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GHOST NEIGHBORHOOD METHOD FOR HANDLING PERIDYNAMIC BOUNDARY EFFECT

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Abstract. Peridynamic is a new formulation of solid mechanics based on integro-differential equations, hence it can deal with discontinuous problems conveniently. When used in boundary-related problems, the theory has some problems comes from a reduction in material stiffness raised by incomplete neighbors of a point nearby the boundary. In this paper, a correction method of resultant force density is made to ensure that the resultant force density is independent of horizons. Based on this independence, a "ghost neighborhood" method is proposed to handle the boundary effect. Finally, the theoretical verification and numerical verification are accomplished to indicate that this method is reasonable and effective.

1 INTRODUCTION

The discontinuous of physical field is very hard to deal with by continuum mechanics, since the continuum mechanics relies on partial differential equations. The reason of this problem is that continuum mechanics assumes the interaction in materials only happens between two "neighboring" particles [2], so the government equations of mechanical behavior are some partial differential equations.

Sometimes, some kind of ill-conditioned continuum mechanical method is proposed to model the discontinuous phenomena. For instance, for simulating the crack propagation behavior, traditional method need some special techniques that have been developed in the field of fracture mechanics. For example, FEM requires remeshing after each incremental crack growth. Although those newly improved method such as cohesive zone element [1]and extended FEM [2], eliminates the need of remeshing, they still rely on the external kinetic relations for injection of such elements while predicting crack growth [5].

In 1998, Silling [3] advanced a new method, Peridynamic, which could overcome the shortcoming of the previous continuum methods, he introduced a model of long-range force between material points over finite distances in a body, and then the partial differential equations can be replaced with the integro-differential equations. This creative modification makes Peridynamic can deal with the continuous media, the discontinuous defects (such as crack) and particles in a single theoretical frame work without additional techniques. Many researchers used it to deal with the discontinuous and made some improvement of this method. Erkan Oterkus studied damage in metallic and composite structures [4][5] . Silling and his companion used this theory to study damage of composite and concrete structures [6][7][8] . The states based peridynamic was proposed by Silling in 2007, this model makes a large improvement of peridynamic. For example, the limitation of poisson ratio is removed [10]. YU [11] made an improvement of a new adaptive integration method which brings an error reduction.

However, peridynamic has some theoretical problems and computational techniques which need to do further investigation. Especially, the boundary effect discovered by Oterkus [5] is a problem when we need do some simulations related to the surface or boundary. The cause of this boundary effect is that the neighborhood for these boundary effect is incomplete, while resultant force density is calculated with an integration on a complete neighborhood [9]. The boundary effect will lead to a reduction in material stiffness near the boundary [5].

In this paper, our work will focus on the handling of peridynamic boundary effect with a "ghost neighborhood" method. Firstly, we will expound the boundary effect and its causes. Then a correction of bond force density is made which makes the resultant force density independence of horizons. Finally, the ghost neighborhood method is advanced and verified in both theoretical and numerical.

2 PERIDYNAMIC BOUNDARY EFFECT

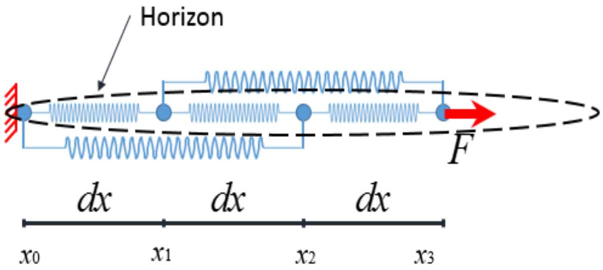
By introducing the "long-range" force concept, the motion differential equation of a body can be converted to an integro-differential equation as equation (1). In fact, peridynamic theory can be thought as a continuum version of molecular dynamics [9].

$$\rho\ddot{\mathbf{u}}(\mathbf{x}_A,t)=\int_{\mathrm{H_x}}\mathbf{f}(\mathbf{u}(\mathbf{x}_B,t)-\mathbf{u}(\mathbf{x}_A,t),\mathbf{x}_B-\mathbf{x}_A)dV_B+\mathbf{b}(\mathbf{x}_A,t) \tag{1}$$

