11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
E. Oñate, J. Oliver and A. Huerta(Eds)

ANALYSIS OF CRACKS IN BI-MATERIALS/COMPOSITES WITH VARIABLE ORDER SINGULARITY USING MESHLESS METHOD

NELSON M.MUTHU[*], SURJYA K.MAITI[†] AND WENYI YAN[#]

[*] IITB-Monash Research Academy
Indian Institute of Technology, Bombay
Mumbai, Powai, 400076, India
e-mail: mnelson1986@gmail.com

[†] Indian Institute of Technology, Bombay
Mumbai, Powai, 400076, India
e-mail: skmaiti@iitb.ac.in

[#] Department of Mechanical and Aerospace Engineering,
Monash University, Clayton,
VIC 3800, Australia
e-mail: wenyi.yan@monash.edu

Key Words: *Meshless Method, Crack propagation, Bi-material interface, Particle-matrix composite*

Abstract. In this paper, a micromechanical model is presented, using the element-free Galerkin (EFG) method, to simulate the propagation of the damage, based on linear elastic fracture mechanics. The first case study involves precise modelling of a static crack with a crack tip normal to a bi-material interface. The stress intensity factor (SIF) obtained from the displacement technique is in good agreement with the published results. The second case involves a study of the propagation of a crack under tensile load. The angle of crack propagation is determined by the maximum tangential principal stress (MTPS) criterion in conjunction with the energy release rates in a non homogenous medium. The crack path which arises out of the movement through the particle-matrix interface for various locations of initial crack, are plotted and compared with published results.

1 INTRODUCTION

Reinforced composites play an important role in the aerospace and automobile industries, where the strength-to-weight ratio is a crucial factor. Significant research has been reported on the study of failure mechanisms of these materials to facilitate proper application in design and usage. A larger number of analytical investigations [1] and limited experimental ones have been reported in the past. More attention is now focused on computational methods to help reduce developmental costs. This is also driven by outgrowth of sophisticated modelling techniques and the decreasing cost of hardware [2,3].

Damage often initiates at micro scale from micro-sized flaws in composites. This may eventually lead to one or more dominant cracks, which can result in catastrophic failures. The extent of propagation of a crack depends on multiple parameters such as the fibre volume fraction, number of neighbouring pre-existing cracks, the shape of the fibre reinforcements, their locations in the matrix, the fracture resistances of the fibre, the matrix and the interface. In general, the fracture in composites consists of a combination of matrix failure, fibre failure and interface decohesion/fracture. It is sometimes observed that a dominant crack restricts to itself in the matrix [4-6]. Failures in such cases are qualified by brittle fracture. In this paper, the possibilities of matrix and interface fractures at a micro scale are investigated using the principles of macro fracture mechanics.

In the case of crack propagation through bi-materials and composites, depending on the location of the crack tip with respect to the interface, the order of singularity, λ, varies, that is, $\sigma_{ij} \propto r^{\lambda-1}$. If the crack tip is located in a homogenous material, $\lambda = 0.5$. If it is located at the interface, λ can have a single real value or two real or complex values depending on the material combination and angle made by the crack with the interface [7]. λ can be obtained analytically in the case of bi-materials by solving the transcendental equation [8].

Although the finite element method (FEM) and the boundary element method (BEM) are the established methods in the modelling of crack propagation in composites, they pose challenges as they require re-meshing which is computationally costly at every instance of crack propagation. The eXtended Finite Element Method (XFEM) and Meshless Methods (MMs) have been developed to overcome some of these shortcomings. The use of MMs in such cases remains almost unexploited. The current work deals with the application of the EFG method, which is modified, to model the variable order singularity problem. In addition, crack propagation is examined using the proposed methods. The details, of the formulation as well as some case studies, are presented in the paper.

As the crack propagates in composites, the order of singularity varies depending on the location of the crack. The instantaneous angle of crack propagation is decided by MTPS or zeros shear stress criteria [9]. These criteria do not require computation of the SIFs. MTPS is proven to work well in the homogenous medium. Many criteria were established for crack kinking analysis in the case of bi-materials: the maximum energy release rate criterion, the maximum hoop stress criterion and zero K_{II} criterions. No single criterion offers a full solution for the crack kinking problem from an interface [10]. In this work, when the crack tip is at an interface, the angle of crack propagation is decided by MTPS in conjunction with the energy release rate technique. The performance of the scheme is presented here.

