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NUMERICAL STUDY OF TEMPERATURE AND STREAMFUNCTION PATTERNS BEFORE FULL CONVECTION IN GEOTHERMAL CELLS OF BÉNARD TYPE

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Abstract. Numerical solution of steady state flow and heat transfer problem in scenarios type Bénard for Rayleigh numbers under the threshold of convection is presented. For this range of Rayleigh, patterns of temperature are horizontal lines regularly distributed throughout the domain, however, although Bénard cells do not emerge, two non-symmetric eddies of negligible flow velocity appear at the lateral boundaries. As Rayleigh increases in the range studied, eddies increases their sizes towards the center of the domain as well as their flow velocities. Simulations are carried out by the network simulation method.

1 INTRODUCTION

Under certain geometry and boundary conditions, coupled fluid flow and heat transfer processes in porous media between horizontal layers heated from below and cooling from above, typical theoretical geothermal scenarios, give place to organized structures of a same length – named Bénard cells – that repeat along the large horizontal domain [1,2].

When the onset of convection is reached, the initial 'apparent' motionless system starts to move and a pattern of 2-D rolls – convection cells – can be clearly distinguished; rotation of adjacent is opposite so that flows near vertical boundaries have the same sign. These cells emerge along the domain with small irregularities in the cells nearer to the lateral sides due to asymmetry. However, if the convection is not triggered, the system remains in the static no-flow situation set up before heating; this steady-state solution means a heat transfer of pure conduction. We are interested in this work with flow patterns under the onset of convection. Figure 1 shows the physical scheme of the problem. Dependent dimensionless variables are temperature (T) and streamfunction (ψ), the last has the advantage that iso-ψ lines show the path of the fluid particles. Since no analytical or semianalytical solutions exist in the literature, the problem is solved by the network simulation method [3], a numerical tool that has been successfully applied to many problems of science and engineering in different fields, corrosion, tribology, elasticity and others [4,5].

This problem, in the case of isotropic 2-D media is characterized by an only dimensionless group of coefficients, the so named Rayleigh number, Ra. For Ra greater than 41 (approximately)[6], clear convection regular cells emerge along the domain with small disturbances at the ends due to the finite length of the domain, while for Ra lower than 41 iso-temperature pattern is formed by straight horizontal lines regularly distributed along the height of the domain, a distribution that reflects a pure-conduction pattern. However, when refining the calculation, two eddies of negligible flow appear in the pattern, being the centre of these eddies near the lateral sides of the domain; as Ra increase within the range 0÷41, the flow increases and extends towards the center of the domain retaining the same pattern for the temperature. This means that the hypothesis of pure-conduction is approximate and that a change in Ra forces a change in steady state patterns, nearly imperceptible for the temperature variable.

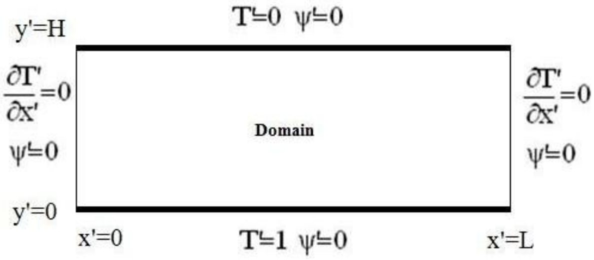

Figure 1: Geometry of the problem

2 MATHEMATICAL MODEL

Mathematical model is formed by the flow and energy equations plus those related to boundary conditions – Boussinesq approximation is assumed. Being u and v are the expressions of the horizontal and vertical components of the flow velocity, these equations are:

$$u = -\left(\frac{K_x}{\mu}\right)\left(\frac{\partial P}{\partial x}\right), \qquad v = -\left(\frac{K_y}{\mu}\right)\left\{\left(\frac{\partial P}{\partial y}\right) + \rho g\right\} \tag{1}$$

$$\left(\rho_f c_{p,f}\right)u \left(\frac{\partial T}{\partial x}\right)+\left(\rho_f c_{p,f}\right)v \left(\frac{\partial T}{\partial y}\right)= k_{m,x}\left(\frac{\partial^2 T}{\partial x^2}\right) + k_{m,y}\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{2}$$

$$T_{(x,y=0)} = 1,\ T_{(x,y=H)} = 0, \left(\frac{\partial T}{\partial x}\right)_{(x=0,y)} = \left(\frac{\partial T}{\partial x}\right)_{(x=L,y)} = 0 \tag{3}$$

$$\psi_{(x,y=0)} = \psi_{(x,y=H)} = \psi_{(x=0,y)} = \psi_{(x=L,y)} = 0 \tag{4}$$

As regards temperature, boundary conditions assume that the top and bottom sides of the domain are isothermal, while the left and right sides are adiabatic; as regards flow, all sides are impermeable to water. T and ψ are the dependent variables temperature (ºC) and streamfunction (m^2/s), respectively, P (N/m^2) the pressure, K (m^2) the permeability, ρ (kg/m^3) the fluid density, g (m/s^2) the gravitational acceleration, μ the fluid viscosity (kg $m^{-1}s^{-1}$), $c_{p,f}$ (J/KgºC) the specific heat, k the thermal conductivity (W/mºC); finally, x (m) and y (m) are the independent variables.