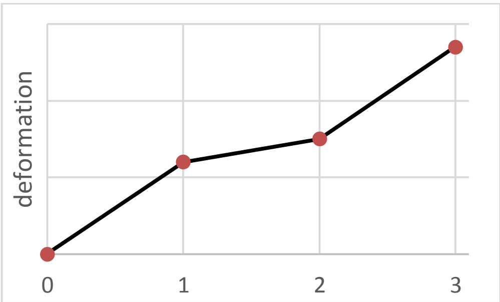
Where H_x is a neighborhood of $\mathbf{x}$, $\mathbf{U}$ the displacement vector field, $\mathbf{b}$ a prescribed body force density field, ρ mass density in the reference configuration, and $\mathbf{f}$ a pairwise force function whose value is the force vector(per unit volume squared) that particle $\mathbf{x}'$ exerts on the particle $\mathbf{x}$.

The materials points in Peridynamic body have two types. For the points far away from the boundary, their neighborhood distribution is symmetrical, all the bond forces are balanced under a uniform deformation; for the points nearby the boundary, their neighborhood distribution is unsymmetrical, so the bond forces are not balanced anymore. The second one is obviously unreasonable and nonphysical.

This problem can be expounded by a simple model. In this model, we applied a pure tension load on a bar with a length of 3dx, and discretized the bar into four particles for simplicity and intuitive. Then we choose a horizon of $2dx$, as shown in Figure 1(a). In this model, the neighborhood of x_1 and x_2 are not complete, which will cause a reduction in material stiffness near the boundary. So the deformation under a pure tension load will not be uniform, shown in Figure 1 (b).



(a)



(b)

Figure 1: Pure Tension of a Four Particles Bar

It should be noted that this boundary influence the whole body, not only those areas nearby boundaries. This will be shown in the next section.

3 METHOD TO HANDLE THE BOUNDARY EFFECT

We will propose a “ghost neighborhood” method to handle the boundary. But before we doing this, a correction method of bond force density calculate should be derived. Because the “ghost neighborhood” method is based on an independence between resultant bond force density and horizons size, while the independence doesn’t exist in available resultant bond force density calculate method.

To illustrate this point, we calculated a resultant force density of a deformed bar with a prototype microelastic brittle (PMB) material model [9]. This model defined the bond force to depend only on the stretch of this bond. The relation between bond force and bond stretch can be found in [5].

$$f = cs\mu \tag{2}$$

$$\begin{aligned}\mathbf{F} &= \int_{x_0-\delta}^{x_0+\delta} \mathbf{f}(\xi) A d\xi = A\int_{x_0-\delta}^{x_0+\delta} cs(\xi)\cos\theta d\xi \\ &= Ac\left(\int_{x_0}^{x_0+\delta} s(\xi)d\xi - \int_{x_0-\delta}^{x_0} s(\xi)d\xi\right)\end{aligned} \tag{3}$$

Where F -resultant force density

x_0 -location of a point in 1D problem

μ -a history dependent scalar valued function that takes on values of either 1or 0 to judge whether a bond is failure

δ -horizon size

ξ -the relative position of two points defined by $\xi = x' - x$ in 1D problem

A -bar's section area

c -a material constant related to elasticity modulus

s -bond stretch between two point defined by $s = \dfrac{|\xi+\eta| - |\xi|}{|\xi|}$

Where $\eta = u(x',t) - u(x,t)$ means the relative displacement. And $u(x,t)$ is the displacement of point x at time t .

It should be noted that in 1D problem the location of a point, the relative position of two points, the displacement of a point and the relative position of two points can be treated as a scalar.

From the eq.(3), we know that the result of the resultant bond density contains an integration of the bond stretch on the whole neighborhood. Which makes the resultant bond force density depends on the horizon size.

3.1 The correction method of bond force density calculate

An infinitesimal segment of deformation bar with a length of l is considered in this section (Figure 2 (a)). The strain of the segment is assumed to be linearly distributed (Figure 2 (b)). The PMB model is adopted in the following analysis.

