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A 2-D NUMERICAL MODEL TO ANALYZE STRESS DISTRIBUTION IN A SOIL MASS DUE TO APPLIED LOADS

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Abstract. The stress field in a soil due to external loads is basically determined by the size and shape of the loaded area and the thickness, uniformity and stress-strain properties of the soil mass. Predictions of stress and settlements in soils when the stress levels are far from failure and stress-strain dependence is linear can be derived assuming that soil is a homogeneous, isotropic and linear elastic material. In this work, numerical solution based on network method is presented for this kind of problems. For cases of linear and triangular load distribution over soils of infinite thickness, where theoretical solutions based on Boussinesq works exist, comparisons of steady state vertical stress within the soils are carried out to check the reliability of the proposed model in its application to more complex 2-D problems.

1 INTRODUCTION

Interrelationships dependency of stresses and strains [1] have been of great potential value to civil engineers when settling the continuous stress redistribution within the soil caused by externally applied loads [2]. For the elastic theory to be applied in soils there must be constant ratios between stresses and the corresponding strains, a requirement that goes beyond the scope of the lineal elasticity. Stress distribution depends on the thickness and homogeneity of the soil mass, the size and shape of the area to be loaded and the mechanical properties of the soil. Stress arrangement in depth determines the magnitude of the settlements; this have to be numerically determined, except in few cases in which it can be analytically solved from the works of Boussinesq [3].

For any kind of 2-D distributed loads, this communication presents a numerical model capable of providing accurate and fast computationally, steady state solution of stresses and strains in a finite or semi-infinite mass of soils.

The model is based on network simulation method [4], a powerful numerical tool that has already been successfully applied in other engineering problems; heat transport, electrochemical reactions and transport through membranes [5-7]. The start point for the design the network model is the spatially discretized finite-difference governing equations that come from the PDEs that set the mathematical model of the problem. Each term of those equations is an electric current of a suitable branch that is balanced with the currents of the other branch in a common node. So, on the one hand, the network model of a volume element is formed by as many branches as terms are in the equation; on the other hand, $N_x \times N_y$ volume elements are connected each other (by ideal electric contacts) to form the whole network model of the domain.

The model contains resistors to implement lineal terms of the equation and controlled current sources to implements coupled terms. The last, whose output current is specified by software as an arbitrary function of the dependent variables, are also used to implement certain types of boundary conditions. In this way, very few components are required and, as a consequence, very few programming rules are used for the implementation of the model. Once this is designed, it is run in the electric circuit simulation code Pspice. Output data are post-processing with Matlab for a graphical representation. To check the reliability of the model and its related numerical simulation two applications are presented, one concerning an uniform pressure (load) and other concerning a triangular pressure distribution, both on an infinite strip. Comparisons are made with the semianalytical solution derived from Boussinesq [3].

2 THE GOVERNING EQUATIONS

Navier equation [1],

$$\mu \nabla^2 \mathbf{u} + (\lambda + \mu) \nabla (\nabla \cdot \mathbf{u}) = \mathbf{0} \tag{1}$$

represents the solution of the linear elastostatic problem in terms of displacements **u**, with λ and μ the Lamé's constant and the shear modulus, respectively. For the general or mixed case, the displacements at the boundary u_i^b are directly applied, while tractions t_i^b must be indirectly applied as a function of displacements, Figure 1.

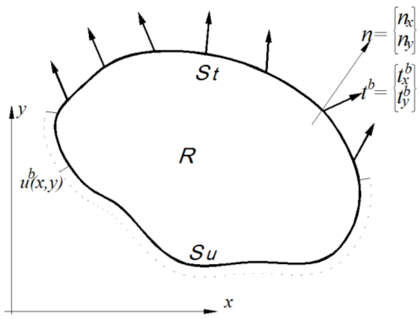

Figure 1: Boundary conditions for the 2D-elastic problem in rectangular coordinates

The governing equations for the 2D plane strain problem can be expressed in rectangular coordinates in the form

$$\left.\begin{aligned}\mu\nabla^2 u_x + (\lambda+\mu)\frac{\partial}{\partial x}\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}\right)=0\\ \mu\nabla^2 u_y + (\lambda+\mu)\frac{\partial}{\partial y}\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}\right)=0\end{aligned}\right\} \tag{2}$$