We will use the dimensionless variables defined by

$$x' = \frac{x}{l_x^*} \qquad (5)$$

$$y' = \frac{y}{H}$$

$$v' = \frac{v}{v^*} = \frac{v\mu}{[\rho_0 g\beta(\Delta T)K_y]}$$

$$u' = \frac{uH\mu}{[\rho_0 g\beta(\Delta T)K_y(l_x^*)]}$$

with

$$l_y^* = H \qquad (6)$$

$$v^* = -\left(\frac{K_y}{\mu}\right)(\Delta\rho)g = -\left(\frac{K_y}{\mu}\right)\rho_0 g\beta(\Delta T)$$

$$u^* = v^*\left(\frac{l^*_{hidden,x}}{H}\right) = -\left(\frac{K_y}{\mu}\right)\rho_0 g\beta(\Delta T)\left(\frac{l^*_{hidden,x}}{H}\right)$$

where l_x^* is the horizontal length of the typical convection cell repeated throughout the domain; H the vertical length; -(K_y/μ)(Δρ)g (Darcy´s velocity) the vertical characteristic velocity and, finally, v^* ($l^*_{hidden,x}$/H), as derived from mass conservation equation, the horizontal characteristic velocity.

With these equations, the only dimensionless parameter that characterizes this problem in isotropic porous media is the so-named Rayleigh number [7]

$$Ra = \left(\frac{Kg\Delta\rho H}{\mu\alpha}\right) \qquad (7)$$

3 THE NUMERICAL MODEL

For the application of network method two steps are followed [3]: firstly, designing the model (for which some knowledge of convectional circuit theory is required by the programmer), and secondly, its simulation in a suitable code such as Pspice [8].

For the first step, the coupled partial differential equations that govern the problem must be written in its finite-difference differential form by discretizing the spatial independent variables, retaining time as a continuous variable. The network model, which has as many circuits as governing equations or dependent variables (two for the problem studied) is then designed assuming that the addends of each equation are electric currents that balance each other in the principal node of the circuit.

So, each term is implemented in its respective branch by a suitable electric device, whose constitutive equation is just that of such term. The addends that realize the coupling between equations are also easily implemented by a special kind of device, called ‘current controlled source’, which is able to provide an output current specified as an arbitrary mathematical function whose arguments can be voltages and/or currents at any point or any device of the network. Besides, the network of adjacent cells are inter-connected by ideal electrical contacts to form the entire model of the domain. Boundary conditions are finally implemented by suitable electrical devices.

The equivalence between thermal and electric variables is defined in the form

j (electric current, W/m^2) ~ heat flux density (W/m^2)
V (electric potential, V) ~ temperature (K)

for the heat transport circuit, and

j (electric current, W/m^2) ~ flow velocity (m/s)
V (electric potential, V) ~ stream function variable (m^2/s)

for the fluid flow circuit. Using the nomenclature of Figure 2, each term of the equations

$$\frac{1}{\Delta y}\left(\frac{\Delta\psi}{\left(\frac{K_x}{K_y}\right)\left(\frac{\Delta y}{2}\right)}\right)_{out} - \frac{1}{\Delta y}\left(\frac{\Delta\psi}{\left(\frac{K_x}{K_y}\right)\left(\frac{\Delta y}{2}\right)}\right)_{in} + \frac{1}{\Delta x}\left(\frac{\Delta\psi}{\left(\frac{\Delta x}{2}\right)}\right)_{out} - \frac{1}{\Delta x}\left(\frac{\Delta\psi}{\left(\frac{\Delta x}{2}\right)}\right)_{in} = -\left(\frac{\rho K_y g\beta}{\mu}\right)\left(\frac{\Delta T}{\Delta x}\right) \quad (8)$$

$$\left(\rho_f c_{p,f}\right)\left[\left(\frac{\Delta\psi}{\Delta y}\right)\left(\frac{\Delta T}{\Delta x}\right) - \left(\frac{\Delta\psi}{\Delta x}\right)\left(\frac{\Delta T}{\Delta y}\right)\right] = \frac{1}{\Delta x}\left(\frac{\Delta T}{\left(\frac{\Delta x}{2k_{m,x}}\right)}\right)_{out} - \frac{1}{\Delta x}\left(\frac{\Delta T}{\left(\frac{\Delta x}{2k_{m,x}}\right)}\right)_{in} + \frac{1}{\Delta y}\left(\frac{\Delta T}{\left(\frac{\Delta y}{2k_{m,y}}\right)}\right)_{out} - \frac{1}{\Delta y}\left(\frac{\Delta T}{\left(\frac{\Delta y}{2k_{m,y}}\right)}\right)_{in} \quad (9)$$

