

FUNDAMENTAL STUDY OF FLUID-SOIL-SEEPAGE FLOW COUPLED ANALYSIS BY A PARTICLE METHOD BASED ON THE MIXED FLOW THEORY

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Abstract.

Damage mechanisms of port structures such as breakwater and seawall have been studied in the past, and there are mainly three causes; I. horizontal force due to the water level difference between the front and rear breakwater, II. soil scour and erosion behind the seawall during overflow and III. piping destruction associated with the decline of the bearing capacity by seepage flow. In this study, a particle simulation tool based on the SPH has been developed to solve the different soil damage mechanisms; soil scour and seepage flow problem. These simulations should treat the Fluid-Soil and Fluid-Seepage flow interactions, and the particle simulation tool has been modified and improved to solve each interaction problem. For the Fluid-Soil interactions in the soil scour problem, soil is modeled by a Bingham flow model which is one of the non-Newtonian fluids, and the Mohr-Coulomb criterion is applied in the plastic yield judgment. On the other hand, in the seepage flow analysis, surface flow and seepage flow are described by the same government equation “Darcy-Brinkman equation”, and simultaneous analysis is carried out. These different simulations have been implemented by modifying the standard SPH method.

1 INTRODUCTION

In 2011, Tohoku-Kanto earthquake tsunami caused serious damage on the port structures such as breakwater and seawall. Damage mechanisms of these structures have been studied in the past, and there are mainly three causes; I. horizontal force due to the water level difference between the front and rear breakwater, II. soil scour and erosion behind the seawall during

overflow and III. piping destruction associated with the decline of the bearing capacity by seepage flow.

Fluid-Structure-Soil coupling simulation is desired for a systematic comprehension of seawall collapse mechanism, and it may help to develop next disaster prevention method. In this study, a particle simulation tool based on the SPH has been developed to solve the different soil damage mechanisms; soil sour and seepage flow problem. These simulations should treat the Fluid-Soil and Fluid-Seepage flow interactions, and the particle simulation tool has been modified and improved to solve each interaction problem.

For the Fluid-Seepage flow interaction analysis, in this study, as a governing equation of Fluid-Seepage flow, Darcy-Brinkman equation is introduced. This equation has been widely used to study flows in porous media in various contexts, and its derivation is based on the volume-averaging method, focusing on the underlying assumptions and on its relationships with the Navier-Stokes and Darcy equations. Using this equation, simultaneous analysis is carried out. These different simulations have been implemented by modifying the standard SPH method.

Finally, efficiency and adequacy of the proposed simulation technique has been validated through an application to two experimental tests.

2 SMOOTHED PARTICLE HYDRODYNAMICS (SPH) FORMULATION

In this paper, smoothed particle hydrodynamics (SPH) method^[1] was adopted. A basic concept in SPH method is that any function ϕ attached to particle “ i ” at a position r_i is written as a summation of contributions from neighbor particles

$$\phi(r_i) \approx \langle \phi_i \rangle = \sum_j \frac{m_j}{\rho_j} \phi_j W(r_{ij}, h) \quad (1)$$

Note that, the triangle bracket $\langle \phi_i \rangle$ means SPH approximation of a function ϕ . The divergence of a vector function can be assumed by using the above defined SPH approximation as follows

$$\nabla \cdot \bar{\phi}(r_i) \approx \langle \nabla \cdot \bar{\phi}_i \rangle = \frac{1}{\rho_i} \sum_j m_j (\bar{\phi}_j - \bar{\phi}) \cdot \nabla W(r_{ij}, h) \quad (2)$$

and the expression for the gradient can be represented by

$$\nabla \phi(r_i) \approx \langle \nabla \phi_i \rangle = \rho_i \sum_j m_j \left(\frac{\phi_j}{\rho_j^2} + \frac{\phi_i}{\rho_i^2} \right) \nabla W(r_{ij}, h) \quad (3)$$

In this study, Incompressible SPH (ISPH) method^{[2],[3],[4]} developed in the incompressible fluid analysis was adopted. In this method, the pressure is calculated implicitly and the velocity is calculated explicitly.