2 ELEMENT-FREE GALERKIN METHOD

In MMs, the domain is discretized with nodes. Each node interacts with other nodes through weight functions which may vary in size and profile. Fig. 1 shows a geometry with a crack and discretised with nodes with regular circular weight functions. These weight functions affect the behaviour of the nodal shape functions.

In the EFG method, any scalar field ($\mathbf{u}(\mathbf{x})$) variable can be represented as

$$\mathbf{u}(\mathbf{x}) = \sum_{I=1}^{n} \Phi_I(\mathbf{x})\mathbf{u}_I \tag{1}$$

where $\Phi_I(\mathbf{x})$ are the nodal shape functions. The shape functions $\Phi_I(\mathbf{x})$ in eq. (1) are approximants and do not satisfy the Kronecker delta property.

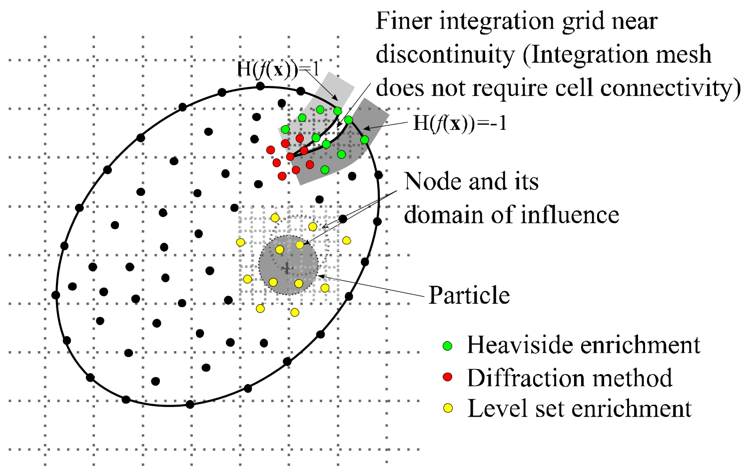


Fig. 1. Nodal discretisation for a domain with a crack and a inclusion.

The moving least squares (MLS) technique is used to formulate the shape functions. The shape functions are developed as follows:

$$\Phi_I(\mathbf{x}) = p^T(\mathbf{x}).[A(\mathbf{x})]^{-1}.B(\mathbf{x}_I) \tag{2}$$

$$A(\mathbf{x}) = \sum_{I=1}^{n} w(\mathbf{x} - \mathbf{x}_I)p(\mathbf{x}_I)p^T(\mathbf{x}_I) \tag{3}$$

$$B(\mathbf{x}) = [w_1(\mathbf{x} - \mathbf{x}_1)p(\mathbf{x}_1),..., w_i(\mathbf{x} - \mathbf{x}_i)p(\mathbf{x}_i),..., w_n(\mathbf{x} - \mathbf{x}_n)p(\mathbf{x}_n)] \tag{4}$$

The linear polynomial basis, $p = [1 \;\; x \;\; y]$, is used here. $A(\mathbf{x})$ is the moment matrix and $w(\mathbf{x} - \mathbf{x}_I)$ is the weight function. The cubic B-spline with a circular domain of influence (d_I) is used as the weight function.

$$w(r) = \begin{cases} \frac{2}{3} - 4r^2 + 4r^3, & r \le \frac{1}{2} \\ \frac{4}{3} - 4r + 4r^2 - \frac{4}{3}r^3, & \frac{1}{2} < r \le 1 \\ 0, & r > 1 \end{cases} \tag{5}$$

where r ($\| \mathbf{x} - \mathbf{x}_I \| / d_I$) is the radial distance of point $\mathbf{x}$ from node I, and is obtained as a function of the characteristic size, d_I, of node I.

2.1 Crack representation (Diffraction Method)

In the diffraction method, the analogy is the diffraction of light around a sharp corner, where the weight function is taken into account. Fig. 2 shows the weight functions of the nodes that influence the crack, with weight function 'diffracting' around the crack tip.