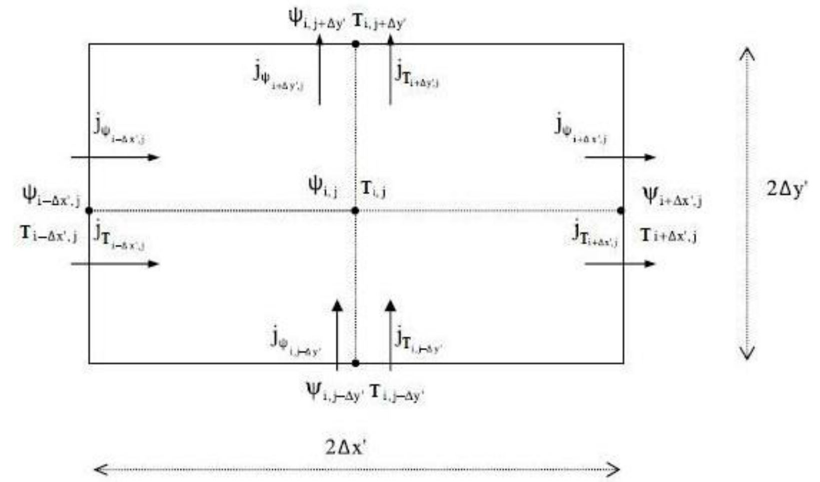

Figure 2: Nomenclature of the volume element

can be implemented in the network model by a suitable branch. Lineal terms are implemented by lineal electric components (resistor, coils and capacitors) while non-lineal and coupled terms are implemented by software in a suitable device contained in the libraries of the circuit simulation codes named current controlled source. Figure 3 shows the network model. A total number of N×M volume elements are connected in series to represent the whole domain. Boundary conditions, whatever they are, are implemented in the model by simple constant voltage or current sources or by controlled sources [3]. Once the model is completed, it is run in Pspice with no other mathematical manipulation; programming routines in MATLAB are used to show the solution graphically.

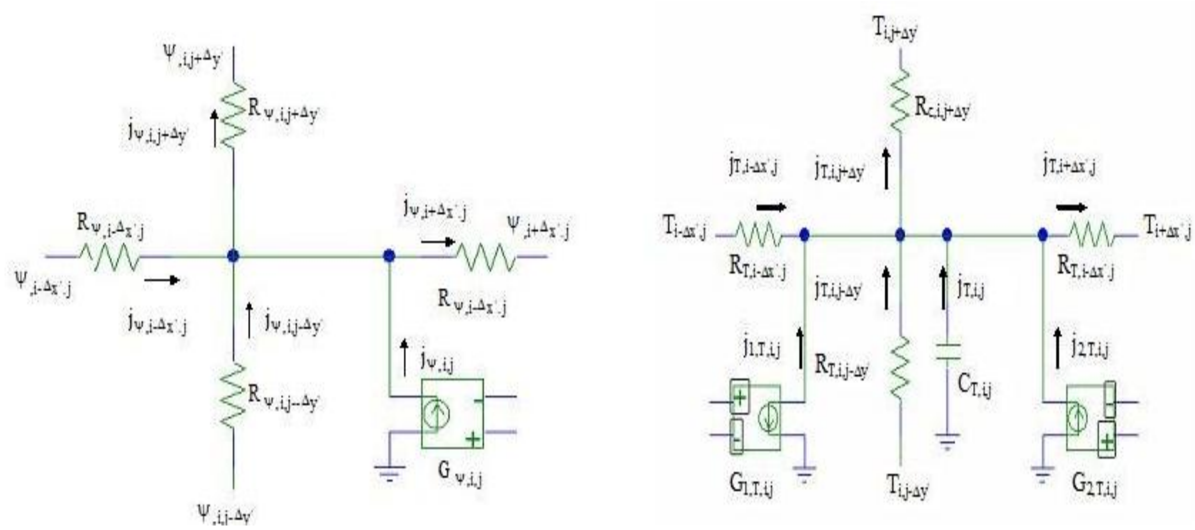

Figure 3: Network model. a) streamfunction variable, b) temperature

4 SIMULATIONS

The values of the geometrical and physical parameters used are: L = 360 m, H = 90m, μ = 0.0002 Kg/m/s, g = 9.81 m/s^2, $\Delta\rho$ (maximun fluid density change) = 230 Kg/m^3, $K_x = K_y$ (permeability) = $4.8\cdot10^{-14}$ m^2, and $\alpha_x = \alpha_y$ (diffusivity): 10^{-6} m^2. Finally, a grid of 80 (horizontal)$\times$20(vertical) volume elements has been used. Any of the former values can be suitably changed to get the required Ra according to equation (7). The onset of convection takes place for Ra $\approx$ 41, so that we are interested in patterns resulting from Ra<41, in order to investigate the apparent motionless of the flow assumed by most researchers, its potential structure in cells and the distortion of the temperature pattern.