On the other hand, 2D plane stress problems can be expressed with the same equations that the plane strain case, replacing the value of the constant λ in equation (1) for an adequate value [1]. In the mixed case, the complete mathematical model needs relations between tractions imposed at the boundary and the unknown displacements. From the stress relations

$$\left.\begin{aligned}\sigma_{xx}n_x+\sigma_{xy}n_y=t_x^b\\ \sigma_{xy}n_x+\sigma_{yy}n_y=t_y^b\end{aligned}\right\}, \tag{3}$$

the necessary relations, in terms of displacements, are:

$$\left.\begin{aligned}t_x^b=\left[\lambda\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}\right)+2\mu\frac{\partial u_x}{\partial x}\right]n_x+\mu\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right)n_y\\ t_y^b=\mu\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right)n_x+\left[\lambda\left(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}\right)+2\mu\frac{\partial u_y}{\partial y}\right]n_y\end{aligned}\right\} \tag{4}$$

3 THE NETWORK MODEL

Equations (2) expressed in finite difference form is the base for the design of the network model which is implemented in such a way that its equations are formally equivalent to those of the physical process. A detailed description of the design can be found in Morales et al [3]. The entire domain is substituted by two electrical circuits formed by N*x* x N*y* volume elements that reproduce the equilibrium forces in both components of the displacements, Figure 2. The electric potential in each circuit represents a rectangular component of the displacement.

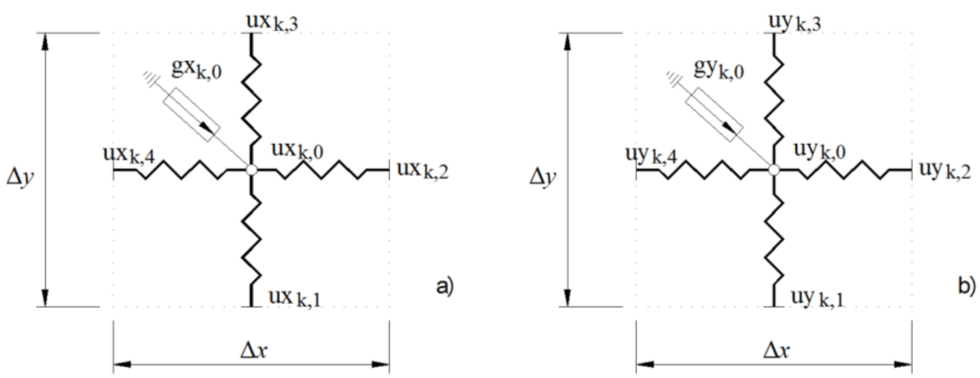

Figure 2: Circuits for equilibrium equation at *x*-direction (a) and *y*-direction (b).

To simulate the elastic relation is enough the use of simple resistors and a controlled current source, a special component contained in the libraries of the circuits simulation codes capable of assuming both, non-linearities as well as coupled conditions in the equations. As regards displacements boundary conditions, these are easily implemented by a constant voltage sources, Figure 3; tractions at the boundary are implemented by controlled voltage sources, which reproduced the coupled terms in the equations (3), Figure 4.

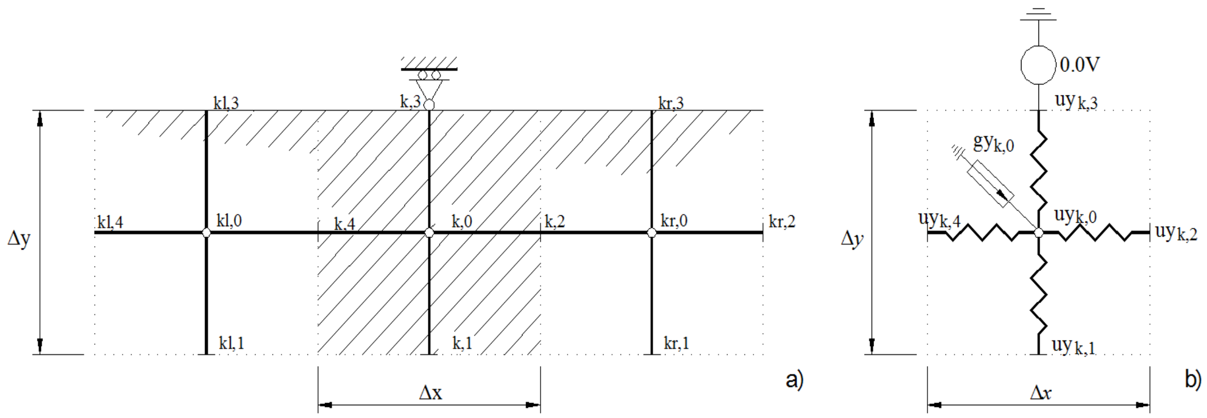

Figure 3: Implementation of a boundary displacement: a) Physical model, b) network model.

