span length	2 m
Web width	4 m
Young's Modulus	150 MPa
Poisson's ratio	0.35
Web speed	500 mm/s
wrap angle	60 degree
Friction ratio	0.5

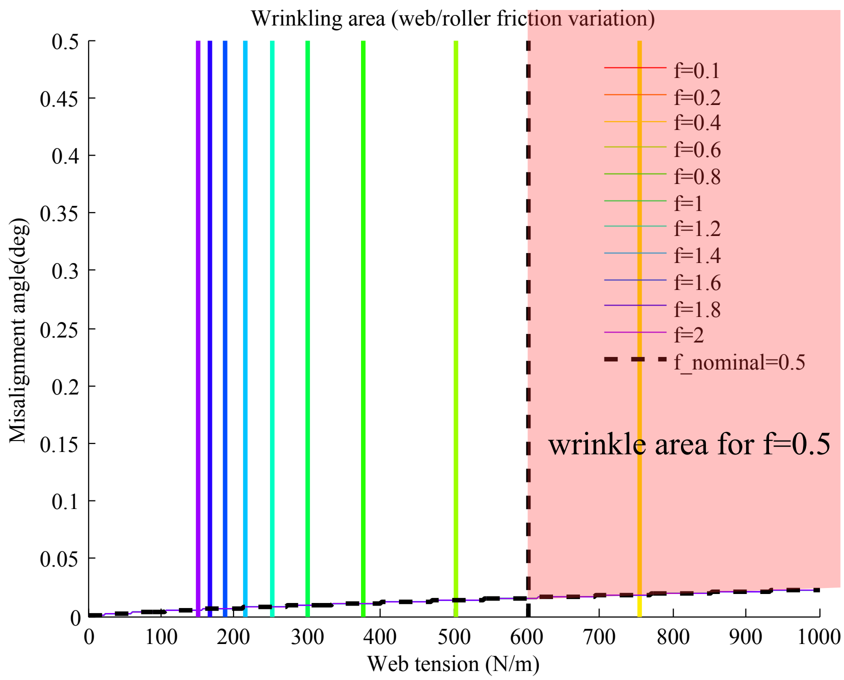

Figure 3: classical buckling theory: web/roller friction variation

4 FINITE ELEMENT MODELING

In this part we used the Recurdyn software, a finite element modeling software including multibody dynamics modeling. We use the wrinkle prediction theory as a reference in order to validate our finite element model. The wrinkle prediction theory has been validated experimentally [5].
We change the web tension and the misalignment angle. In this study, only the third roller can be misaligned (by an angle of 0.5 degree). 6 DOF quad-4 shell elements are used for the web whereas rollers are rigid bodies. The solver used for the integration of the motion equation is an implicit general-alpha method
On figure 4, we can see the wrinkle prediction theory with the parameters used in table 3.
The setting of the simulations 1-4 are given on figure 3. Simulation 5 has similar settings than simulation 4 but the ration span length/width is changed (0.7). The buckling theory indicates that wrinkles should be observed only in the simulations 4 and 5.
We analyse the equivalent stresses (von Mises) in order to have an idea of potential plastified area and of maximum lateral stresses. A compressive lateral stress will appear around the wrinkle
On the simulation 1 (see Figure 5 the maximum equivalent stress is 13.24 MPa and the maximum lateral stress is 1 MPa $< \sigma_f$=10.5 MPa. σ_f represents the compressive lateral

Table 3: Parameters table

Web thickness	0.58 mm
Web span length	0.8 m
Web width	4 m
Young's Modulus	150 MPa
Poisson's ratio	0.35
Web speed	500 mm/s
wrap angle	90 degree
Friction ratio	0.5 and 0.8