The numerical simulations provides, for Ra = 10, 30, 35, 40, 41 and 42, the temperature and streamfunction patterns show in Figures 5 to 9, respectively. Only the right half of the domain has been represented for a better lecture.

From these figures some interesting aspects can be observed. First of all, for Ra < 42, patterns of temperature provides a map of pure conduction but really the fluid is flowing within de cell, although its velocity is quite negligible if we compare with the velocities that result for Ra>42, several orders of magnitude lower than that from Ra<41 (note that even for Ra = 41 the thermal stratification is regular).

In addition, the core of the flow non-symmetric eddy is located near the center of the lateral boundaries, extending its influence progressively as Ra increases. The second eddy emerges for Ra $\approx$ 30 and is maintained for greater values of Ra until to reach the onset of the convection, which occurs for Ra $\approx$ 42.

Note that four convection cells are clearly appreciate for Ra = 40, a cells number that is maintained up to Ra=42 (when the convection has already been triggered); the difference, more quantitative than qualitative, between the patterns for Ra = 40 and Ra = 42 is reflected in the order of magnitude of the flow velocity. The number of cells is justified by the value of l^*_x, a characteristic length that is function of the geometrical and physical parameters of the problem [x].

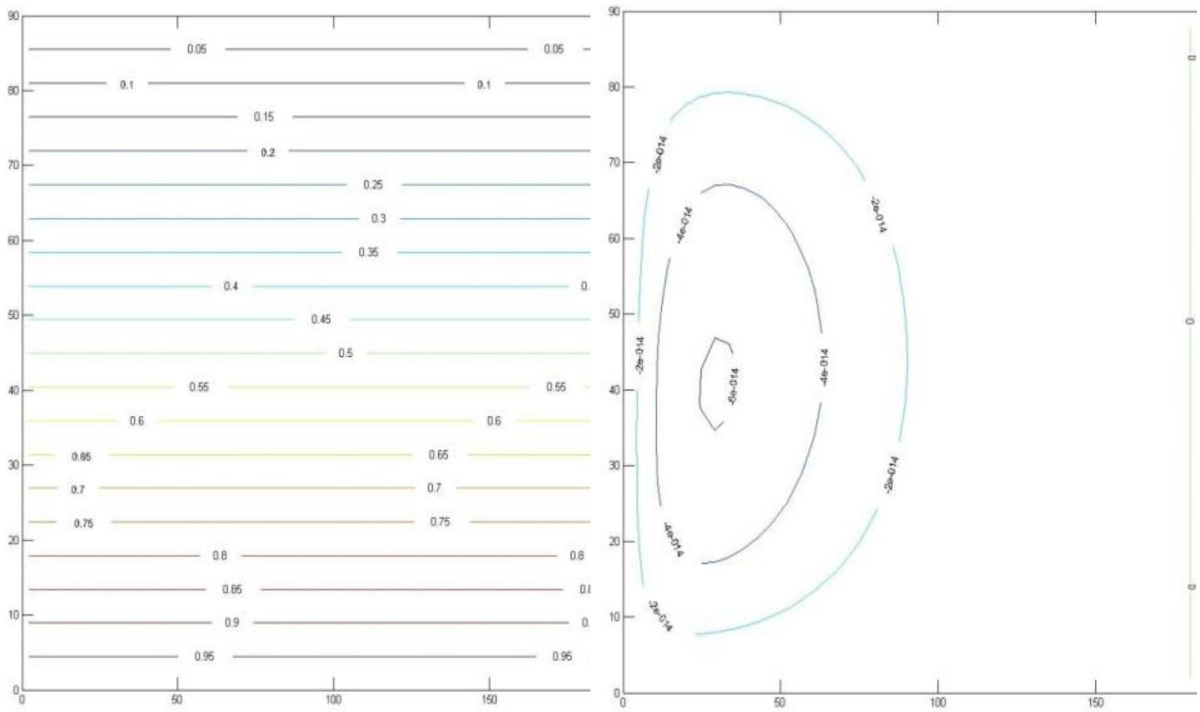

Figure 4: Steady state temperature (upper) and streamfunction patterns for Ra =10