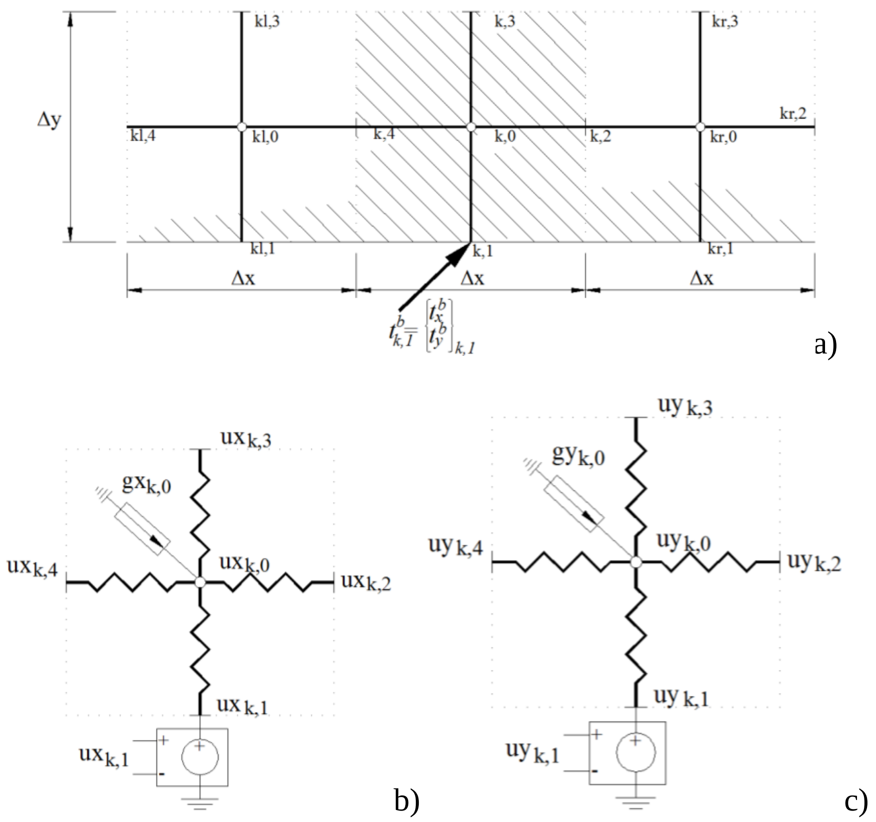

Figure 4: Implementation of traction boundary condition: a) Physical model, b) and c) network model.

4 APPLICATIONS

4.1 Uniform pressure on an infinite strip

Figure 5 (left) shows the scheme of this application. The domain is a square sample of 5 m (case a) and 10 m (case b) side, with a strip (B=1) symmetrically located at its top. Q= 100 kN/m. A grid size of 20×20 (case a) and 40×40 (case b) volume elements is assumed. Young modulus, E=3500 kN/m^2 and Poisson coefficient, ν=0.3. For this application, Figure 6 and 7 show, at the left, the vertical stress for cases a and b, respectively, while analytical solutions are depicted, for comparison, at the right of each figure. In addition, Von misses stresses obtained numerically are shown for each case in Figure 8. As expected, for the larger length numerical solution is closer to analytical, the last applied to a soil of infinite thickness.