3 GOVERNING EQUATION

3.1 Modelling of Fluid and Soil

Fluid is generally modeled as Newtonian fluid, and described Navier-Stokes equation as

$$\frac{D\mathbf{v}}{Dt} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{v} + \mathbf{g} \quad (4)$$

In addition to the above equation, mass conservation law was also used as governing equation.

$$\frac{D\rho}{Dt} + \rho\nabla \cdot \mathbf{v} = 0 \quad (5)$$

On the other hand, soil is generally modeled as solid and nonlinear elastic-plastic material. However, in this study, soil was modeled as Bingham fluid^{[5],[6]} which is one of Non-Newtonian fluid because of expressing flow failure of ground. Bingham fluid has shear strength and can be written as following equation

$$\boldsymbol{\tau} = \mu_s^0 \dot{\boldsymbol{\gamma}} + \boldsymbol{\tau}_y \quad (6)$$

Here, $\boldsymbol{\tau}_y$ is yield shear stress and μ_s^0 is viscosity after yield. Note, a Bingham fluid has behavior such as a rigid body and dose not deform but when the shear stress surpasses the yield stress, flow failure occurs resulting in large deformations. The Mohr-Coulomb criterion is introduced for the yield shear strength in the Bingham model for a given soil as

$$\tau_y = c + p \tan \varphi \quad (7)$$

Where, c is the cohesion, and φ is the internal friction angle. Then, the soil viscosity should be expressed in Bingham model as

$$\mu_s = \frac{\boldsymbol{\tau}}{\dot{\boldsymbol{\gamma}}} = \begin{cases} \frac{c + p \tan \varphi}{\dot{\boldsymbol{\gamma}}} (= \mu_E) & \dot{\boldsymbol{\gamma}} \leq \dot{\boldsymbol{\gamma}}_y \\ \mu_s^0 (= \mu_p) + \frac{c + p \tan \varphi}{\dot{\boldsymbol{\gamma}}} & \dot{\boldsymbol{\gamma}} > \dot{\boldsymbol{\gamma}}_y \end{cases} \quad (8)$$

Here, drawing the graph of above Eq.(8), relationship $\boldsymbol{\tau}$ and $\dot{\boldsymbol{\gamma}}$ is expressed as

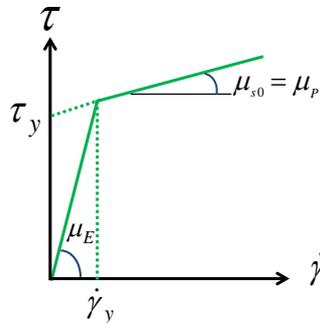


Figure1. Relationship $\boldsymbol{\tau}$ and $\dot{\boldsymbol{\gamma}}$

3.2 Modelling of Fluid and Seepage flow

Fluid (Surface flow) and Seepage flow are generally described by Navier-Stokes equation and Darcy's law, however, in case water particles penetrate into soil mound and change surface flow to seepage flow, there needs to be a unified formula between them. So, in this research, as a governing equation of Fluid-Seepage flow, Darcy-Brinkman equation^{[7],[8]} is introduced and shown below.

$$\frac{\partial}{\partial t} \mathbf{v} + \mathbf{v} \cdot \nabla \left(\frac{\mathbf{v}}{\chi} \right) = -\frac{\chi}{\rho_l} \nabla p + \nu \nabla^2 \mathbf{v} + \chi \mathbf{g} - \frac{\nu \chi}{k} \mathbf{v} \quad (9)$$

Here, χ is the liquid volume fraction (porosity) and k is the permeability. ν is the liquid velocity in Fluid phase and Darcy velocity in Darcy phase. Where we solve the Darcy-Brinkman equation with changing porosity χ and permeability k , we can solve Navier-Stokes equation in the pure fluid and Darcy's equation in the porous matrix.

