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FLOW UNDER RETAINING STRUCTURES: A NEW APPLICATION OF NETWORK METHOD

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Abstract. The network method is here applied with sufficient precision and negligible computing time, to simulate seepage flow on permeable soils located under concrete dams. Differential equations of the problem are spatially discretized and implemented by a network model. The numerical solution is provided by standard electric circuit simulation codes. Programming routines using Matlab allow graphical solutions in terms of streamflow and potential iso-lines. Seepage loss is evaluated for isotropic and anisotropic soils as well as under the influence of sheet piling scenarios.

1 INTRODUCTION

Steady state groundwater flow under concrete dams, weirs founded and cofferdams on non-homogeneous, anisotropic and permeable soils, is governed by Laplace equation in terms of the total head (or piezometric level) variable. The relation between the soil and the percolated water is important in the design of foundations and failures due to piping because of excess pressure of water.

Therefore, understanding the hydraulic conditions is important to design structures correctly [1,2]. In these scenarios, water flows and pore pressure changes adjust very rapidly reaching steady state conditions nearly instantaneously.

In most cases, the geometry of the typical scenarios is a rectangular 2-D domain, which upper layer is separated in two lateral sides by the dam, with impermeable vertical wall at the foot of the dam. Although the boundary conditions are generally of the first (Dirichlet) or second (homogeneous Neumann) type, the complex semianalytical solution is formed by mathematical series of slow convergence. Flow of water through earth masses is in most of cases three dimensional, but it is too complicated and flow problems are usually solved on acceptance that the flow is two-dimensional. Thus, flow lines are parallel to the plane of the structure.

Only a small number of seepage problems have been solved analytically. They have a lot of difficulties due to the boundary conditions of the flow equation that cannot be satisfied in all cases. Harr [3] has solved some simple hydraulic structures and Mandel [4] developed a conformal mapping technique to solve analytically seepage related to excavations and cofferdams.

As an alternative, civil engineers make used of graphical solutions based on the so called flow net construction. The flow net represents two orthogonal families of curves, the flow lines (a line along which particle travel from upstream to downstream) and the equipotential lines (a line along that join points have the same potential head). The equipotential lines intersect flow lines at right angles. Calculation of seepage from a flow net can be tedious and require a lot of time. The main unknowns of interest reached are the steady state seepage loss, the upthrust on the base of the dam and the maximum exit hydraulic gradient. Standard commercial codes can also be used for the numerical solution.

The present work investigates a model, based on network method and seepage theory, capable of solving these problems with sufficient accuracy and negligible computing time, using a standard circuit simulation code such as Pspice [5]. Network simulation method [6] is a numerical tool widely used for the solution of non-lineal, coupled or uncoupled problems, in many engineering fields such as heat transfer, tribology, corrosion, elastostatic and vibrations [7-9]. The method goes beyond the scope of classical electric analogy that is currently used in many text books of different engineering fields, particularly in heat transfer, since it is capable of working with non-lineal and coupled problems type. For the first time, it is applied in the field of geosciences in this work; particularly to groundwater.

The proposed model uses as dependent variable the piezometric head (h), related to the saturated water flow through the Darcy´s equation. This lineal relation allows of deriving the value of the four resistors that form the network model of the elementary cell or volume element. Flow conservation is directly assumed by the Kirchhoff´s theorem referred to the conservation of electric current in the network. Once solved the state steady field h(x,y), programming routines of MATLAB [10] provides the flow lines or the field of values of the streamfunction, $\psi(x,y)$. The representation of suitable flow nets from the solution, with an arbitrary number of iso-lines for each variable, provides a clear vision of the flow and piezometric head distribution through the domain. Applications to isotropic and anisotropic soils are presented.

2 PHYSICAL AND MATHEMATICAL MODELS

The basic problem-scheme is presented at the left of Figure 1, while physical representation is shown at the right. A concrete structure is confined between two finite regions with of different piezometrichead that causes water flows underground from the larger water level (left) to the lower level (right).