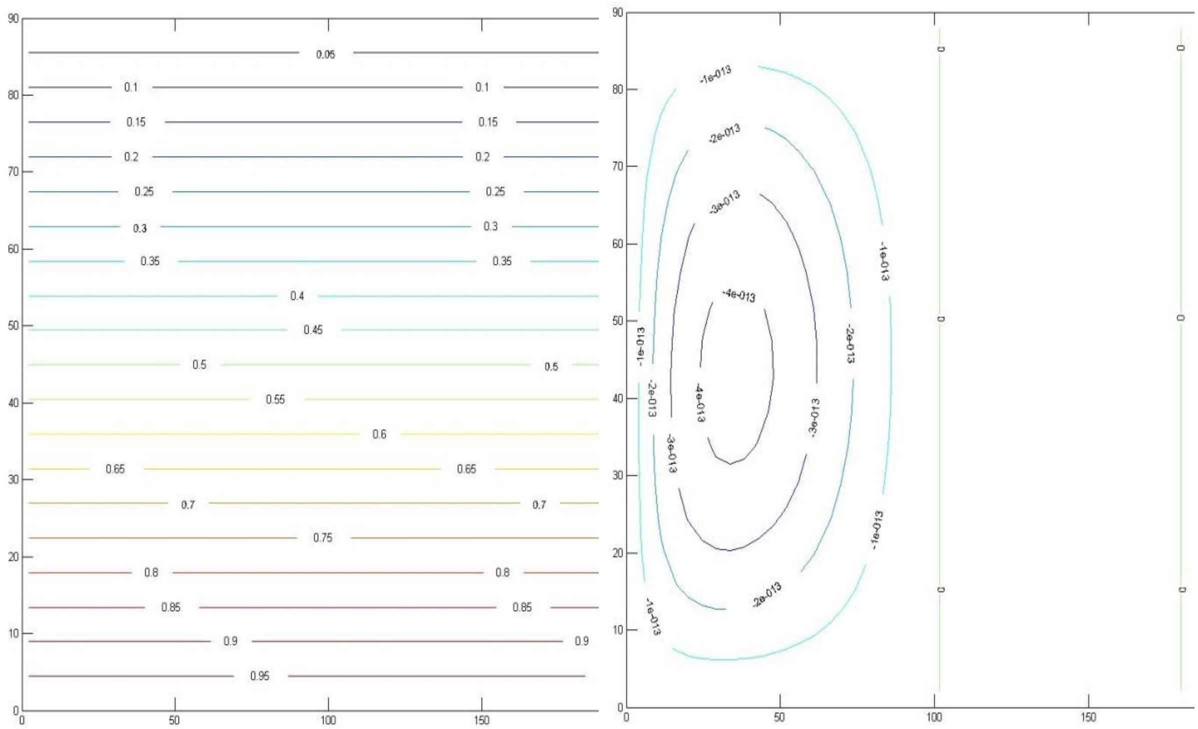

Figure 5: Steady state temperature (upper) and streamfunction patterns for Ra =30