4 VALIDATION TEST

In the following section, the numerical examples have been conducted to validate the current scheme. In this chapter, two hydraulic experiments each about Fluid-Soil interaction problem and Fluid-Seepage flow interaction problem were adopted.

4.1 Soil scouring problem

As a validation test of Fluid-Soil interaction analysis, a hydraulic experiment done by Yamamoto^[9] was adopted. Fig.2. and Table.1 present the schematic diagram and analysis parameters for the experiment. This experiment is return flow scouring experiment and its scale is showing below. As for soil parameters, the density ratio between soil density ρ_s and water density ρ_w is taken as $\rho_s/\rho_w=1.6$. The soil viscosity after yield is set to $\mu_s^0 = 5\text{Pa s}$. The internal angle and cohesion are taken as $\phi=8^\circ$ and $c=0\text{kPa}$. Here, gradient of sandy beach and slope is one-fifteenth, and there is a seawall at the front of sandy beach. There is a velocity meter away from the seawall by 6cm. Arrows in left side and right side are respectively water supply pipe and drainage pipe, and tank in right hand side is rectification tank. Under these conditions, numerical experiment was carried out.

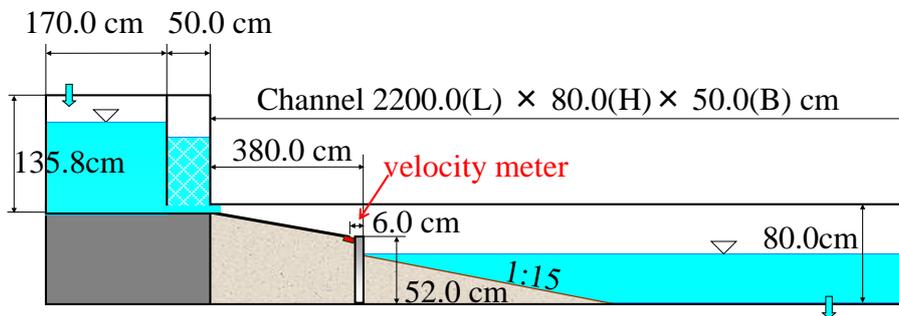
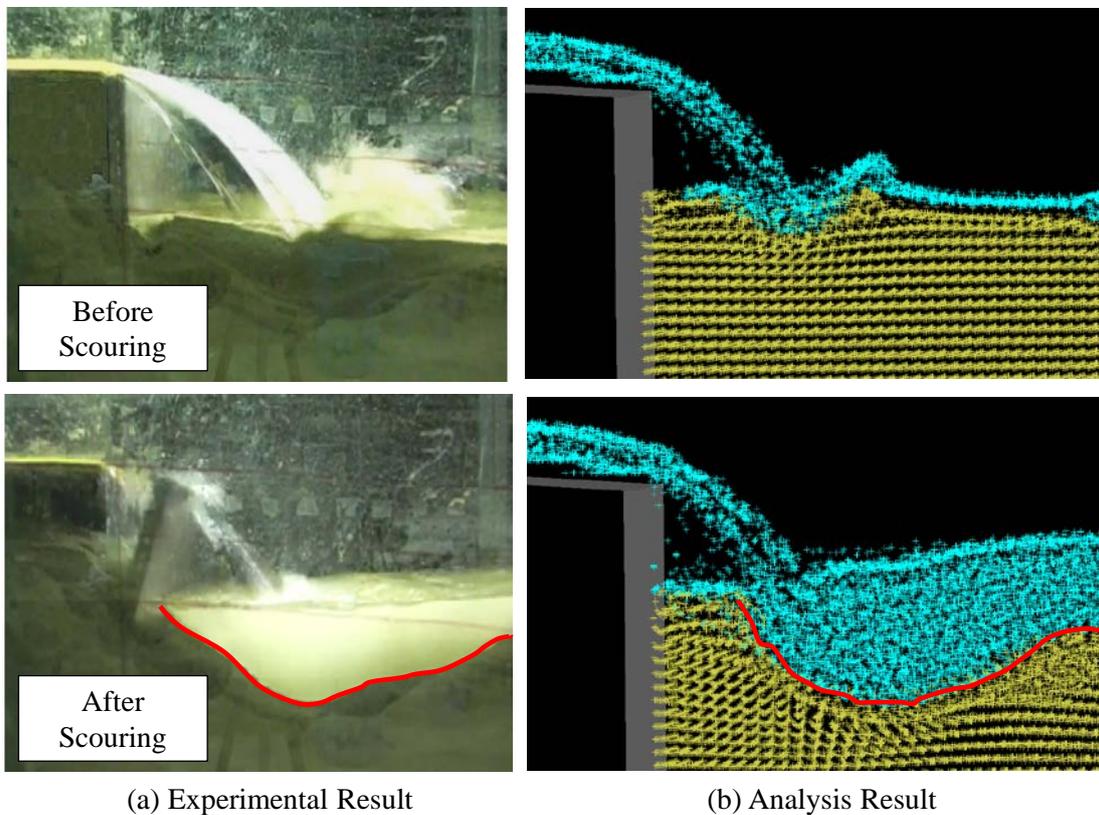