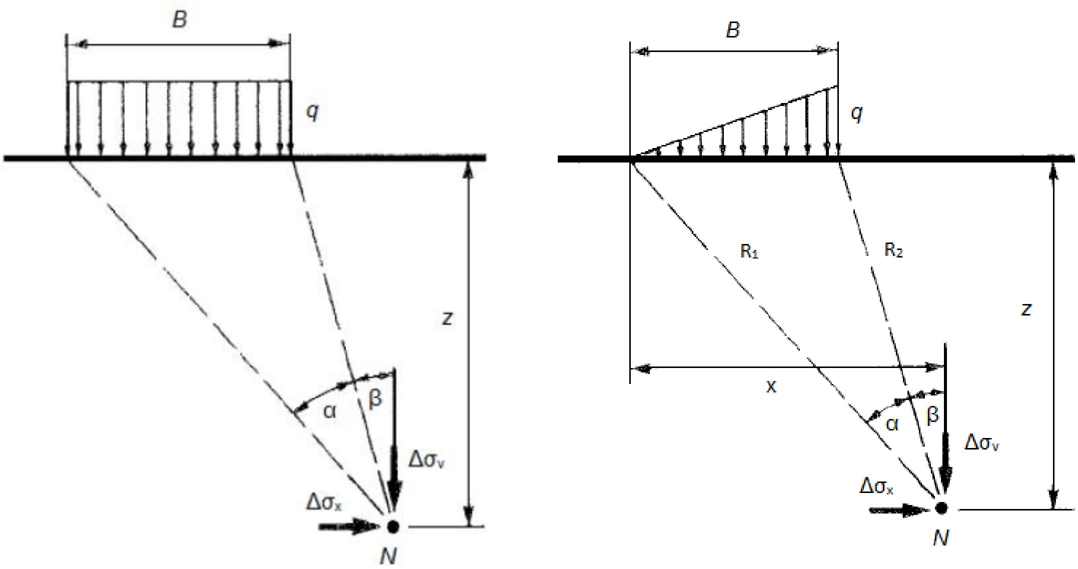

Figure 5: Uniform load (left) and triangular load (right) schemes

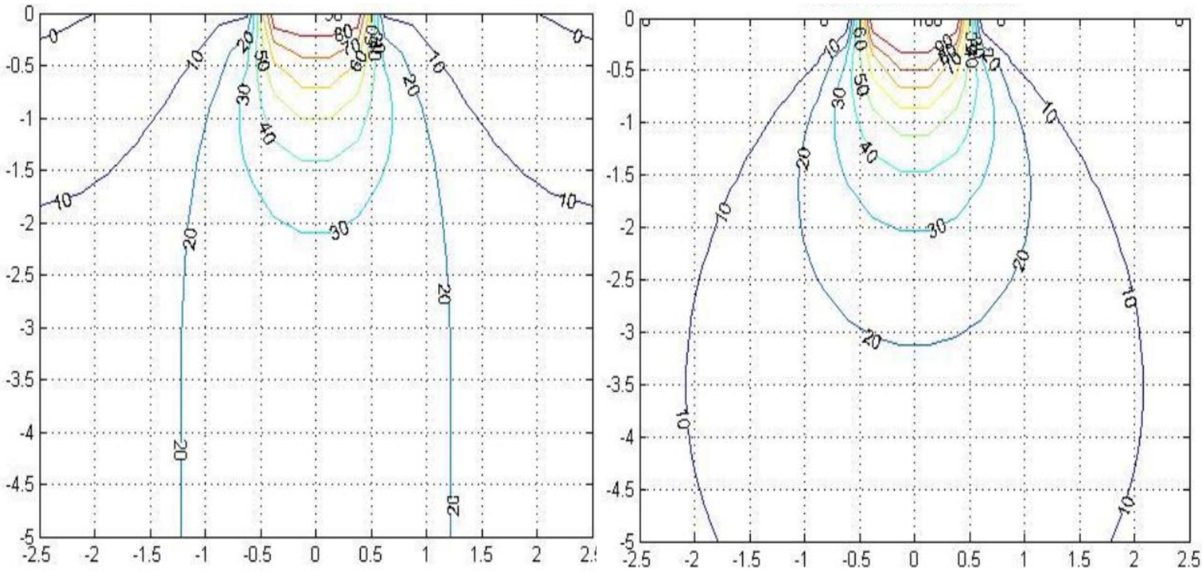

Figure 6: Vertical stress. Network method (left), analytical solution (right). L=H=5 m