In classical continuum mechanics, the resultant force density (Figure 2(c)) at the central point can be given by

$$F = A\cdot\frac{\partial\sigma}{\partial x}\cdot dx/(Adx) = \frac{\partial\varepsilon}{\partial x}\cdot E \tag{4}$$

Where $\dfrac{\partial\varepsilon}{\partial x}$ is a constant in this segment because of the linearity of strain variation.

In Peridynamic, the displacement, the stretch (Figure 2(d)) and the resultant force density can be given by

$$u_x = \int_{x_0}^{x} \varepsilon x_0 dx = \int_{x_0}^{x} (\varepsilon_{x_0} + \frac{\partial\varepsilon}{\partial x}x)\,\mathrm{d}\,x = x\varepsilon_{x_0} - x_0\varepsilon_{x_0} + \frac{1}{2}\frac{\partial\varepsilon}{\partial x}x^2 - \frac{1}{2}\frac{\partial\varepsilon}{\partial x}x_0^2 \tag{5}$$

$$s_{i,j} = (\mathrm{u}_i - \mathrm{u}_j)/(\mathrm{x}_i - \mathrm{x}_j) = \frac{1}{2}\frac{\partial\varepsilon}{\partial x}(\mathrm{x}_i + \mathrm{x}_j) + \varepsilon_{x_0} \tag{6}$$

$$F = \int_{x_0-\frac{l}{2}}^{x_0+\frac{l}{2}} fA\cos\theta dx = \int_{x_0-\frac{l}{2}}^{x_0-\frac{l}{2}} csA\cos\theta dx = cA\cdot(\frac{1}{2}\frac{\partial\varepsilon}{\partial x}\frac{l}{2})\cdot\frac{l}{2} \tag{7}$$

Where $\cos\theta$ is used to expression the direction of the force, and $\cos\theta$ takes on values of either 1 or 0 in 1D problem

Let $\delta = \frac{l}{2}$. The eq.(7) can be replaced by

$$F = cA \cdot (\frac{1}{2}\frac{\partial \varepsilon}{\partial x}\delta) \cdot \delta \tag{8}$$

Combination the result from Peridynamic eq.(9) with that from classic theory eq.(6) leads to the "spring constant" in the PMB model.

$$c = \frac{2E}{A\delta^2} \tag{9}$$

The "spring constant" can be used in any non-uniform deformation, and the resultant force density is independent of the horizon size since only the displacement of adjacent particles is used to calculate the resultant force density on one material point.