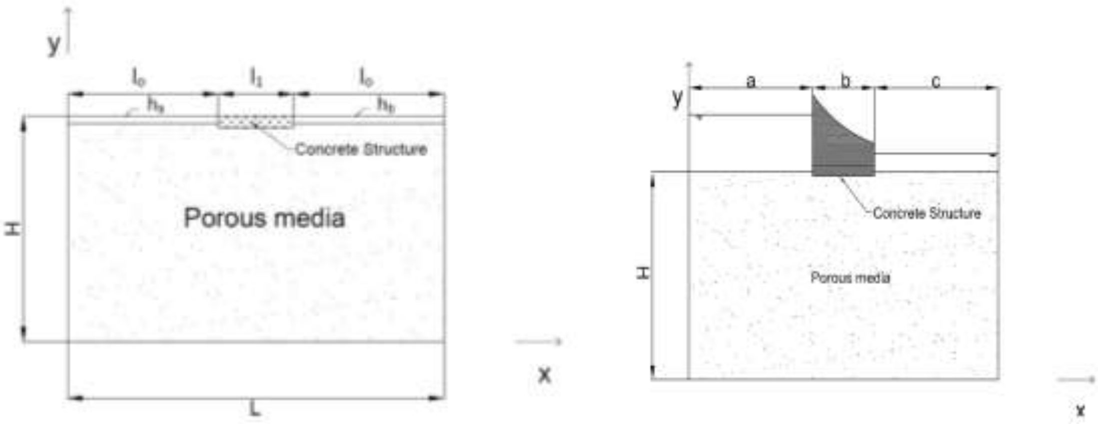

Figure 1: Scheme (left) and physical (right) representations of the problem

Under the hypothesis of incompressible fluid and no volume changes in the soil, the governing equation is given by that of Laplace; assuming an anisotropic hydraulic conductivity (permeability), this is written as

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \qquad (1)$$

while the boundary conditions are described by the equations:

$$\begin{aligned}
&(y=H,\ 0<x<l_0) && h=h_a \qquad (2)\\
&(y=H,\ l_0\le x\le l_1) && \frac{\partial h}{\partial y}=0\\
&(y=H,\ l_0+l_1\le x\le L) && h=h_b\\
&(x=0,\ y)\ \text{ and }\ (x=L,\ y) && \frac{\partial h}{\partial x}=0\\
&(y=0,\ x) && \frac{\partial h}{\partial y}=0
\end{aligned}$$

where k_x and k_y are the horizontal and vertical permeability, respectively, and h the piezometric level; h_a and h_b denote the values of Dirichlet conditions applied to the left and right sides of the dam. The streamfunction variable (ψ), whose iso-lines show the flow paths of the fluid particles, is defined as

$$v_x = \frac{\partial \psi}{\partial z} \quad \text{and} \quad v_y = -\frac{\partial \psi}{\partial x} \quad , \qquad (3)$$

3 THE NETWORK MODEL

This, based on the electro-hydraulic analogy, establishes a formal equivalence between the variables piezometric level (in the real process) and electric potential (in the network model), as well as between the variables water flow and electric current. The design of the network starts from Darcy equation

$$\mathbf{j}_w = -k(\nabla h) \qquad (4)$$

which relates the water flow density with the gradient of the piezometric head through the permeability parameter.