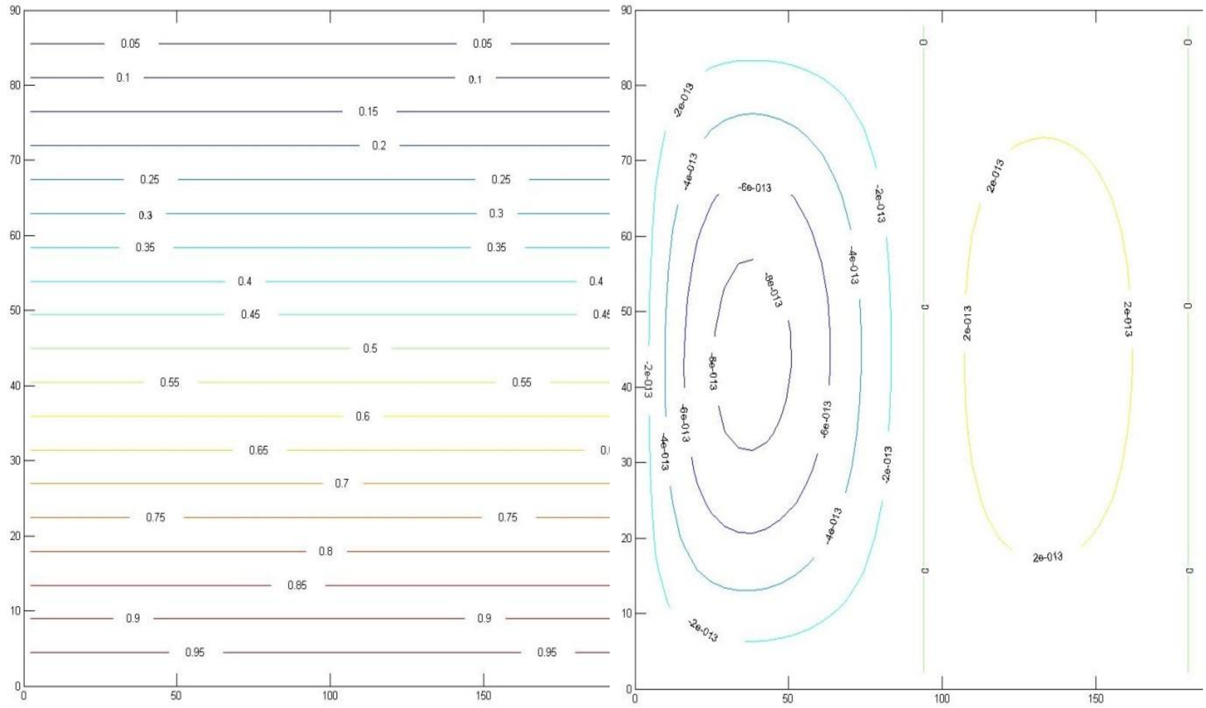

Figure 6: Steady state temperature (upper) and streamfunction patterns for Ra =35

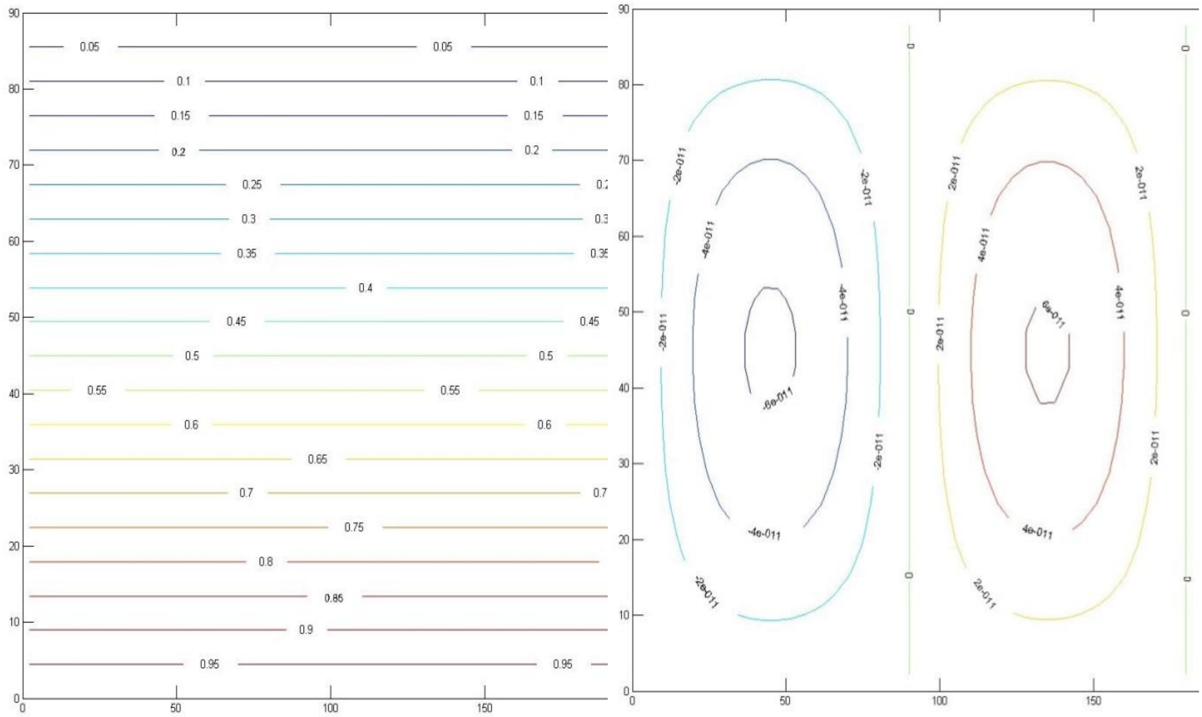

Figure 7: Steady state temperature (upper) and streamfunction patterns for Ra =40