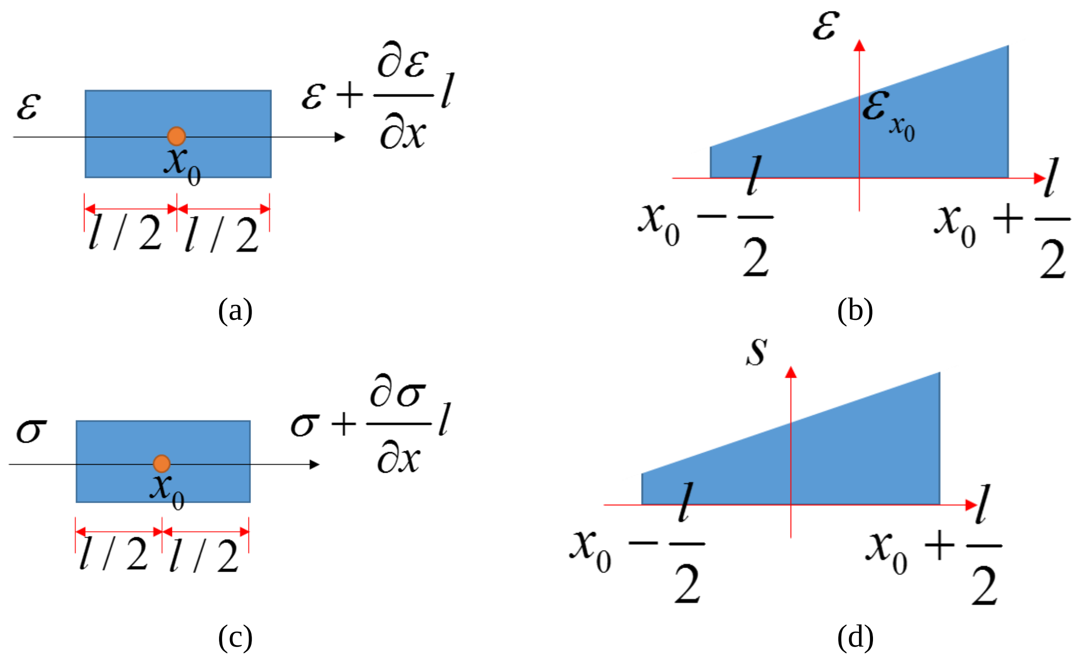


Figure 2: the assumption of the strain distribution in an infinitesimal segment

3.2 ghost neighborhood method

We consider a homogeneity deformation bar with uniform section (shown in Figure 3). Some interpolating points were placed on the bar. This points will be used to calculate related variables (such as the bond stretch, the displacement). A point $\mathbf{x}_i$ which is nearby the boundary is taken to be studied in this section. We choose two different horizon size δ_1 and δ_2 for this point. This two different size makes the point have two different neighborhood, shown in green and in dark blue in the picture. We used the stretch between the point $\mathbf{x}_i$ and its adjacent points $\mathbf{x}_{i-1}$ and $\mathbf{x}_{i+1}$, which is named as $s_{i-1,i}$ and $s_{i+1,i}$, to calculate the stretch in the whole neighborhood with an interpolation algorithm. It should be noted that only the stretch between $\mathbf{x}_i$ and its adjacent points $\mathbf{x}_{i-1}$, $\mathbf{x}_{i+1}$ are used (stretch between $\mathbf{x}_i$ and $\mathbf{x}_{i+2}$ is ignored) to ensure the validity of this method since we used an assumption of linearly variation. Besides, the neighborhood of $\mathbf{x}_i$ is incomplete under the horizon size of δ_2 , because this size is larger than the distance between point $\mathbf{x}_i$ and the boundary. We create a ghost neighborhood (shown in Cambridge blue) to complete its neighborhood. The stretch between points in the ghost part and describe is obtained with an extrapolation algorithm.

In addition, we used the relative location ξ as a variable to describe the variation function of stretch. And, the function is not defined when $\xi = 0$, $s_{0,i}$ in this figure mean the stretch between the $\mathbf{x}_i$ and its infinitely near point.

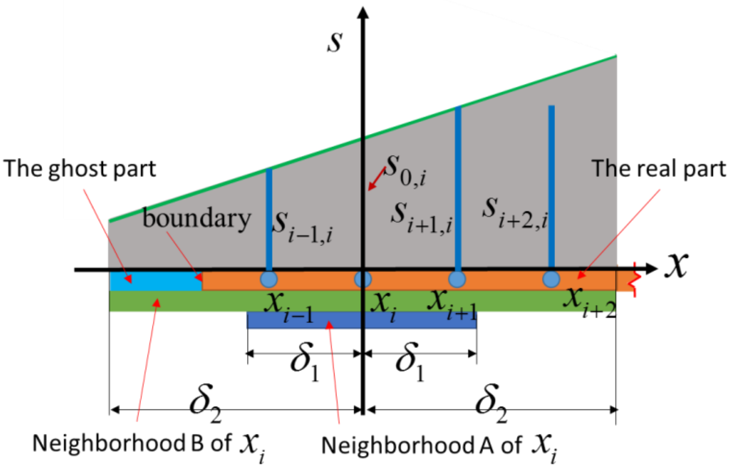


Figure 3: A point with different neighborhoods

The stretch variation function and $s_{0,i}$ is defined by

$$s(\xi)|_{\xi\neq 0} = s_i + (s_{i+1} - s_{i-1}) / (|x_{i+1} - x_{i-1}|) \cdot \xi \tag{10}$$

$$s_i = (|\mathrm{x}_{i-1} - \mathrm{x}_i| \cdot \mathrm{s}_{i-1} + |\mathrm{x}_{i+1} - \mathrm{x}_i| \cdot \mathrm{s}_{i+1}) / (|\mathrm{x}_{i+1} - \mathrm{x}_{i-1}|) \tag{11}$$

The resultant force density is calculated with an integration of all bond force between x_i and all material points in the neighborhood.