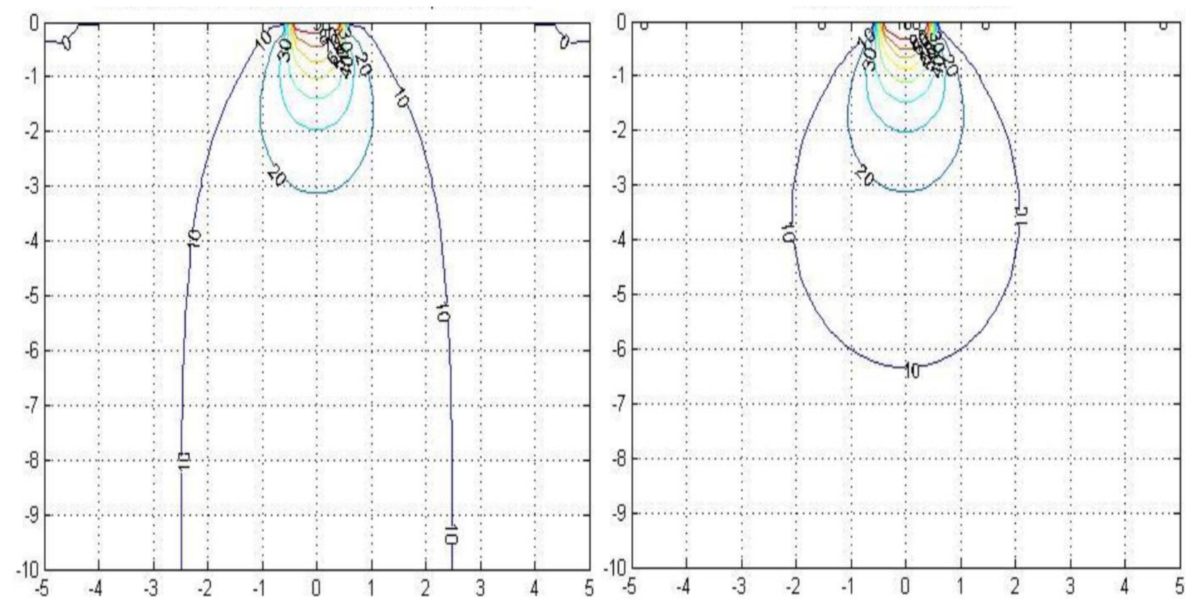

Figure 7: Vertical stress. Network method (left), analytical solution (right). L=H=10 m

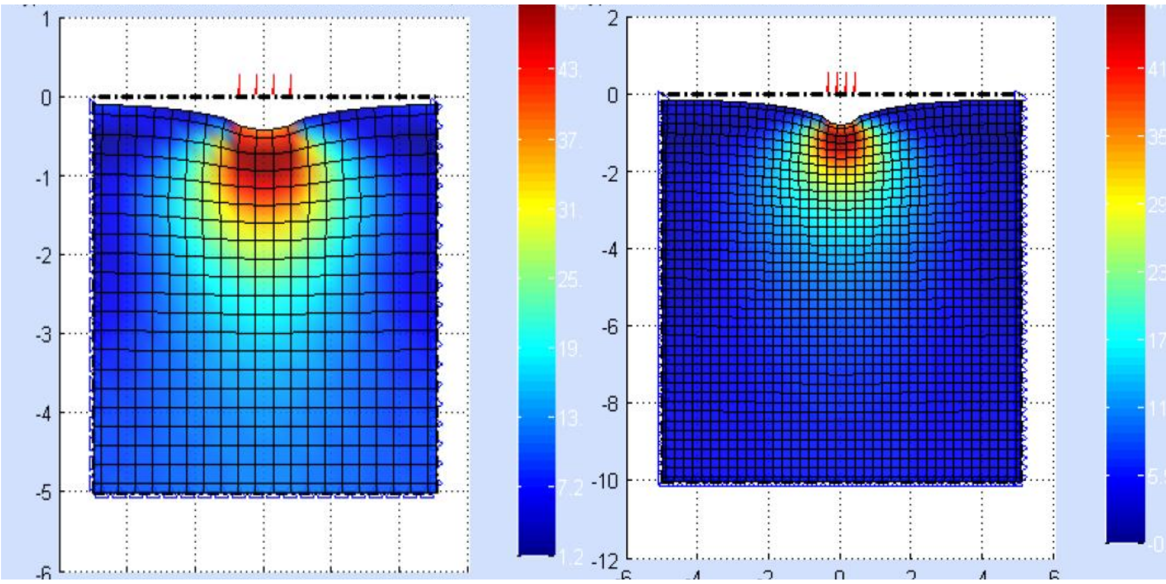

Figure 8: Von Mises stress for the uniform load. L=5 m (left; L=10 (right)