Figure2. Initial schematic diagram for hydraulic experiment

Table1. Analysis option

Particle size	Total particles	Time step	Real time
2cm	1371475	0.0001s	12s

The analysis result is summarized in this blow. Fig.4 shows the analysis result compared to experimental result. Left hand side pictures are after scouring snap shot of hydraulic experiment and right hand side pictures are those of numerical experiment. In the right hand side pictures, the soil particles are transported by the flow. From these comparisons, the predicted express soil erosion and scouring behaviors shows a good agreement with the hydraulic experiment test. The shape of soil erosion is, however, slightly different, then permeability in the soil domain and the other factors may be necessary to improve our numerical results.

**Figure4. Comparison between experiment and analysis result**

4.2 Seepage flow problem inside soil mound

As a validation test of Fluid-Seepage flow interaction analysis, a hydraulic experiment done by Kasama^[10] was adopted. Fig.5. presents the schematic diagram for the experiment. In this model, soil mound is fixed, and velocities of seepage flow particles are displayed on the mound particles. In experiment, value of Δh is set to be 40mm, 80mm, 120mm and 145mm. In the case of Δh is 145mm, soil mound was collapsed like piping phenomenon, and caisson

was tumbled. So, in this analysis, as a first step, in order to check seepage flow behavior, Δh is set to be 120mm.

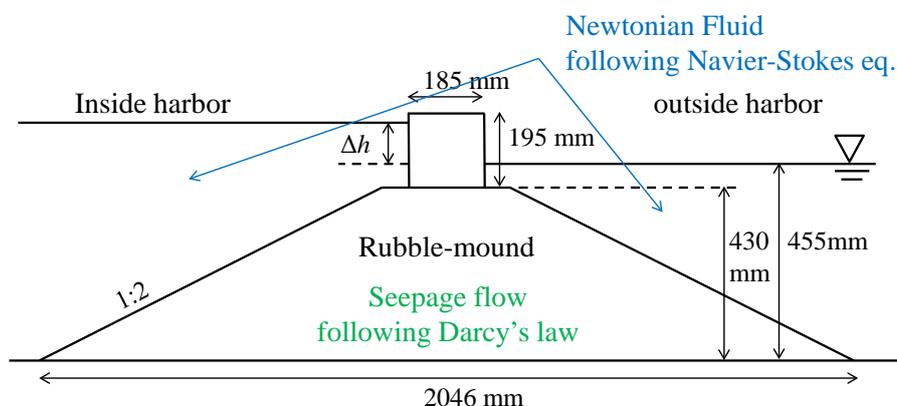


Figure5. Initial schematic diagram for seepage flow experiment

Fig6. and Fig.7 show analysis results. Figure.6 is distribution of piezo water head and Figure.7 is that of velocity in left fig's dot line region. The distribution of analytical piezo water head is similar to experimental one, and it confirmed that seepage flow particles flow from high piezo water head region to low region. These differences may be caused by treatment of the boundary between Newtonian fluid and seepage flow region governed by the Darcy's law.