The resultant force density under this two different horizon sizes are calculated in the following equations.

$$\begin{aligned} F &= \int_{x_i}^{x_i+\delta_1} f(\xi) A d\xi - \int_{x_i-\delta_1}^{x_i} f(\xi) A d\xi = \int_{x_i}^{x_i+\delta_1} cs(\xi) A d\xi - \int_{x_i-\delta_1}^{x_i} cs(\xi) A d\xi \\ &= cA\delta_1^2 (s_{i+1,i} - s_{i-1,i}) / (x_{i+1} - x_{i-1}) = 2E(s_{i+1,i} - s_{i-1,i}) / (x_{i+1} - x_{i-1}) \end{aligned} \tag{12}$$

$$F=\int_{x_i}^{x_i+\delta_2} f(\xi)Ad\xi-\int_{x_i-\delta_2}^{x_i} f(\xi)Ad\xi=\int_{x_i}^{x_i+\delta_2} cs(\xi)Ad\xi-\int_{x_i-\delta_2}^{x_i} cs(\xi)Ad\xi$$
$$=cA\delta_2^2(s_{i+1,i}-s_{i-1,i})/(x_{i+1}-x_{i-1})=2E(s_{i+1,i}-s_{i-1,i})/(x_{i+1}-x_{i-1}) \quad (13)$$

The resultant force density of x_i got from eq.(12) is considered to be right, because this equation is calculated under a horizon sizes of δ_1 . And the neighborhood of x_i is complete under this horizon size.

Besides, the result of resultant force density is same under this two different horizon size.

Considering this two facts mentioned in the last two paragraph, we can make a conclusion that the boundary effect can be handled with the "ghost neighborhood" method.

In addition, the bond force density on end points is calculated in the classic theory with the assumption that the stretch is constant between an end particle and its adjacent particle. Shown as in Fig.4

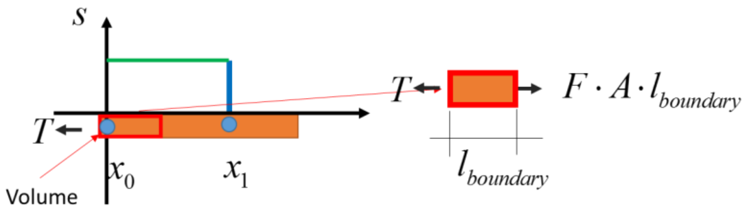


Figure 4: force calculate for points at the end of a bar.

In this figure, $l_{boundary}$ is a characteristic length of the end particle. It will influence the volume of this particle and the extern force density on it. The bond force density and extern force density is given by;

$$F=\frac{E\varepsilon A}{Al_{boundary}}=\frac{EsA}{Al_{boundary}} \quad (14)$$

$$F_{extern}=\frac{T}{Al_{boundary}} \quad (15)$$

Where ε -location of a point in 1D problem

T - extern force effects on the end particle.

3.3 Theoretical Verification

The verification will be done in both static tense of a section bar and free oscillation of it. Firstly, a static model is construct to We start at a pure static tense of a uniform section bar, shown in Figure 5.

The length of the Undeformed bar is L_0 , and it will be L_1 after deformation. Let the elasticity modulus of the bar is E, and the section is A. Using the equilibrium equation for point x_0 (17) and x_i (18), we can get the displacement of the bar (19).

$$F = cA\delta^2 (s_{i+1,i} - s_{i-1,i}) / (x_{i+1} - x_{i-1}) = 0 \tag{16}$$

$$F = \frac{E\varepsilon A}{Adx_{boundary}} = \frac{EsA}{Al_{boundary}} = F_{extern} = \frac{T}{Al_{boundary}} \tag{17}$$

$$u(x) = \frac{T}{EA} x \tag{18}$$

Where F_{extern} is the extern force density.