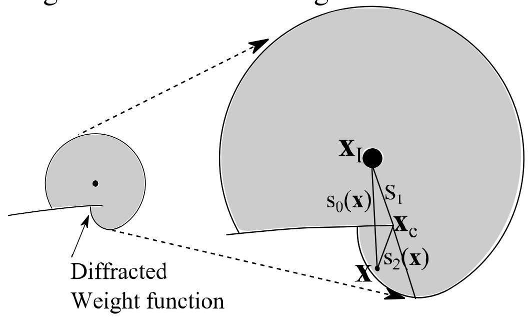


Fig. 2. The diffraction method

The modified distance in the diffracted region is given by

$$r = \left(\frac{s_1 + s_2(\mathbf{x})}{s_0(\mathbf{x})}\right)^{\rho} s_0(\mathbf{x}) \tag{6}$$

where $\mathbf{x}$ is the sampling point, $\mathbf{x}_i$ is the node and $\mathbf{x}_c$ is the crack tip. $s_0(\mathbf{x}) = \|\mathbf{x} - \mathbf{x}_i\|$, $s_1 = \|\mathbf{x}_i - \mathbf{x}_c\|$ and $s_2(\mathbf{x}) = \|\mathbf{x} - \mathbf{x}_c\|$. The parameter ρ is mostly set to either 1 or 2. In this work, it is taken as 2.

2.2 Enriching the diffracted nodes

A new weight function is developed by intrinsically enriching the cubic weight function for diffracted nodes. It is given by

$$w_c(r) = r^{\lambda}\, w(r) \tag{7}$$

where λ is the order of the singularity for the crack tip and r is the radial distance from the crack tip. This weight function, $w_c(r)$, usurps the behaviour of the displacement and stress field variation around the crack tip. Therefore, in addition to the nodal degrees of freedom (DOF), there will be also phantom/additional DOF, associated with the diffracted nodes, corresponding to the new weight function.

Consequently, the displacement approximation in the proposed XEFG method [11], in the presence of a crack (strong discontinuity) and inclusion boundaries (weak discontinuity) takes the form

$$\mathbf{u}(\mathbf{x}) = \sum_{I \in w(\mathbf{x}), w_c(\mathbf{x})} \Phi_I(\mathbf{x})\mathbf{u}_I + \sum_{I \in w_j(\mathbf{x})} \Phi_I(\mathbf{x})\{\mathbf{a}_I \mathrm{H}(f(\mathbf{x}))\} + \sum_{I \in w_b(\mathbf{x})} \Phi_I(\mathbf{x})\mathbf{c}_I \Psi_I(\mathrm{x}) \tag{8}$$

where the function $\Psi_I(\mathrm{x})$ is used for displacement continuity across the material interface of the particle where $\Psi_I(\mathrm{x}) = \mathrm{F}^{\mathrm{I}}(\mathrm{x}) - \mathrm{F}^{\mathrm{I}}(\mathrm{x}_I)$ and $\mathrm{F}^{\mathrm{I}}(\mathrm{x}) = \sum_{I \in w_c(\mathrm{x})} |\zeta_I| \Phi_I(x) - \left| \sum_{I \in w_c(\mathrm{x})} \zeta_I \Phi_I(x) \right|$. The Heaviside enrichment for displacement discontinuity, $\mathrm{H}(f(\mathbf{x}))$, is given by

$$\mathrm{H}(f(\mathbf{x})) = \begin{array}{ll} 1 \text{ if } & f(\mathbf{x}) \geq 0 \\ -1 \text{ if } & f(\mathbf{x}) < 0 \end{array} \tag{9}$$

3 INSTANTANEOUS ANGLE OF CRACK PROPAGATION

3.1 MTPS criteria

The maximum tangential stress (MTS) criterion proposed by Erdogan and Sih [12] was based on a stress field approximated by the first term of the Williams' stress function expansion. In this case, the direction of crack extension, $(\theta_c)_{MTS}$, is given by the condition:

$$\frac{\partial \sigma_{\theta\theta}}{\partial \theta} = 0, \frac{\partial^2 \sigma_{\theta\theta}}{\partial \theta^2} \leq 0$$
$$\theta_c = 2\tan^{-1}\left(\frac{-2K_{II}/K_I}{1+\sqrt{1+8(K_{II}/K_I)^2}} \right) \tag{10}$$