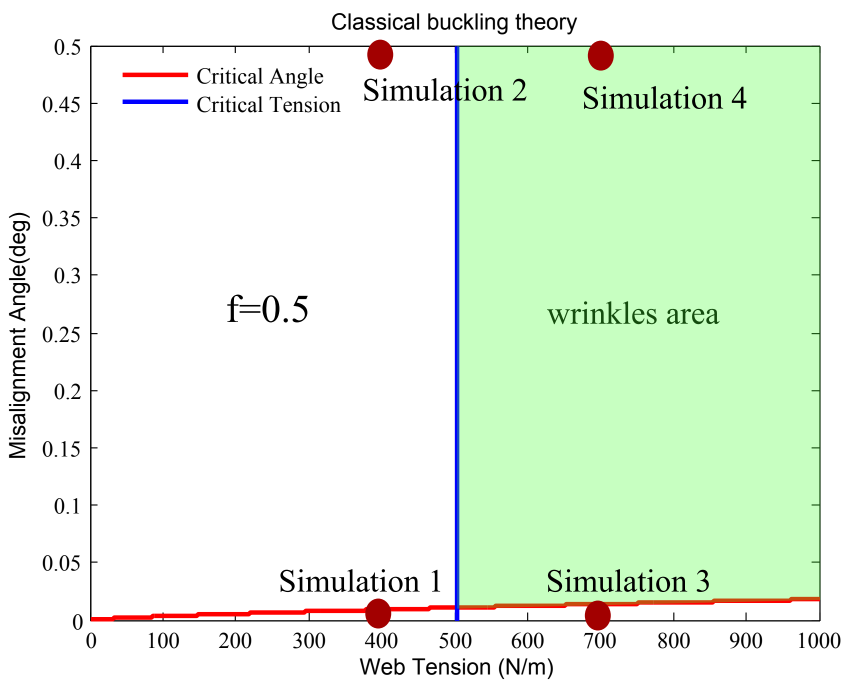

Figure 4: Classical buckling theory

stress necessary to create wrinkles. It is obtained by the buckling theory. We can see on figure 5 that the stresses are uniformly distributed and with a low mean value (5 Mpa).

On the simulation 2 the maximum equivalent stress is 16.3 MPa and the maximum lateral stress is 6 MPa $< \sigma_f$. The figure 6 shows some non-uniformly distributed stresses and with a low mean value (7 Mpa). The effect of a misaligned roller is a loss of uniformity of stresses in the web width.

On the simulation 3 (see Figure 7) the maximum equivalent stress is 22 MPa and the maximum lateral stress is 9 Mpa$< \sigma_f$. The equivalent stresses are uniformly distributed but with a higher mean value (11 Mpa) than Figures 5 and 6.