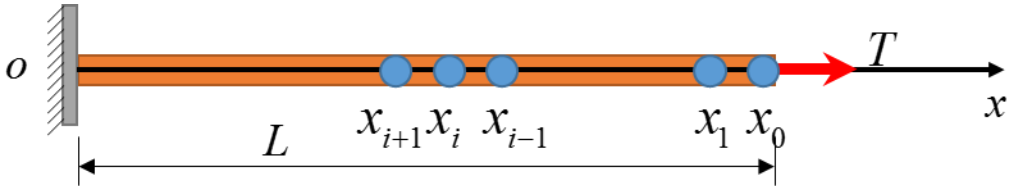


Figure 5: Stress state of a bar

This new method gives a same result with the classic theory under a static pure tension. Then a free oscillation model (as shown in Figure 6) is built to verify the dynamic behavior of this new method.

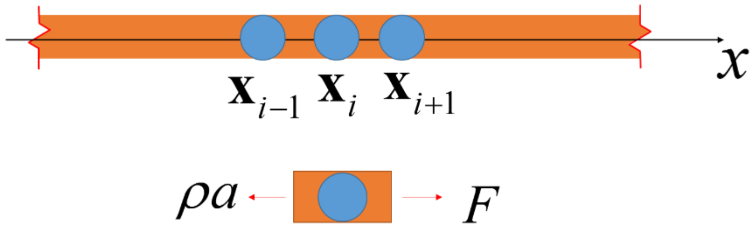


Figure 6: Free Oscillation Of A Bar

Using the equilibrium equation for point p_i , and forcing the distance dx between each adjacent particles to approach zero

$$F = \rho a = cA\delta^2 \frac{s_{i+1} - s_{i-1}}{2dx} = cA\delta^2 \frac{\frac{u_i - u_{i-1}}{dx} - \frac{u_{i+1} - u_i}{dx}}{2dx} \tag{19}$$

When $dx \to 0$, $$F = cA\delta^2 \frac{\frac{u_i - u_{i-1}}{dx} - \frac{u_{i+1} - u_i}{dx}}{2dx} = \frac{cA\delta^2}{2} \frac{\partial^2 u}{\partial x} = E \frac{\partial^2 u}{\partial x} \tag{20}$$

Where ρ is the mass density of the bar. And E is the elasticity modulus of the bar.
The kinematic equation can be obtained from equation (20)

$$E\frac{\partial^2 u}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2} \tag{21}$$

Which is same to the result in classic theory as well.
This two verification state that this new method can give a correct solution in both static and dynamic problems

3.4 Numerical Verification

The numerical is based on the tension of a bar shown in Figure 7. A uniform tension force $f = 100 / m^2 / m$ is applied on the same bar. The length L of the bar is set to be 0.9m, and the elasticity modulus is 100pa. One end point is fixed.

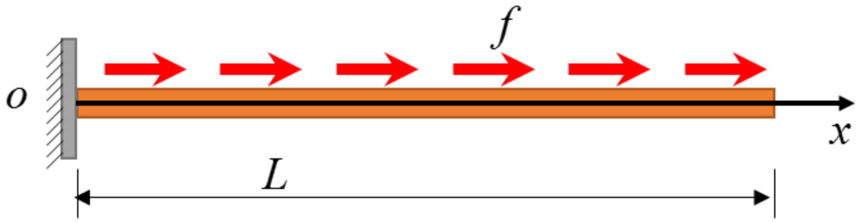


Figure 7: Deformation of a bar with different horizons

The calculation was carried out with all parameters held constant, but for different discrete particles, the deformation is given under five discrete particle numbers $nx = 10, 20, 30, 40$ *and* 100. The displacement of all points of the bar is shown in Figure 8. Generally, the "ghost neighborhood" method gives a good result of the displacement, and the displacement curve gets closer to the theory displacement curve. We then used a displacement of the end point as a measurement of errors. This errors and displacements under different discrete particles, as well as the theory is shown in Figure 9. As shown in this figure, the error declines significantly when the particle number increase and is tending to stability when the particle number are large enough. In addition, the error becomes less than 1% when the number reaches 100.