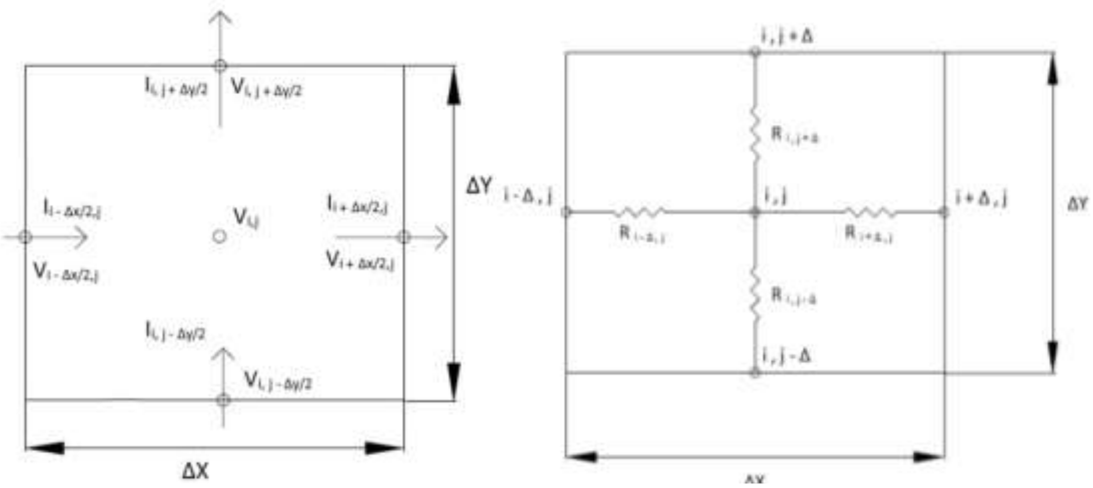

Figure 2: Nomenclature (left) and network model (right) of a volume element or cell

Assuming a 2-D flow and the nomenclature of Figure 2 (left), equation (4) expressed in finite-difference form allows of implementing the network model for the volume element of the anisotropic domain. Four resistors of value

$$R_{i-\Delta,j} = R_{i+\Delta,j} = (\Delta x/2k_x); \qquad R_{i,j-\Delta} = R_{i,j+\Delta} = (\Delta y/2k_y) \tag{5}$$

are symmetrically distributed and connected to the centre of their respective sides. Doing this, the electric current conservation makes that Laplace equation (1) be satisfied: electric current of each branch is balanced with other in a common node. Connecting $N_x \times N_y$ volume elements with ideal electric contacts and adding the boundary with suitable electric components, the complete network model is formed. Dirichlet condition (constant head) is implemented by simple constant voltage generator while homogeneous Neumann condition (impermeable flow) is simply implemented by a resistor of very high value (theoretically infinite). Once the whole network model is designed, it is run in the circuit simulation code Pspice.

Making use of equation (3), the problem may be formulated in terms of the dependent variable ψ with the governing equation

$$k_x \frac{\partial^2 \psi}{\partial x^2} + k_y \frac{\partial^2 \psi}{\partial z^2} = 0 \tag{6}$$

and boundary conditions given by

$$\begin{array}{ll} (y=H,\ 0<x<l_0 \text{ and } l_0+l_1 \leq x \leq L) & \frac{\partial \psi}{\partial x} = 0 \\ (y=H,\ l_0 \leq x \leq l_1) & \psi = \psi_0 \\ (x=0, y),\ (x=L, y) \text{ and } (x, y=0) & \psi = \psi_1 \end{array} \tag{7}$$

These equations are quite equivalent to those (1) and (2) that use the variable h, except for the boundary conditions which are the reverse. In this way, the network model for equations (6) and (7) is also that of Figure 2 (right). For the particular solution related to values h_a and h_b, ψ_0 and ψ_1 must be given; since the reference ψ_0 is arbitrary, we can fix $\psi_0 = 0$ and derive ψ_1 from equation (4) and the solution h(x,y) provides from the simulation of the network model of equations (1) and (2). In this form, the solution of both equivalent (conjugate) problems is compatible and the iso-lines of h and ψ represented at a same domain. Doing this for a same number of lines, both for isotropic and anisotropic media, reliable flow nets can be graphed and contrasted with hand-made nets. From these, the steady state seepage loss and the upthrust on the base of the dam can be exactly known.

4 APLICATIONS

The dam section is shown in Figure 1. The width of the dam is b=10 m, the hydraulic conductivity of the permeable layer in the vertical and horizontal directions is 10^{-6} m/s. Two cases are chosen (Table 1) having different boundaries conditions. In the first (Case 1) the width of study is 50 m while, in the second (Case 2), it is 20 m.