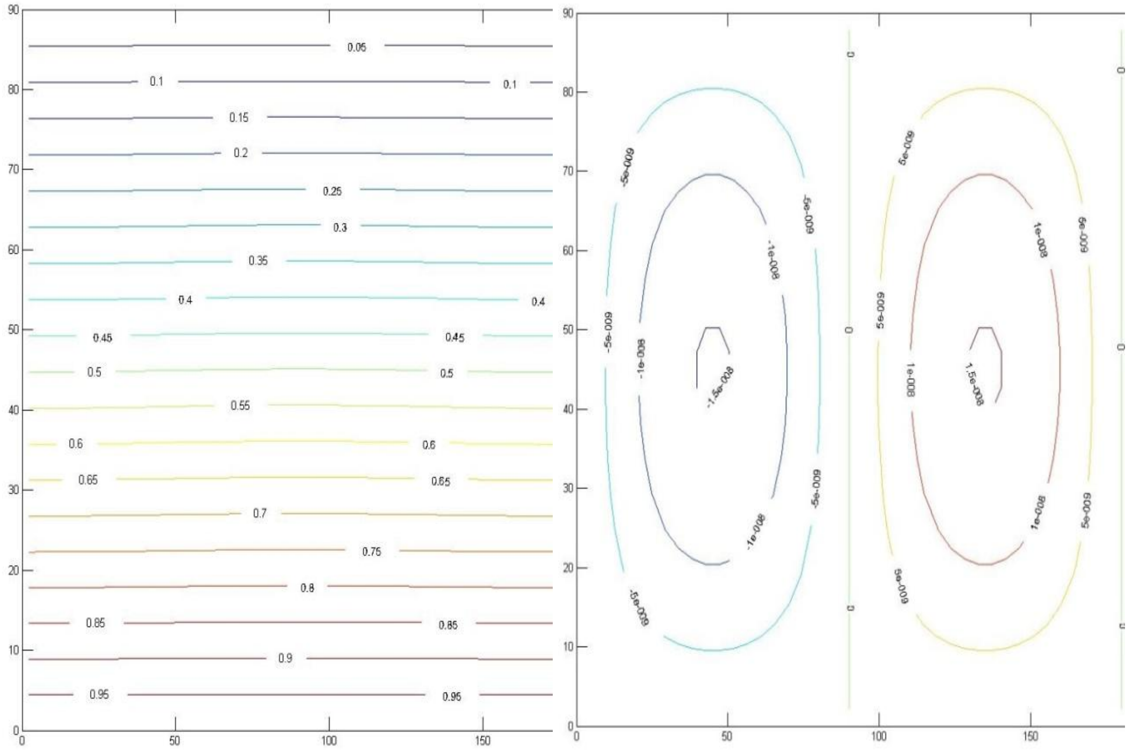

Figure 8: Steady state temperature (upper) and streamfunction patterns for Ra =41

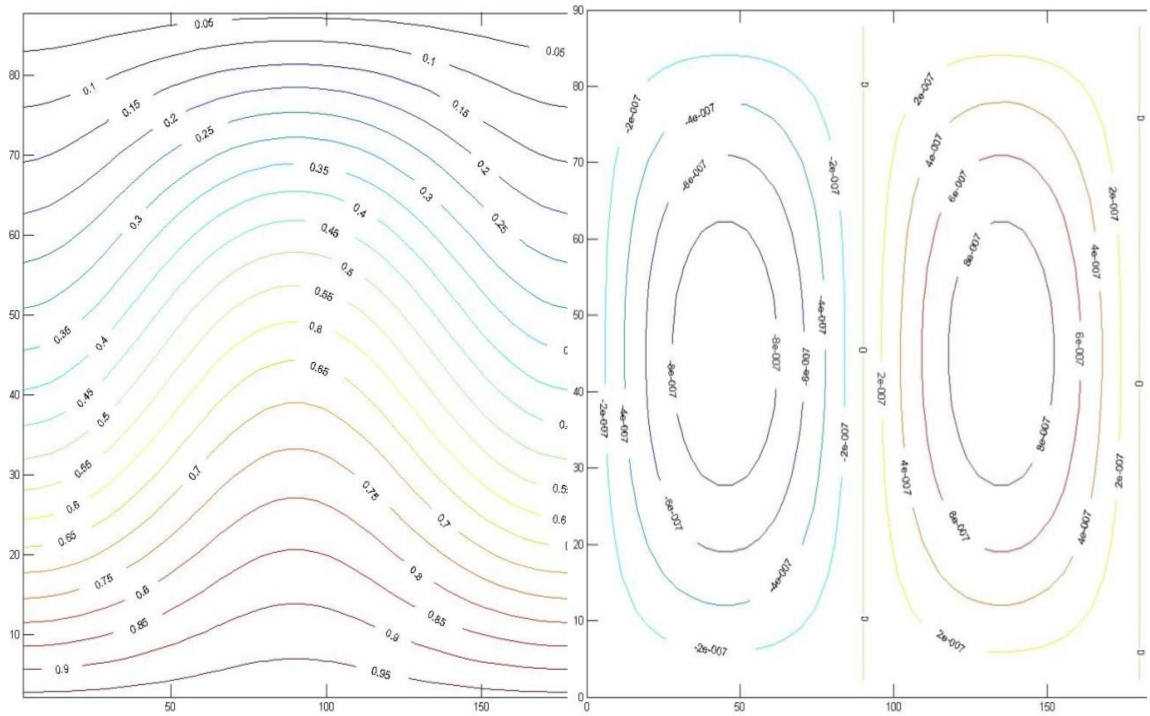

Figure 9: Steady state temperature (upper) and streamfunction patterns for Ra =42

Finally, Figure 10 illustrates the temperature and streamfunction patterns for Ra = 50, a value clearly greater than the threshold of the convection.

