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DYNAMIC RESPONSE OF THE DAMPING PAD FLOATING SLAB TRACK CAUSED BY VEHICLE-TRACK INTERACTION

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Abstract: The paper investigates the dynamic response of the damping pad floating slab systems of railway under the action of vehicle. A vehicle-slab-track model is presented in this paper, which consists of the vehicle and floating slab track subsystem. The vehicle is modeled as a multi-body system, and the track supported by the rail-pads as an Euler beam supported by uniformly distributed springs, and the float slab with the damping pad as a beam with free ends resting on an elastic foundation. The running safety of vehicles on the floating slab track at various train speeds is examined. The resonance mechanism and conditions of vehicle-track system are investigated through theoretical derivations and numerical simulations.

Keywords: *dynamic response; floating slab track; resonance; vehicle*

1 INTRODUCTION

The damping pad floating slab track has excellent performance for reducing track vibration. Now it is widely in service for metro and railway in China, Europe, USA and Japan. Floating slab track basically consists of concrete slabs supported on resilient elements such as rubber bearings. For the damping pad floating slab track, additional dynamic loads are induced at wheel-rail interface due deformation of slab while the train passes.

Therefore, great efforts have been attached to this subject in recent years. Zhai et al (1999) analyzed the effects of track irregularities and the elasticity and damping of cement asphalt mortar under the slab on system dynamics. Steenbergen et al (2007) carried out a parametric study on the slab track on soft soil from a dynamic viewpoint. Yau et al(2002) analyzed the dynamic response of the track and the contact forces between the wheels and track caused by a series of sprung masses Moreover, the vibration problem of floating slab track also attracts lots of attentions (Cui and Chew, 2000; Lombaert et al, 2006; Hussein et al, 2006; Kuo et al, 2008)

This paper focuses on dynamic response of the damping pad floating slab systems of railway under the action of vehicle. The running safety of vehicles on the floating slab track at various train speeds is examined. The resonance mechanism and conditions of vehicle-track system are investigated through theoretical derivations and numerical simulations.

2 ANALYSIS MODEL

The analysis model consists of vehicle subsystem and track subsystem. The vehicle subsystem is modeled as a multi-body system with 10 degrees-of-freedoms (DOFs) running on the track with a constant velocity. The track subsystem comprises the rail and floating slabs, in which the rail is considered as Euler beam with discrete continuous elastic supports resting on slabs, and the slabs are regarded as Euler beams with free ends on continuous supporting foundation, as shown in Figure 1. The properties of vehicle and track are shown in Table 1.

Table 1. Parameters in analysis

Item & Notation	Value	Unit
Mass of car body (M_c)	5.36E+4/2	kg
Mass of bogie (M_t)	3200/2	kg
Mass of wheelset (M_w)	2400/2	kg
Pitch moment of inertia of car body (J_c)	2.7E+6/2	kg.m^2
Pitch moment of inertia of bogie (J_t)	7200/2	kg.m^2
Half length between truck centers (l_c)	8.5	m
Half of wheelbase (l_t)	1.25	m
Vertical secondary stiffness (K_{2z})	4E+5	N/m
Vertical secondary damping (C_{2z})	5E+4	N·s/m
Vertical primary stiffness (K_{1z})	1.04E+6	N/m
Vertical primary damping (C_{1z})	4E+4	N·s/m
Rail bending stiffness (E_r)	2.059E11	N/m^2
Rail bending moment of inertia(I_r)	3.217E-5	m^4
Mass of rail (m_r)	60.8	kg/m
Rail pad spacing (l_p)	0.6	m
Stiffness of rail pad (K_p)	5.0E7	N/m
damping of rail pad (C_p)	7.5E4	N·s/m
Slab bending stiffness (E_s)	3.5E10	N/m^2
Slab bending moment of inertia (I_s)	4.05E-3	m^4
Mass of slab (M_s)	2.5E3	kg
Slab length (L_s)	12	m
Stiffness of slab bearing (K_s)	6.5E6	N/m
Damping of slab bearing (C_s)	7.5E4	N·s/m

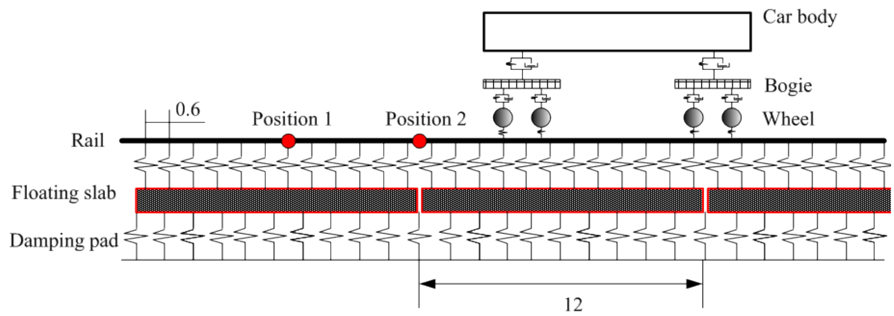

Figure 1. Model of vehicle on slab track

The equations of motion for vehicles can be obtained according to the D'Alembert's principle. The Equations of vehicle are written in reference [1].