Table 1: Phisical and geometrical parameters, and total rate of seepage through the permeable layer per unit length (isotropic soil and diferent lengths upstream and downstream)

	Permebility	(Kx/Ky)	H(m)	a(m)	b(m)	c(m)	$Q(m^3/s)$
Case 1	1,00E-06	1	10	20	10	20	5,27E-06
Case 2	1,00E-06	1	10	5	10	5	4,56E-06

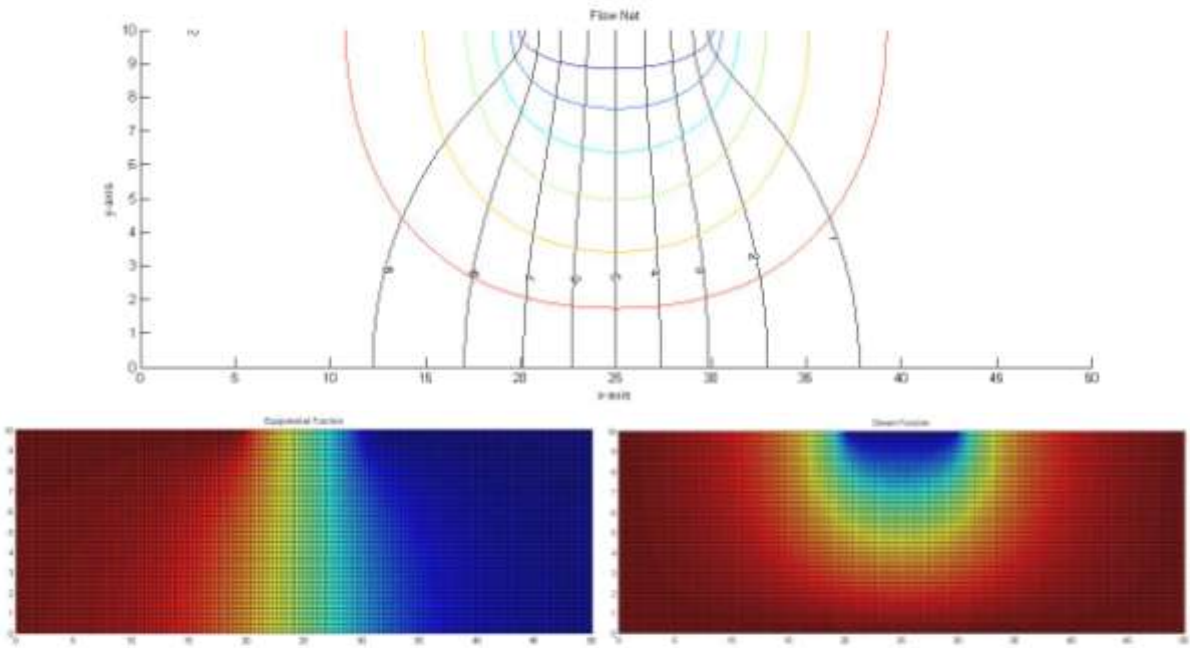

Figure 3: Flownet construction with equipotential and flow lines (up), equipotential and stream functions (down). Case 1

Simulation results are shown in Figures 3, for Case 1, and 4, for Case 2. Equipotential and flow lines are simultaneously represented in the upper graph of each figure for a better appreciation of the solution. Matlab routines illustrate equipotential and stream functions, separately, by color maps. Total seepage losses, read directly in the simulation, are given in Tables 1 and 2 for all cases.