With the result above, we can make a conclusion that the "ghost neighborhood" method is an effective way to handle the boundary effect.

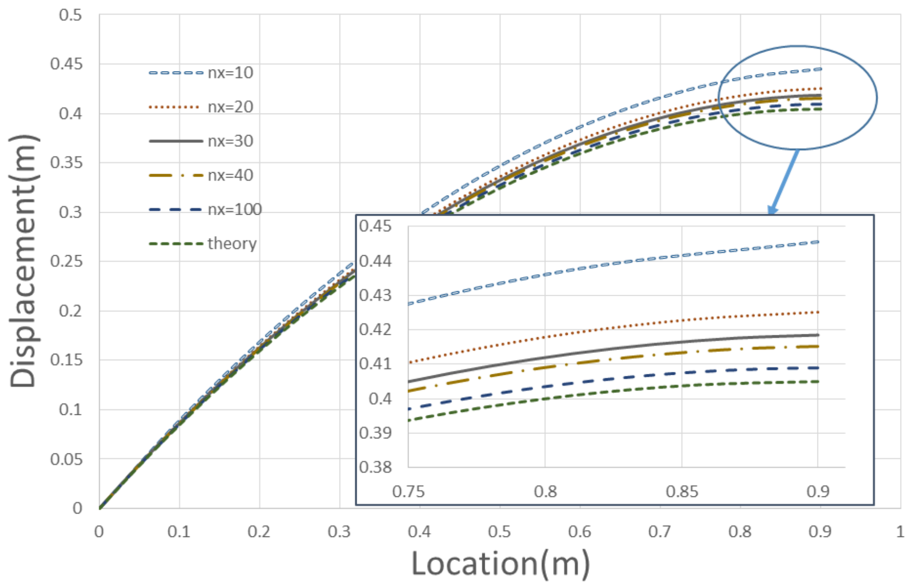


Figure 8: Deformation of a bar under different discrete particles

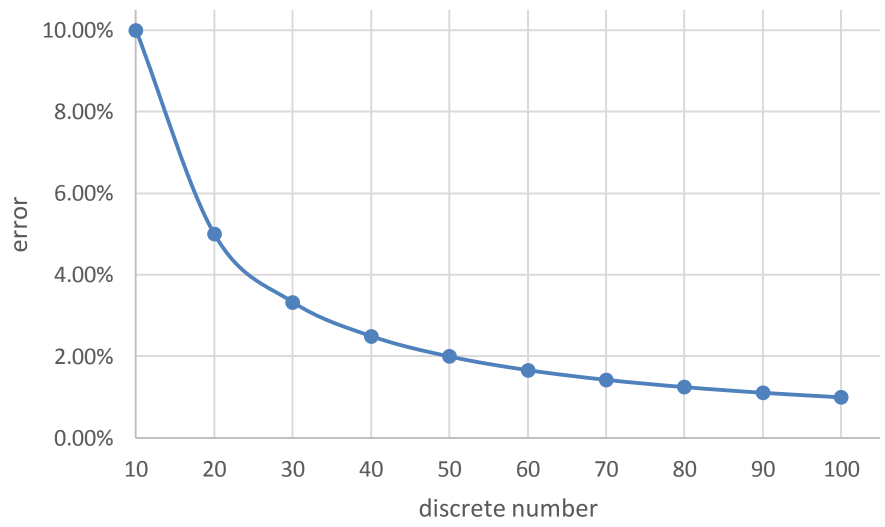


Figure 9: End Point Displacement And The Error Variation

4 DISCUSSION AND CONCLUSION

A correct method of bond force density calculation is advanced in this paper which is independent of horizons. So this correct method can deal with arbitrarily deformation.

The ghost neighborhood, based on the independence of horizons, is proposed later. The theoretical verification and numerical verification indicated that this method gives a very good result of 1D problems, with a significantly declined error when particle number increases.

Furthermore, this ghost neighborhood, designed for handling boundary effect, can deal with those interface problems with a little transform.

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