4.2 Triangular pressure on an infinite strip

Figure 5 (right) shows the physical scheme. For this application the domain is, again, a square sample of 5 m (case a) and 10 m (case b) side, with a strip (B=1) at the centre of the top side. Q_{max} = 100 kN/m. A grid size of 20×20 (case a) and 40×40 (case b) volume elements is assumed while Young modulus´and Poisson coefficient have the same values of the former application. Figure 9 and 10 show, at the left, the vertical stress for cases a and b, respectively; analytical solutions are depicted at the right of the figures. Finally, numerical Von misses stresses are shown, for each case, in Figure 11.

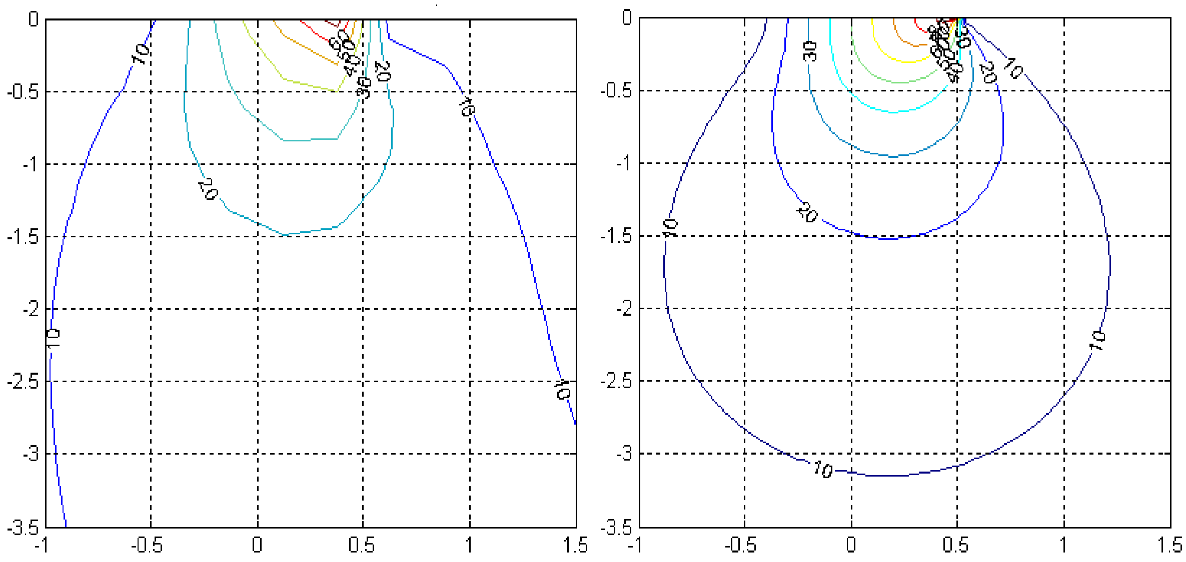

Figure 9: Vertical stress. Network method (left), analytical solution (right). L=H=5 m