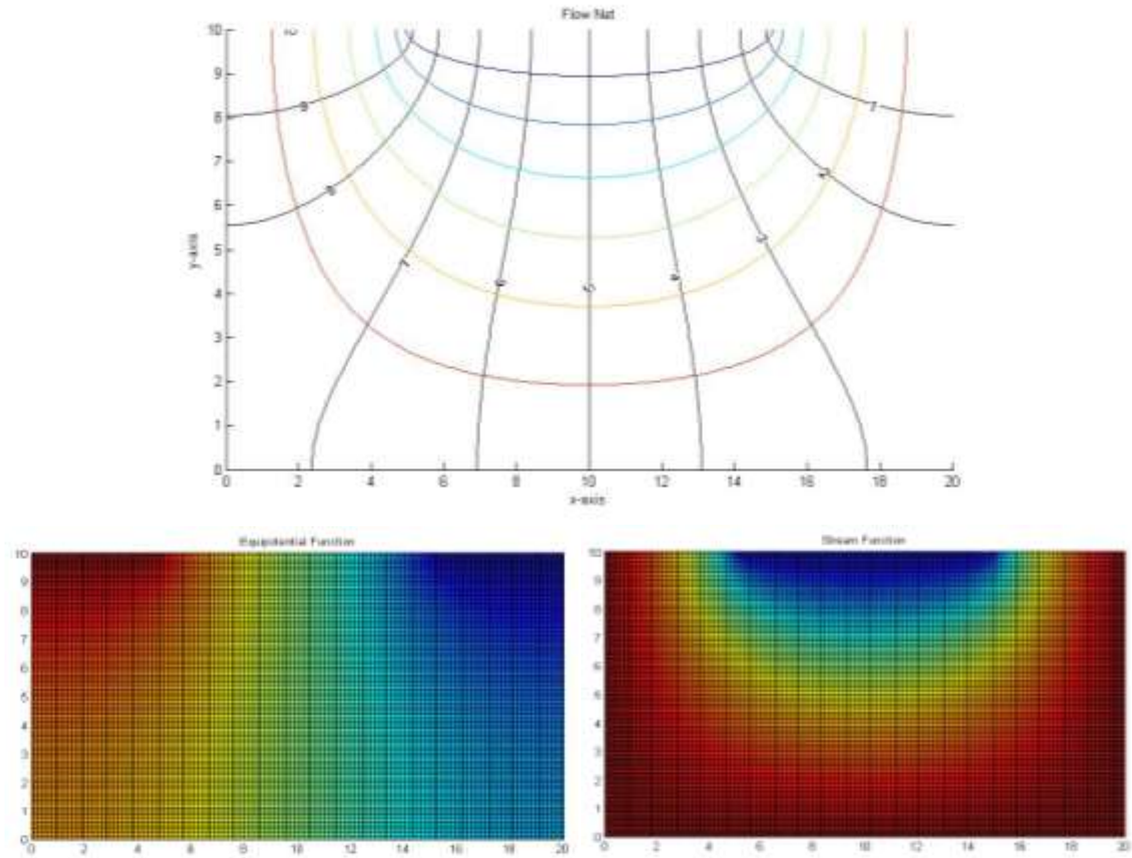

Figure 4: Flownet construction with equipotential and flow lines (up), equipotential and stream functions (down). Case 2

For the next application, permeability of the layer in the vertical and horizontal directions are10^{-6} m/s and 10^{-7} m/s, respectively, for anisotropic case, and 10^{-6} m/s for isotropic. The first case, which has no sheet pile, is separated in Case 1I (for isotropic medium) and Case 1A for anisotropic. Case 3I and 3A have a sheet pile of 3 m length (d) and the Case 4I and 4A have a sheet pile of 7 m.

Simulation results for these cases are given in Figures 5 to 8, for Cases 3I, 3A, 4I and 4A, respectively. Again, in the upper graphs of these figures equipotential and flow lines (up) are drawn together, while in the upper graphs color maps are used for each variable. It is important to note that, as expected, equipotential and flow lines cross each other forming a 90º angle when the medium is isotropic; a condition that is not satisfied for anisotropic medium.

Table 2: Phisical and geometrical parameters, andtotal rate of seepage through the permeable layer per unit length (isotropic and anisotropicsoil, and diferents length of sheet pilling)

	(Kx/Ky)	H(m)	a(m)	b(m)	c(m)	d(m)	Q(m³/s)
P1I	1	10	20	10	20	0	5,27E-06
P1A	10	10	20	10	20	0	2,40E-06
P3I	1	10	20	10	20	3	4,15E-06
P3A	10	10	20	10	20	3	1,56E-06
P4I	1	10	20	10	20	7	2,78E-06
P4A	10	10	20	10	20	7	9,18E-07