The rail is described as Euler beam on a discrete-continuous elastic supports. According to

The control equation of rail is

$$\ddot{q}_k(t)+\omega_k^2 q_k(t)=-\sum_{i=1}^{N}F_{rsi}(t)Y_k(x_i)+\sum_{j=1}^{4}P_{wj}(t)Y_k(x_{wj}) \tag{1}$$

With

$$\omega_k=-\left(k^2\pi^2/l_r^2\right)\sqrt{E_r I_r/m_r} \tag{2}$$

$$F_{rsi}(t)=K_p\left[z_r(x_i,t)-z_s(x_i,t)\right]+C_p\left[\dot{z}_r(x_i,t)-\dot{z}_s(x_i,t)\right] \tag{3}$$

Where F_{rsi} is the ith rail seat force, $z_r(x_i,t)$ is vertical displacement of rail at position of the ith rail seat, $z_s(x_i,t)$ is vertical displacement of slab at position of the ith rail seat, $P_{wj}(t)$ is force between rail and wheel, the force can be expressed as follows:

$$P_{wj}(t)=\frac{1}{G}\left[z_{wj}(t)-z_r(x_{wj},t)\right]^{3/2} \tag{4}$$

$$G=3.86R^{-0.115}\times 10^{-8} \tag{5}$$

Where $z_{wj}(t)$ is vertical displacement of the jth wheel, $z_r(x_{wj},t)$ is vertical displacement of rail at position of the jth wheel, R is radius of wheel.

The deflections of rail $z_r(x,t)$ are then calculated with superposition of M terms of modes.

$$Z_r(x,t)=\sum_{k=1}^{M}Y_k(x)q_k(t) \tag{6}$$

$$Y_k(x)=\sqrt{\frac{2}{m_r l_r}}\sin\left(\frac{k\pi x}{l_r}\right) \tag{7}$$

The slabs are modeled as Euler beams with free ends on continuous supporting foundation. the control equation of slab becomes the form as shown below:

$$\ddot{T}_n(t)+\frac{c_s l_s}{m_s}\dot{T}_n(t)+\frac{k_s+E_s I_s\beta_n^4}{m_s}l_s T_n(t)=\sum_{i=1}^{N}\frac{F_{rsi}(t)}{m_s}x_n(x_i) \tag{8}$$

The vertical deflections of slabs $z_s(x,t)$ are calculated with superposition of NS terms of modes.

$$Z_s(x,t)=\sum_{n=1}^{NS}x_n(x)T_n(t) \tag{9}$$

with

$$\begin{cases} x_1=1 \\ x_2=\sqrt{3}\left(1-2x/l_s\right) \\ x_m=(\cosh\beta_m x+\cos\beta_m x)-c_m(\sinh\beta_m x+\sin\beta_m x)\left(m>2\right) \end{cases}$$

c_m, β_m are constant value which shown in Table 2.

Table 2. constant value of Euler beams with free ends

m	1	2	3	4	5	⩾6
c_m	-	-	0.983	1.001	0.100	1.000
$\beta_m l_s$	0	0	4.720	7.853	10.996	(2m-3) π/2

The system equations of vehicle and track can be solved by Newmark-β method which is used in this study.

3 LOADED TRACK RESONANT SPEED

The loaded track frequency $f_{w/t}$ of a coupled wheel-track system on an elastic foundation of uniform stiffness can be approximately estimated by (Luo et al, 1996; Dong, et al, 1994)

$$\begin{aligned} f_{w/t} &= \frac{1}{2\pi}\sqrt{\frac{k_{tr}}{m_{tr}+m_w}} \\ k_{tr} &= 2\sqrt[4]{4EI\times k_f^3} \\ m_{tr} &= 3m\times\sqrt[3]{\frac{EI}{k_{tr}}} \end{aligned} \tag{10}$$

Where k_{tr} and m_{tr} are the effective stiffness and mass respectively, and k_f is the equivalent stiffness per unit length of the elastic foundation.

For a moving wheel-load system with constant speed V travelling along a track supported by discrete rail-pads of constant intervals, the excitation frequency f_{ext} to the wheel-load system due to the discrete rail-pads is

$$f_{ext} = \frac{V}{L_e} \tag{11}$$

where L_{e} is the effective spacing between two adjacent rail-pads.

When the excitation frequency equals the loaded track frequency, resonance can be excited between the bogies and the rails. The resonant speed V_{res} can be solved as

$$V_{res} = \frac{L_e}{2\pi}\sqrt{\frac{k_{tr}}{m_{tr}+m_w}} \tag{12}$$

It is expected that under the condition of resonance, the response of the track will be built up as there are more vehicles passing the track.

4 RESONANT ANALYSES

To simplify the vehicle-track model used for the vehicle-track resonance phenomena, a vehicle is model as a sequence of moving sprung masses sustaining a concentrated load lumped from the weight of the car body, as shown in Figure 2. No consideration is made of rail irregularities.