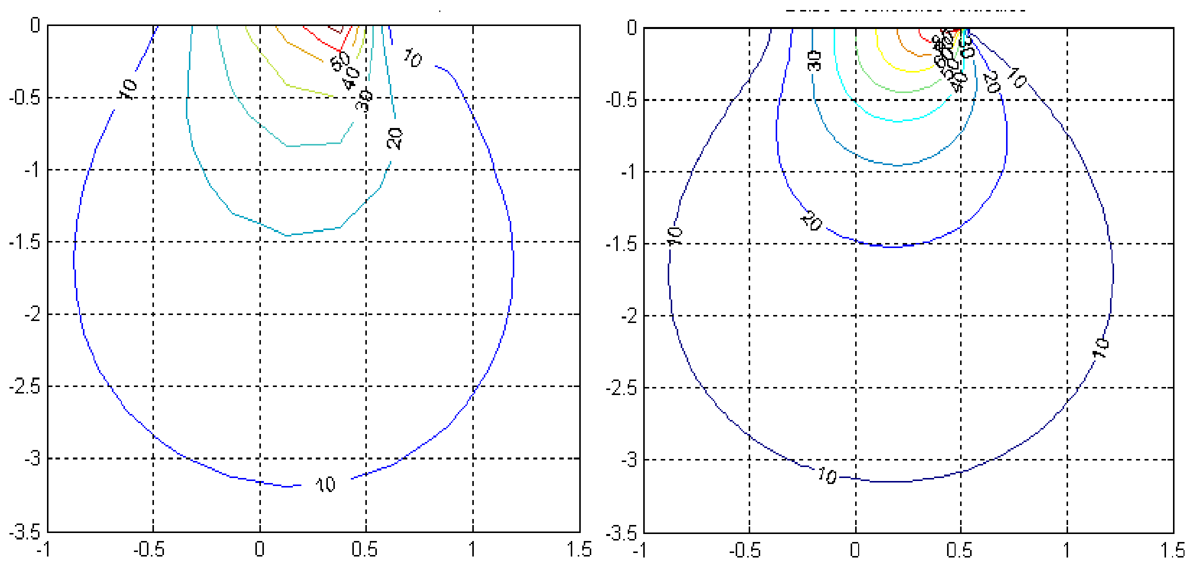

Figure 10: Vertical stress. Network method (left), analytical solution (right). L=H=10 m

As shown, for a square sample of soil of 10 m side, numerical solution is quite close to analytical. This means that, on the one hand, real cases for which the soil thickness is infinite can be simulated by square domains of finite side and, on the other hand, the proposed method is reliable when applied to other load distribution over irregular surfaces.

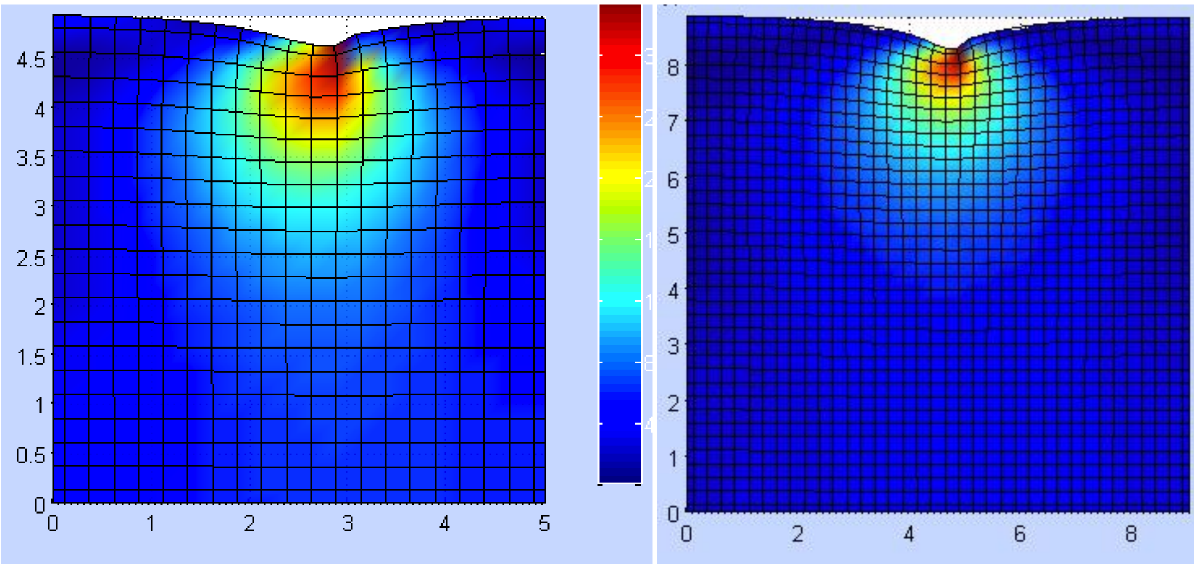

Figure 11: Von Mises stress for the uniform load. L=5 m (left; L=10 (right)

5 CONCLUSIONS

Network method is a powerful numerical tool for the solution of stress distribution in 2-D soils assuming an elastic behavior. The presented applications, based on linear and triangular load distributions over a strip, for which comparisons with theoretical solutions of Boussinesq are carried out, allow of checking the reliability of the proposed method.

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