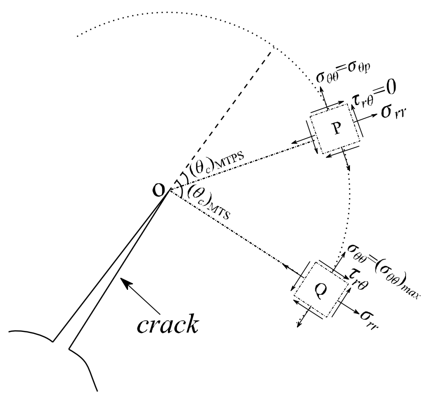


Fig. 3. MTS and MTPS criteria

This direction of crack extension is not a principal direction when more than one term of the eigenfunction expansion is used to calculate crack tip stresses. In this case, the direction given by MTS $\partial\sigma_{\theta\theta}/\partial\theta = 0$, i.e., $(\sigma_{\theta\theta})_{max}$ is not a principal direction.

The MTPS criterion based approach showed that the crack extends in a direction, $(\theta_c)_{MTPS}$, corresponding to $\tau_{r\theta} = 0$, which is close to the direction given by $\partial\sigma_{\theta\theta}/\partial\theta = 0$, but not the same. The corresponding tangential stress is a principal stress. The difference between MTS and MTPS criteria is illustrated in Fig. 3. Other popular criteria, including MTS, are compared with MTPS in [9].

The shear stress ($\tau_{r\theta}$) was computed around a contour of finite radius around the crack tip. The angle of extension was given by the radial direction corresponding to $\tau_{r\theta}=0$. For a crack tip close to the matrix-particle interface, care was taken to ensure that the contour for plotting $\tau_{r\theta}$ does not pass through more than one phase.

3.2 Interaction integral for energy release rates

There are many approaches to find the energy release rate in isotropic and homogenous materials [13,14]. The interaction integral, for the non-homogenous materials in the presence of voids and material interfaces, can be written as

$$I = I_h + I_{nonh}$$
$$I_h = \int_A (\sigma_{ij} u_{i,1}^{aux} + \sigma_{ij}^{aux} u_{i,1} - \sigma_{ik} \varepsilon_{ik}^{aux} \delta_{1j}) q_{,j} dA \tag{11}$$

where I_h is the standard interaction integral for homogenous. I_{nonh} term arises due to heterogeneity.

Many different forms of I_{nonh}, have been proposed in literature [15-17]. Most of the expressions are suitable for the application in which mechanical properties are smooth. An expression for I_{nonh} which is particularly suitable where there is sudden change in material properties such as distinct material interfaces [15], is given by

$$I_{nonh} = \int_A \sigma_{ij} (S_{ijkl}^{tip} - S_{ijkl}(x)) \sigma_{kl,1}^{aux} q dA \tag{12}$$

where σ_{ij}^{aux} and u_i^{aux} are the standard Williams' crack tip solutions for the homogenous medium and $S_{ijkl}(x)$ is a compliance tensor .

3.3 Interaction integral for curved cracks

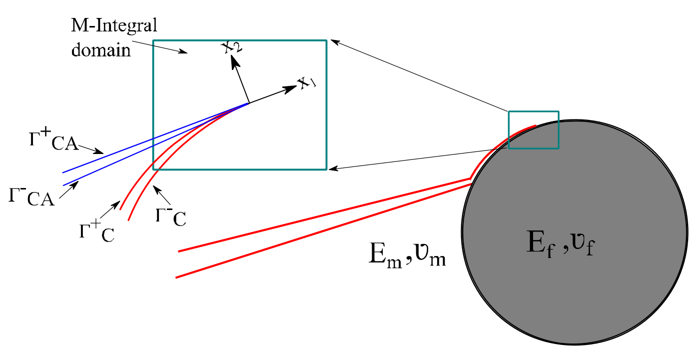


Fig. 4. Crack growth along particle/fibre-matrix interface.