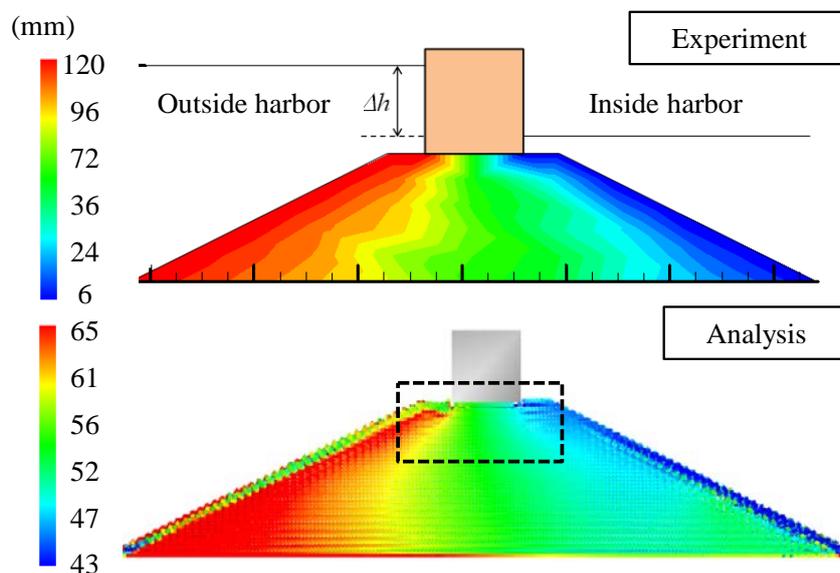


Figure6. Distribution of piezo water head

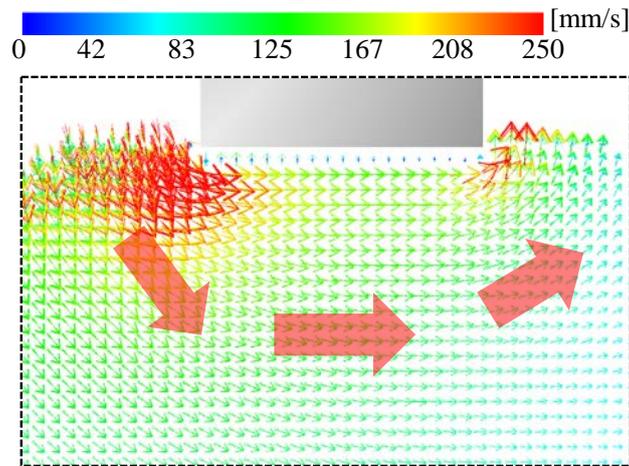


Figure7. Distribution of velocity

5 CONCLUSIONS

In this study, an improved ISPH algorithm has been used to simulate Fluid-soil and Fluid-Seepage flow interactions. The fluid is modeled as a Newtonian fluid, and the soil is modeled as a Bingham model. In this approach, through the validation test, it was confirmed that ISPH method and modeling soil as a Bingham flow can express soil erosion and scouring. On the other hand, fluid and seepage flow were described by unified formula as Darcy-Brinkman equation. Through its validation test, it can be confirmed that Darcy-Brinkman equation is good for Fluid-Seepage flow interaction problem. Finally, as a future work, obtaining validated result from quantitative perspective and combining Fluid-Soil simulator and Fluid-Seepage flow simulator are our goals.

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