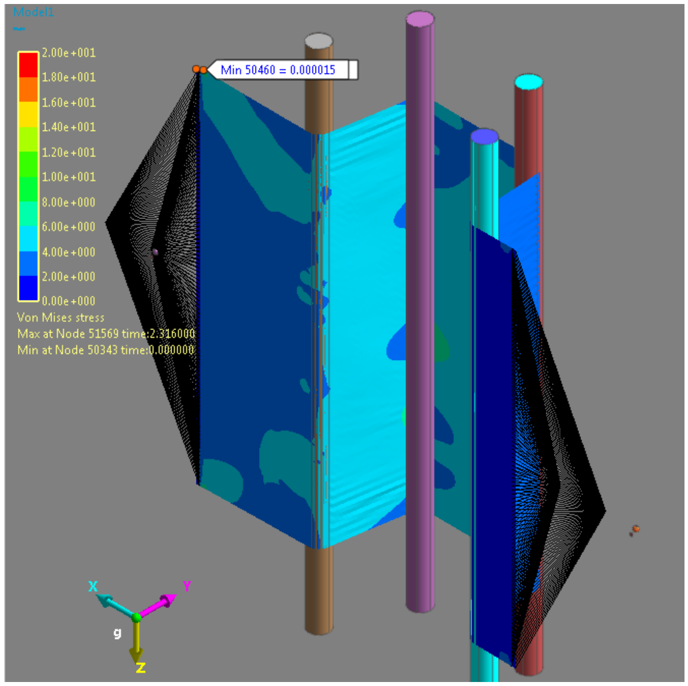

Figure 5: State of equivalent stresses of the web for the simulation 1

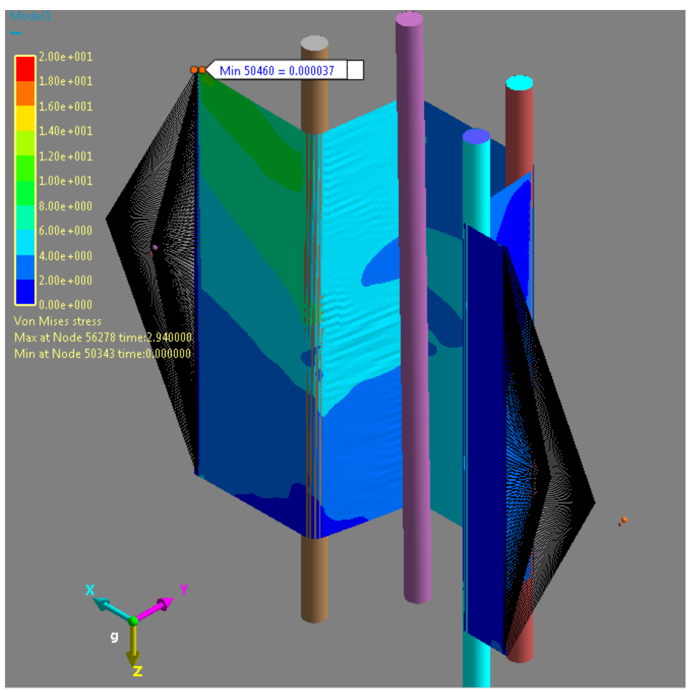

Figure 6: State of equivalent stresses of the web for the simulation 2

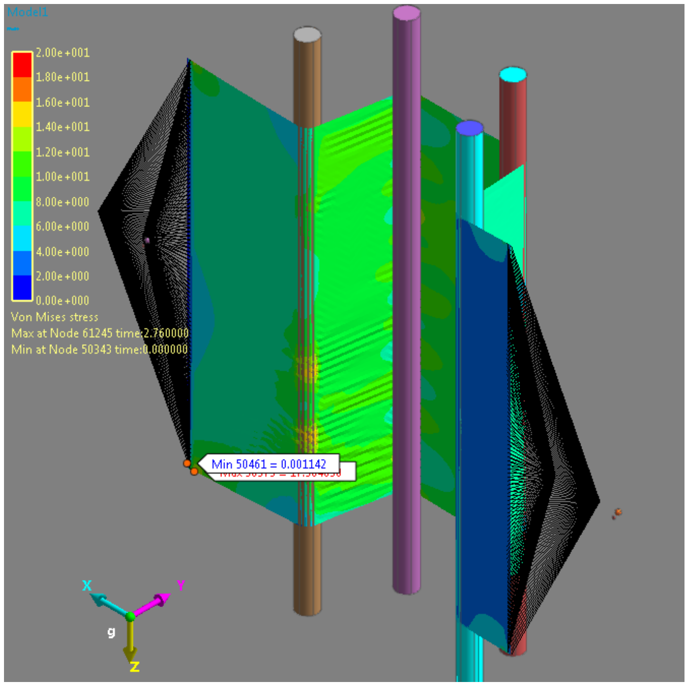

Figure 7: State of equivalent stresses of the web for the simulation 3

On the simulation 4 (see Figure 8) the maximum equivalent stress is 34.5 MPa and the maximum lateral stress is 18 Mpa> σ_f. The stresses are non-uniformaly distributed, with a higher mean value (14 MPa) than Figures 5,6,7 and we can see some high localized stresses. It is a sign of a wrinkle.
One can see on Figure 9 the lateral stresses of simulation 4. The maximum compressive lateral stresses are localized around the wrinkle.

The finite element model coupled with a dynamic solver gives us the expected results. In addition, we can have a lot more of information about the behaviour of the web: state of web stresses, lateral stresses, speed at nodes,etc. We have the state of stresses of the web as shown in the previous figure. We can specificaly study the state of the lateral stresses of the web.
We can also study the influence of the web span length. In fact, industrial knows that this length plays a role in web wrinkles. On Figure 10 the simulation is made with a web span of 0.8 m and a width of 4m. One can see some high localized stresses (34.5 MPa). With a web span of 2.6 m with the same web width, one can see a higher localized stress (42 MPa). By changing only the web span length we create condition even more favourable for wrinkle formation.