If the crack edges near the tip are curved as shown in Fig. 4, the interaction integral has to be modified to accommodate the effects of the curvature. According to [18], an extra term, $I_{crackface}$, appears in the calculation of the integral [19].

$$I_{crackface} = \int_{\Gamma_C^+ + \Gamma_C^-} (\sigma_{ij}^{aux} \varepsilon_{ij} m_1 - m_i \sigma_{ij}^{aux} u_{j,1}) q d\Gamma - \int_{\Gamma_{CA}^+ + \Gamma_{CA}^-} m_i \sigma_{ij} u_{j,1}^{aux} q d\Gamma \tag{13}$$

where m_i is the direction cosines of the normal at the crack surface. σ_{ij}^{aux} and u_i^{aux} are the standard Williams' crack tip solution for bi-materials. The net integral for a curved crack in a non-homogenous medium is given

$$I = I_h + I_{nonh} + I_{crackface} \tag{14}$$

3.3 Criteria for crack extension

When the crack tip is in the isotropic, homogenous medium the angle of instantaneous crack extension is decided solely by the MTPS criterion. The crack will extend if $K > K_{IC}$, where K is the SIF and K_{IC} is the critical SIF.

When the crack tip is at the interface, the angle of crack propagation is decided based on energy release rates and MTPS criteria as shown in Fig. 5.

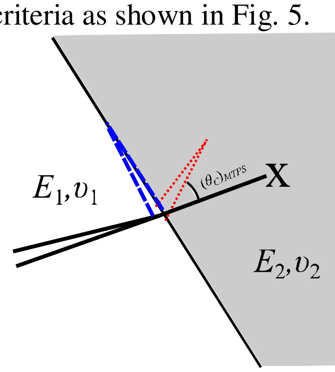


Fig. 5. Possible deflection.

The ERR can be estimated from the interaction integral. The ERR (G_I) along the interface when the crack deflects, and G_0 in the $(\theta_C)_{MTPS}$ direction when the crack penetrates the other material, are given by

$$\begin{aligned} G_I &= \frac{1}{E^*} \frac{K_I^2 + K_2^2}{\cosh^2(\pi\varepsilon)}, \quad \frac{1}{E^*} = \frac{1}{2}\left(\frac{1}{E_m} + \frac{1}{E_f}\right) \\ G_O &= \frac{1}{E^*} \frac{K_I^2 + K_2^2}{2} \end{aligned} \tag{15}$$

where K_1 and K_2 are mode I and mode II SIFs. $E_m(E_1)$ and $E_f(E_2)$ are Young's moduli of the materials. ε is the bi-material constant. The criteria that decides penetration versus deflection depends on the relative standing of the two ratios $\frac{G_I}{\Gamma_f^I}$ and $\frac{G_o}{\Gamma_f^o}$. If

$$\frac{G_I}{\Gamma_f^I} > \frac{G_o}{\Gamma_f^o} \tag{16}$$

where Γ_f^I and Γ_f^o are fracture toughness of the interface and the neighbouring material, respectively. The crack is likely to deflect along the interface if inequality in eq. (16) holds true. Otherwise, it is likely to penetrate into the other material. It is found that, usually, the crack has a tendency to penetrate into the more compliant material.

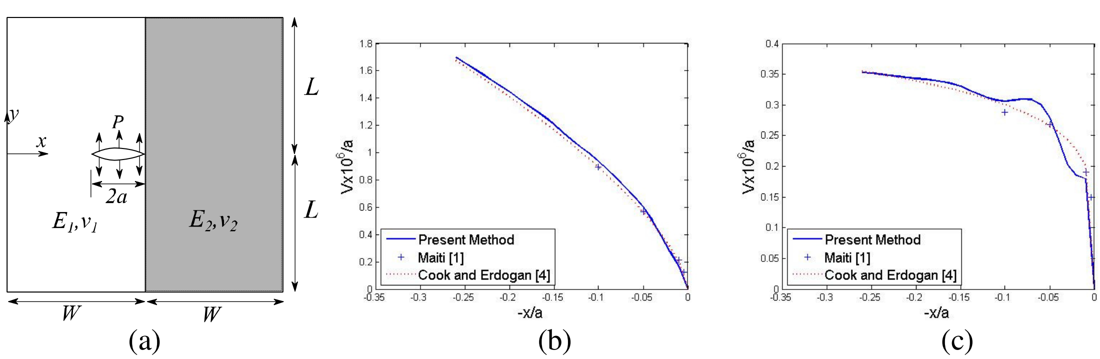
4 CASE STUDIES

4.1 A pressurized crack with a crack tip terminating normal to interface

A pressurised crack, with one of its tip embedded in a homogenous material (Fig. 6a) and the other tip normal to an interface, is studied using the proposed scheme. The stress intensity factor (SIF) is extracted from the crack opening displacement (COD) at a distance of 12% of the crack length (a) from the tip. The SIFs are compared in Table 1 and crack edge profiles are compared in Fig.6b and c for two ratios of shear moduli ($\mu_2 / \mu_1 = 23.07$ and $\mu_2 / \mu_1 = 0.043$).