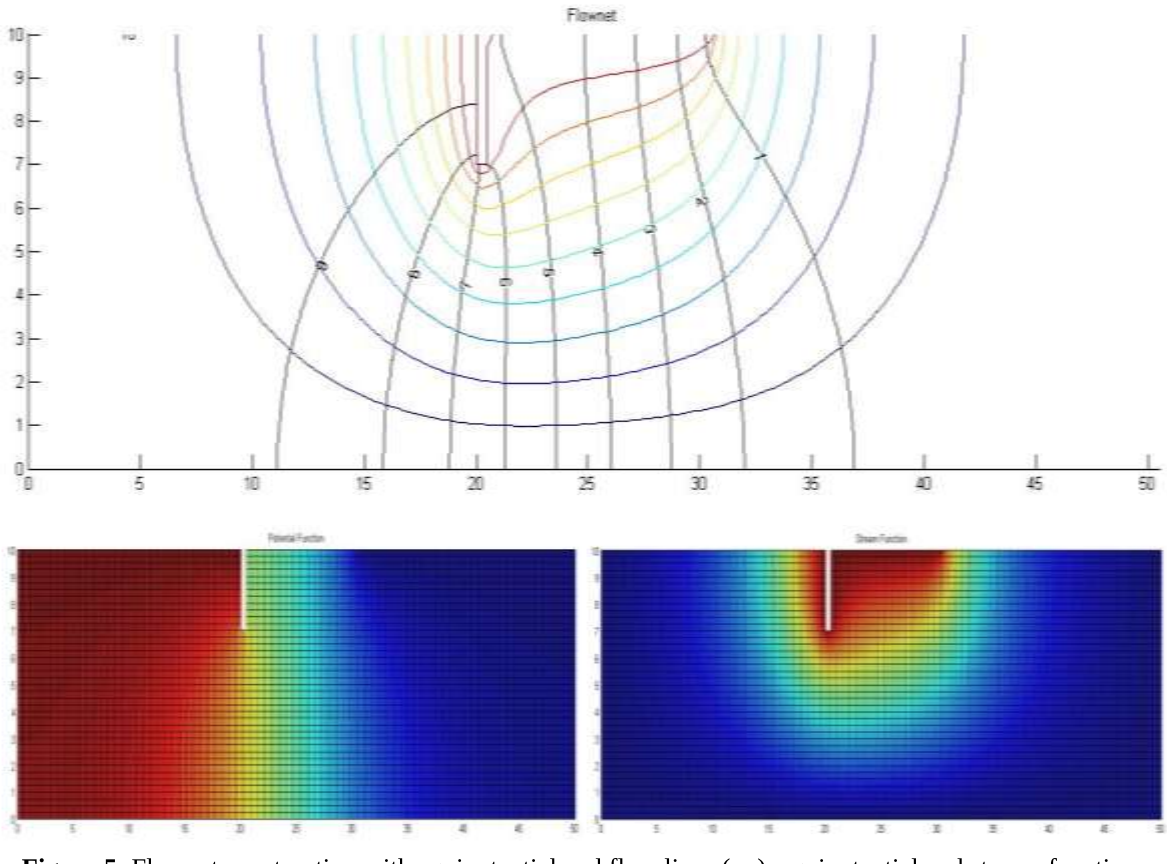

Figure 5: Flownet construction with equipotential and flow lines (up), equipotential and stream functions (down). Case 3I