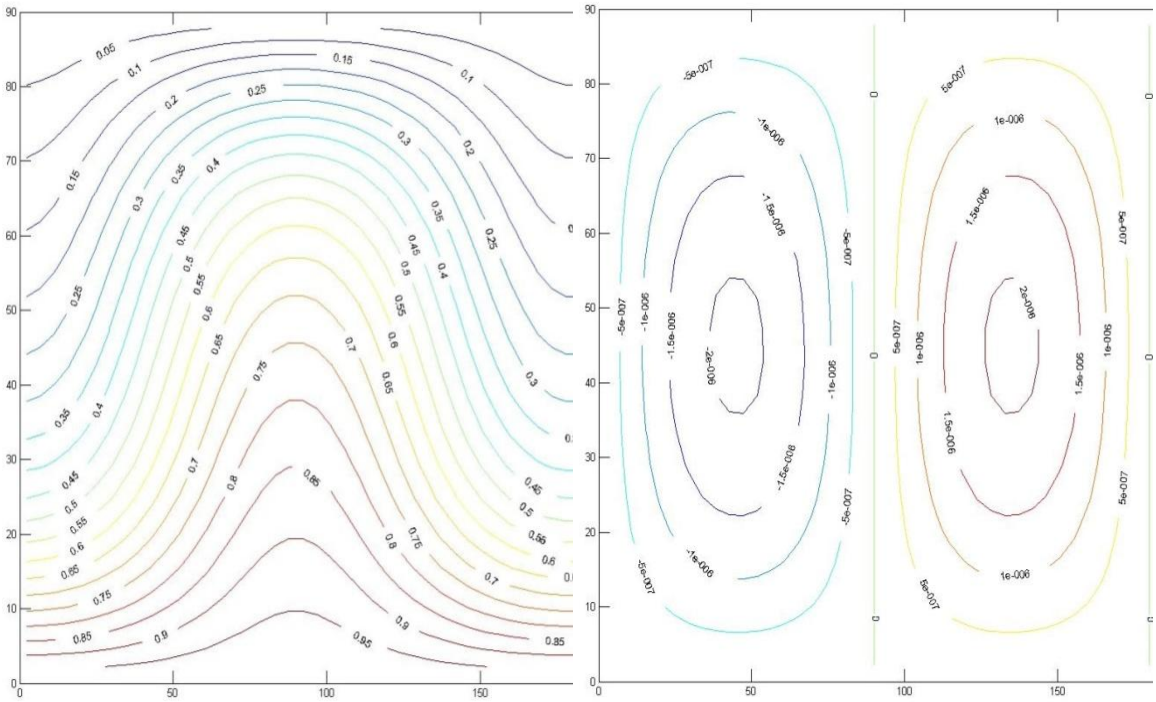

Figure 10: Steady state temperature (upper) and streamfunction patterns for Ra =50

5 FINAL COMMENTS AND CONCLUSIONS

- Network simulation method allows to simulate non-lineal coupled problems of flow and heat transport type in porous media with no other mathematical manipulations different to the correct design of the model.
- When to geothermal problems of Bénard type, the simulation reproduces the regular distribution of cells providing the characteristic length, as well as present the complex phenomena involved as regards the onset of convection that separates the pure conduction pattern from that of convection.
- Particularly, the simulation provides the negligible eddies that emerge in the scenario for values of Rayleigh smaller than the convection threshold, Ra=41. These patterns provides a flow velocity several orders of magnitude lower than those of Ra>41. In addition, qualitatively information of the evolution of this pattern for increasing Ra is also provided by the simulation.

REFERENCES

[1] Holzbecher, E. O. Modeling density-driven flow in porous media: principles, numerics, software (Vol. 1). Springer. (1998).

[2] Pop, I., & Ingham, D. B. Transport Phenomena in Porous Media. Elsevier. (1998)

[3] González-Fernández, C. F. *Heat Transfer and the Network Simulation Method*. Ed. Research Signpost, Kerala, (2002)
[4] Soto Meca, A., Alhama, F. and González-Fernández, C. F. An efficient model for solving density driven groundwater flow problems based on the network simulation method. *Journal of Hidrology* (2007) **339**:39-53.
[5] Morales, J.L., Moreno, J.A. and Alhama, F. Numerical solution of 2D elastostatic problems formulated by potential functions, *Applied Mathematical Modelling* **37 (9)**:6339-6353.
[6] Lapwood, E. R. Convection of a fluid in a porous medium. *Proceedings of the Cambridge,* **44,** 508–521. (1948).
[7] Bejan, A. *Convective heat transfer in porous media*. John Wiley. (1984).
[8] PSPICE 6.0, Microsim Corporation Fairbanks, Irvine, California 92718, (1994)