Dimensions and Loading: a/W= 1/9 *L/W*=1 *W*=0.2286 m *P*=6.895 kPa

Materials: Aluminium-E=68.95GPa ν=0.3, Epoxy-E=3.102GPa ν=0.35



(a) (b) (c)

Fig. 6. (a) Crack normal to the bi-material interface. Crack edge profiles for (b) $\mu_2 / \mu_1 = 23.07$. (c) $\mu_2 / \mu_1 = 0.043$.

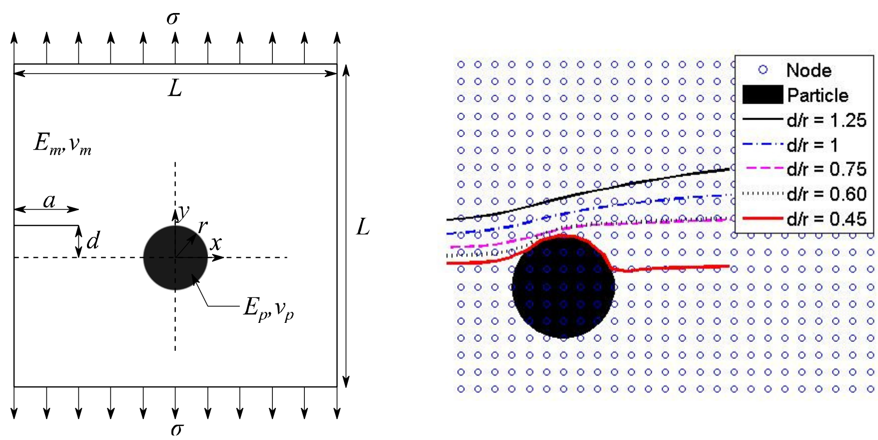
Table 1. Comparison of normalised SIFs $K_I/p(\pi a)^{1-\lambda}$

Shear modulus ratio	$\mu_2/\mu_1 = 23.07$		$\mu_2/\mu_1 = 0.043$	
Singularity constant	$\lambda = 0.5$	$\lambda = 0.6619$	$\lambda = 0.5$	$\lambda = 0.1752$
Present EFG method	0.856 (2.73%)	4.283 (3.23%)	1.487 (0.75%)	0.048 (-12.5%)
Maiti [20]	0.880	4.149	1.476	0.042
Cook and Erdogan [21]	0.882	-	1.355	0.048

4.2 Crack propagation at micro scale

Dimensions and Loading: L=2 mm $2r/L$=0.15 σ=1MPa

Materials: Silicon Carbide-E=6.43×68.95GPa ν*=0.17, Aluminium-E=68.95GPa* ν*=0.3*



(a) (b)

Fig. 7. (a) Crack geometry with a single particle embedded in it. (b) Crack path as per MTPS criteria for various *d/r* ratios.

The second application involves a study of the crack path through a particulate composite (Fig. 7a) for different ratios of *d/r*. The direction of crack path has been determined using well-known, maximum tangential principal stress (MTPS) criteria. For *d/r=0.45*, the crack meets the interface, propagates along it over a span and then moves out of it. Some of these observations (Fig. 7b) are close to the results in [3].

5 CONCLUSIONS

A new, modified EFG method that can capture the asymptotic variation of the stress field accurately in the case of bi-materials and particulate composites is proposed. The SIF for the crack normal to interface obtained by the displacement technique is in good agreement with published results. The angle of the instantaneous crack propagation based on MTPS and ERR gives results comparable with those in the literature. It is found that the crack gets repelled in the presence of a stiff particle. The MTPS criterion reduces the number of instances for which the ERR is to be calculated to find the direction of kinking angle when the crack tip meets an interface.