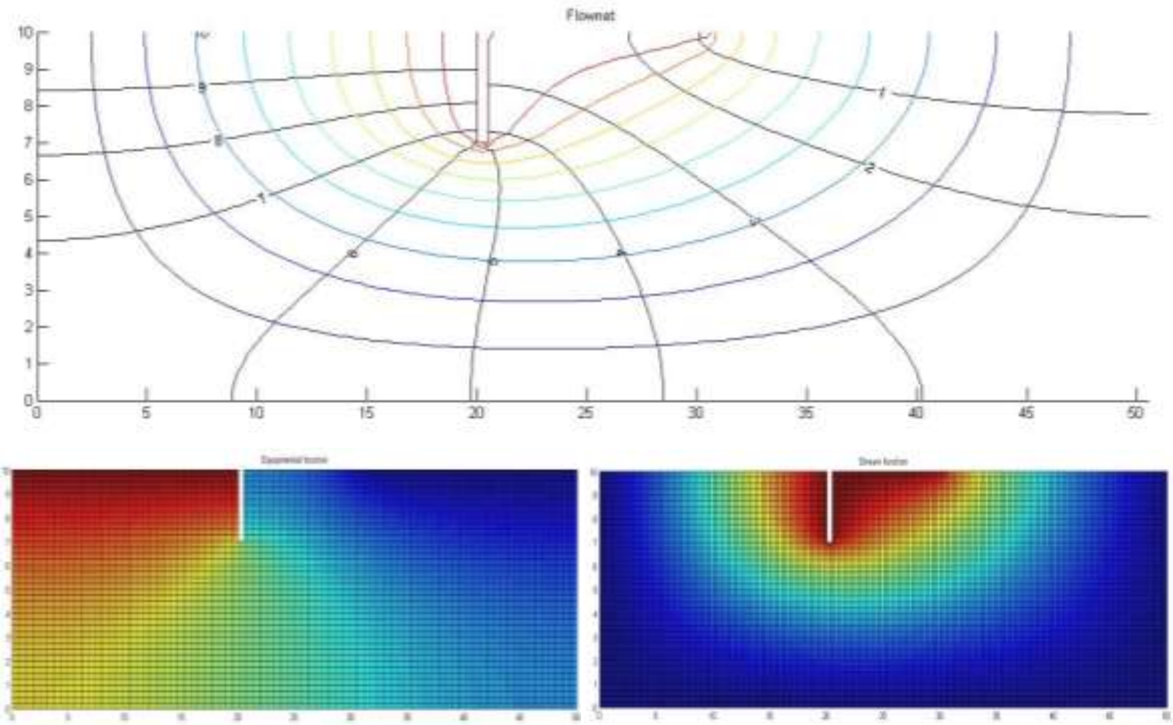

Figure 6: Flownet construction with equipotential and flow lines (up), equipotential and stream functions (down). Case 3A

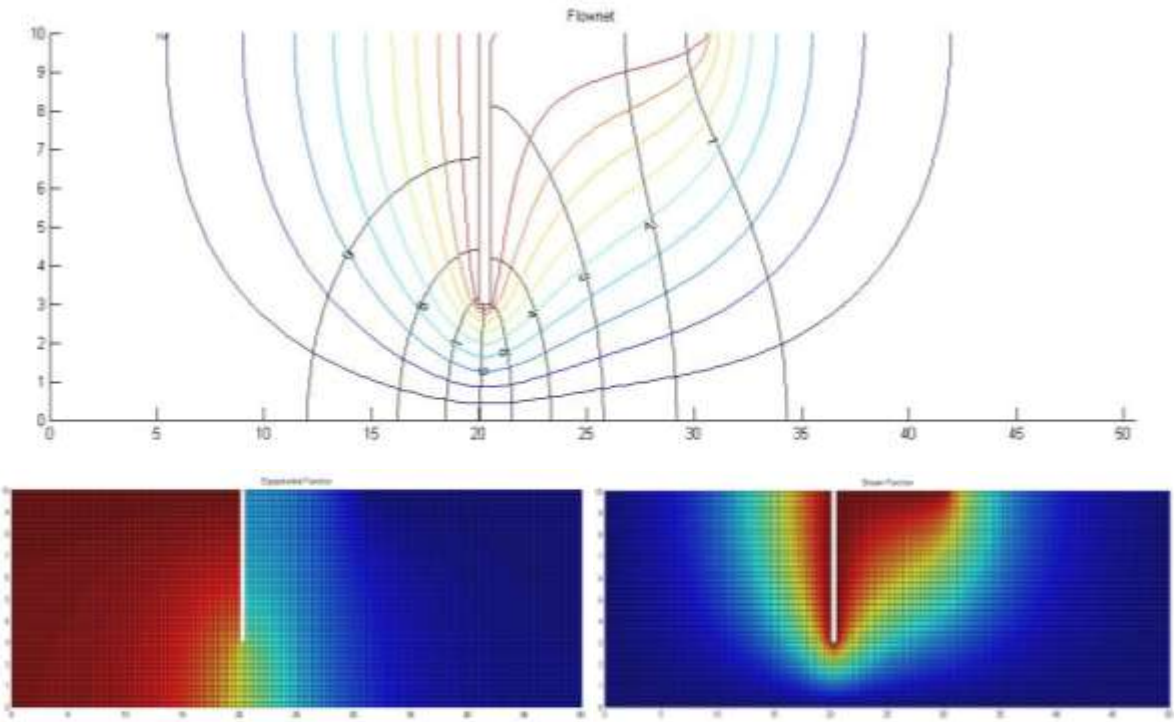

Figure 7: Flownet construction with equipotential and flow lines (up), equipotential and stream functions (down). Case 4I