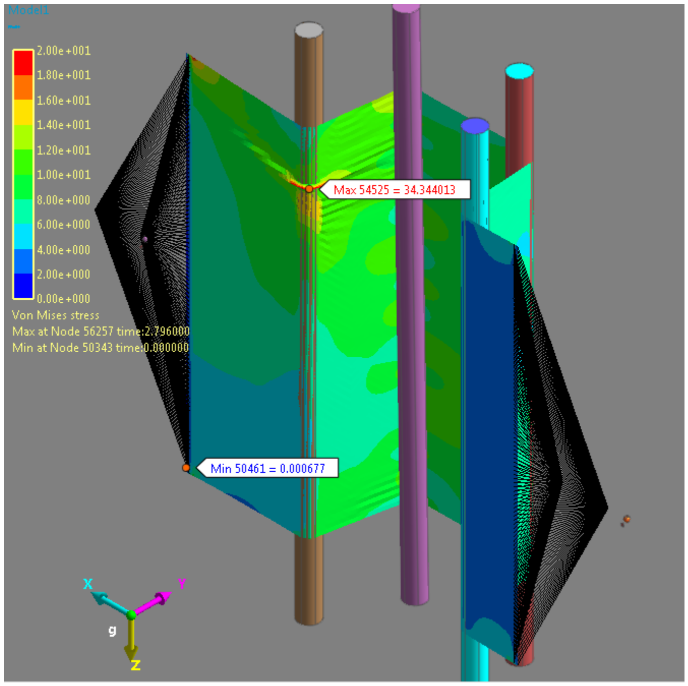

Figure 8: State of equivalent stresses of the web for the simulation 4

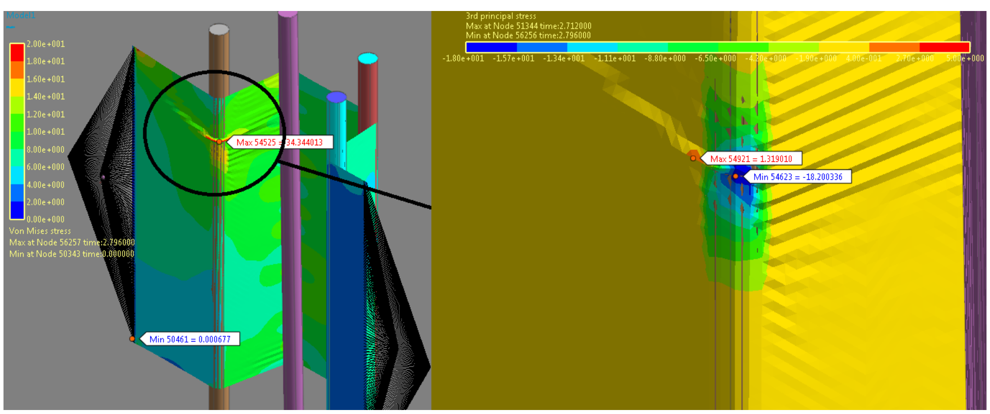

Figure 9: State of lateral stresses of the web for the simulation 4

5 CONCLUSIONS

This paper presents a finite element model coupled with a dynamics solver (software Recurdyn) with the purpose of wrinkle study of elastic webs in roll-to-roll systems. By taking a theory which shows good results as a reference we validated our model. In comparison with the wrinkles theroy, our finite element model gives goods results. In addition, our model permits to obtain more informations: state of web stresses taking into account the web velocity, roller dynamics, slippage, precise web/roller contact. The

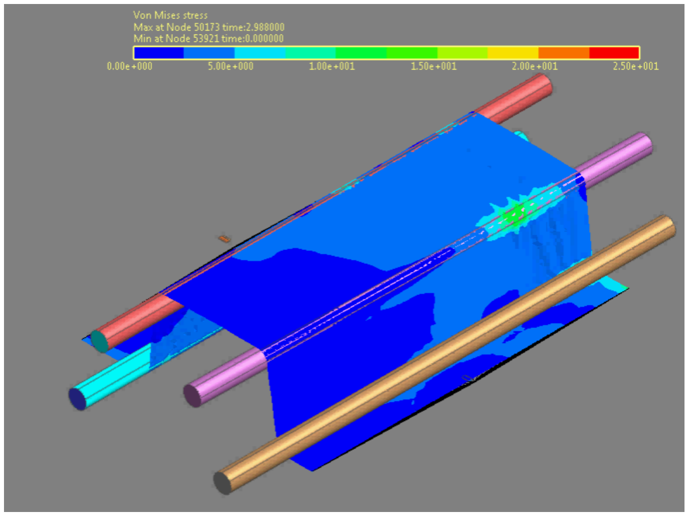

Figure 10: State of equivalent stresses of the web for the Length/Width ratio of 0.2

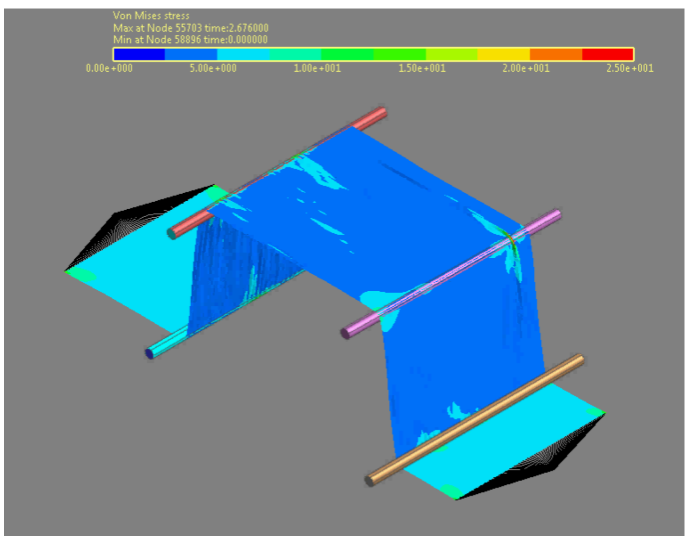

Figure 11: State of equivalent stresses of the web for the Length/Width ratio of 0.7

established model let us to study the effects of different parameters, such as the roller shape for example.

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