REFERENCES

[1] Kassam Z.H.A., Zhang,R.J. and Wang,Z.R., Finite element simulation to investigate interaction between crack and particulate reinforcements in metal matrix composites. *Mater Sci Eng*, 1995; A203:286-299.

[2] Ayyar,A. and Chawla,N., "Microstructure-based modeling of crack growth in particle reinforced composites." in *Compos Sci Technol*, 2006; 66: 1980-1994.

[3] Zhiyong Wang, Li Ma, Linzhi Wu and Hongjun Yu., "Numerical simulation of crack growth in brittle matrix of particle reinforced composites using the XFEM technique" in *Acta Mech Solida Sinica*, 2012; 25:9-21.

[4] Bush,M.B., The interaction between a crack and a particle cluster. *Int. J. Fract.*, 1997, 88(3): 215-232.

[5] Karandikar, PG and Chou TW. Characterisation of aluminium-matrix composites made by compocasting and its variations. *J. Mater. Sci.* 1991; 26, 2255.

[6] Leggoe, JW, Hu XZ and Bush MB "Crack tip damage development and crack growth resistence in particulate reinforced metal matrix composites" in *Eng. Fract. Mech.* 1996; 53:873–895.

[7] Zhen Zhang and Zhigang Suo "Split singularities and the competition between crack penetration and debond at a material interface" in *Int. J. Solids Struct*, 1996; 53:873–895.

[8] Bogy, DB "On the plane elastostatic problem of a loaded crack terminating at a material interface" in *J. Appl. Mech*, 1971; 38:911–918.

[9] SK Maiti, RA Smith "Comparison of the criteria for mixed mode brittle fracture based on the preinstability stress-strain field Part I: Slit and elliptical cracks under uniaxial tensile loading" in *Int J Fract*, 1983; 23:281-295

[10]Ki Ju Kang "Criteria for kinking out of interface crack" in *Eng. Fract. Mech.* 1994; 49:587–598.

[11]Ventura G, Xu JX, Belytschko T. "A vector level set method and new discontinuity approximations for crack growth by EFG" in *Int. J. Num. Meth. Eng,* 2002; 54:923-944.

[12]F. Erdogan and G.C. Sih, Transactions ASME, *Journal of Basic Engineering* 85D (1963) 519-527.

[13]N.Muthu, S.K. Maiti, B.G. Falzon, I. Guiamatsia, "A comparison of stress intensity factors through crack closure integral and other approaches using eXtended element-free Galerkin method" in *Comput Mech* 2013; 52:587-605.

[14]N.Muthu, S.K. Maiti, B.G. Falzon, S. Khoddam, "Modified crack closure integral for extracio of SIFs in meshfree methods" in *Finite Elem Anal Des* 2014; 78:25-39.

[15]Yu,H.J., Wu,L.Z., Guo,L.C., Du,S.Y. and He,Q.L., Investigation of mixed-mode stress intensity factors for non homogeneous materials using an interaction integral method. *Int. J. Solids Struct*, 2009, 46(20):3710-3724.

[16]Kim,J.H. and Paulino,G.H., An accurate scheme for mixed-mode fracture analysis of functionally graded materials using the interaction integral and micromechanics models. *Int. J. Num. Meth. Eng,*2003, 58(10):1457-1497.

[17]Kim,J.H. and Paulino,G.H., Consistent formulations of the interaction integral method for fracture of functionally graded materials. *J. Appl. Mech*, 2005, 72(3):351-364.

[18]Gosz, M and Moran, B "An interaction energy integral method for computation of mixed-mode stress intensity factors along non-planar crack fronts in three dimensions," in *Eng. Fract. Mech.*, 2002, 69:299–319.

[19]Linzhi Wu, Hongjun Yu, Licheng Guo, Qilin He, Shanyi Du, "Investigation of stress intensity factors for an interface crack in multi-interface materials using an interaction integral method" in *J. Appl. Mech*, 2011, 78:(061007)1-11.

[20]S.K. Maiti "A multicorner variable order singularity triangle to model neighbouring singularities" in *Int. J. Num. Meth. Eng,* 1992; 35:391-408.

[21]T.S. Cook and F. Erdogan, "Stresses in bonded materials with a crack perpendicular to the interface" in *Int. J. Engng Si*, 1972; 10:677-697.