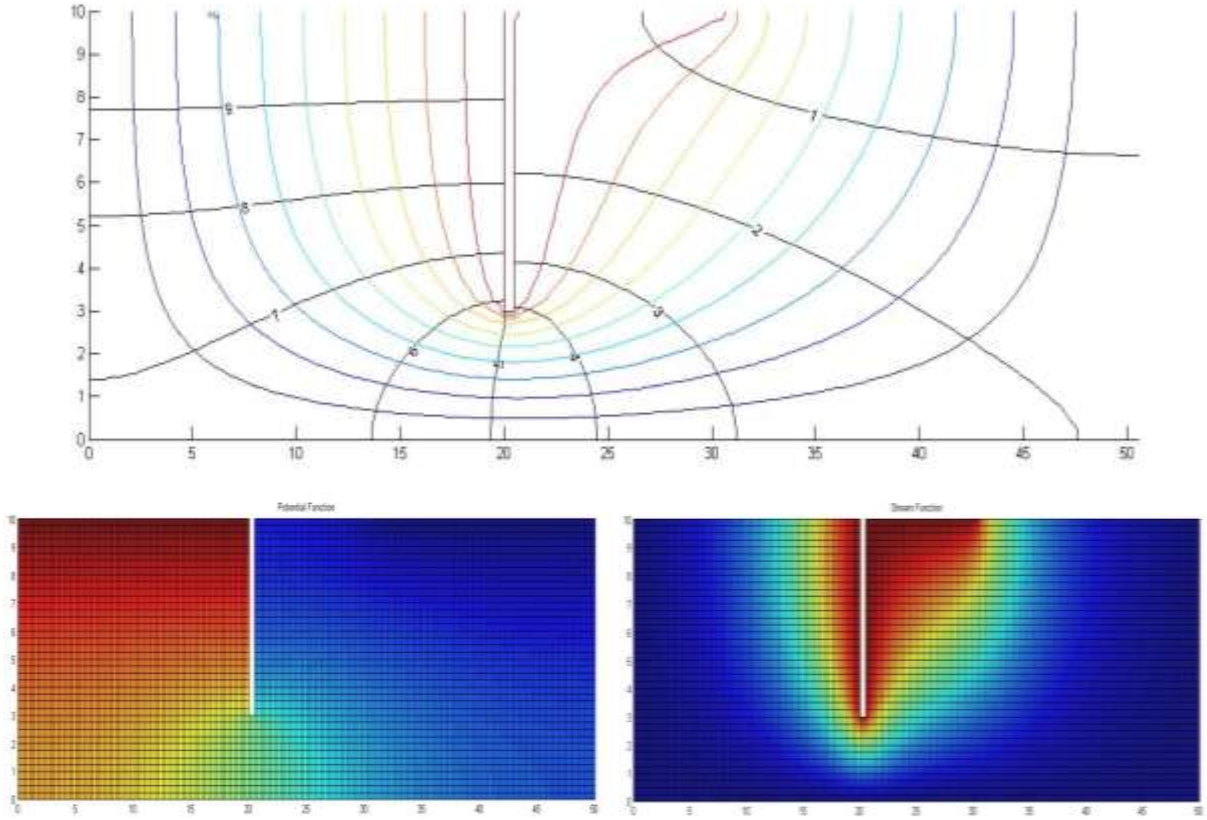

Figure 8: Flownet construction with equipotential and flow lines (up), equipotential and stream functions (down). Case 4A

CONCLUSIONS

- For the first time, network method has been successfully applied to the solution of seepage flow in isotropic and anisotropic media with or without sheetpile. Simulations report seepage losses and graphical 2-D representations of the main unknowns, streamfunction and piezometric head.
- The simulation of the model in conjunction with programming subroutines in Matlab, allow us to make easier and faster the construction of flownet.

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