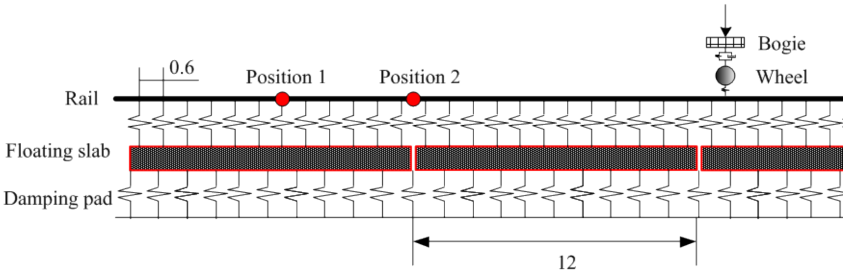

Figure 2. Model of moving sprung mass on slab track

The displacement of rail solved and the increase rate of the displacement have been plotted with respect to the train speed in Figures 3. As can be seen, there exist multiple resonant peaks for the rail displacement. This is mainly due to the coincidence of some of the excitation frequency implied by the moving sprung mass model at different speeds with the coupled frequency of the wheel/track/rail-pad system.

Based on the parameters of section 2 and equation (12), the resonant speeds can be computed as 108, 216, 324 km/h. 108km/h is consistent with the resonant peak shown in

Figures 3. Other resonant peaks for the rail displacement don't completely correspond with the resonant speed in section 2.This is because the model take nonlinear contact force between wheel and rail into consideration and the nonlinear contact leads to separation between the wheels and the track.

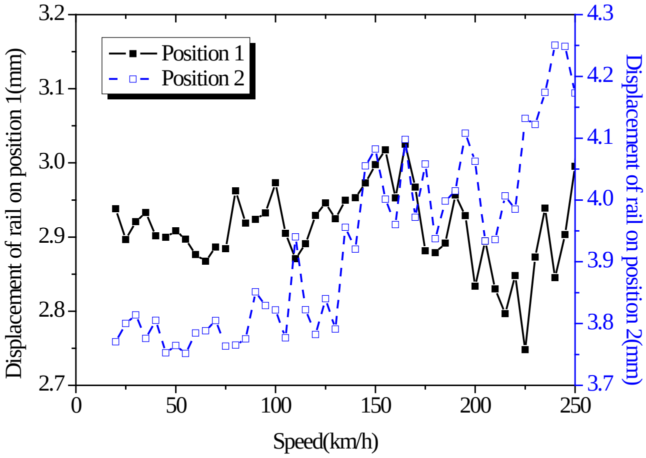

Figure 3. Increase of rail displacemt based on moving sprung mass model

5 DYNAMIC RESPONSE OF VEHICLE-TRACK SYSTEM

Analyses are conducted with the model of vehicle-floating slab track. Figures 4 to 7 show the track dynamic responses at the running speed from 20 to 300 km/h. It is illustrated that the dynamic response increases with the speed increases. Displacement and acceleration of rail at joint of slabs is greater than those of other position. From figure 8, the contact force between wheels and rail increases obviously when the vehicle passes the slab joint. We can conclude that the soft damping pad result in enlarging the dynamic response at joint of slabs. The acceleration of vehicle is almost unaffected at 20~150km/h, while at higher moving speed, the acceleration of vehicle increase obviously.

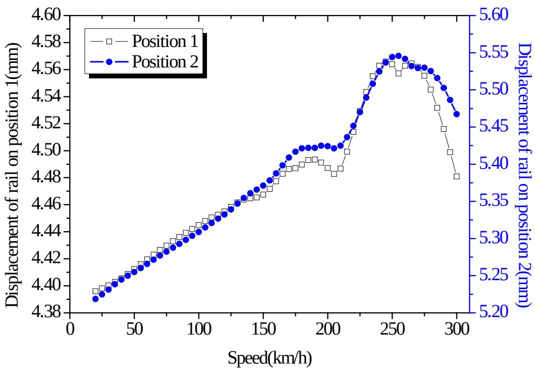

Figure 4. Increase of rail displacemt based on vehicle-track model

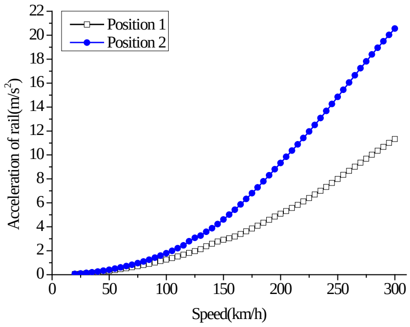

Figure 5. Increase of rail acceleration based on vehicle-track model

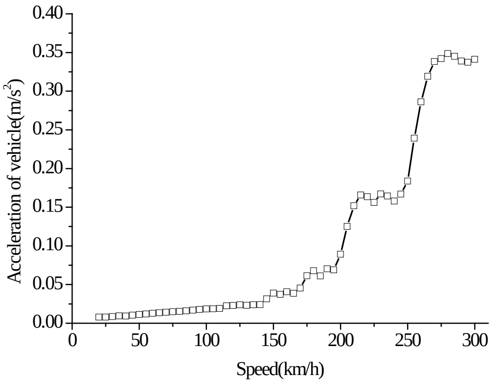

Figure 6. Increase of vehicle acceleration based on vehicle-track model

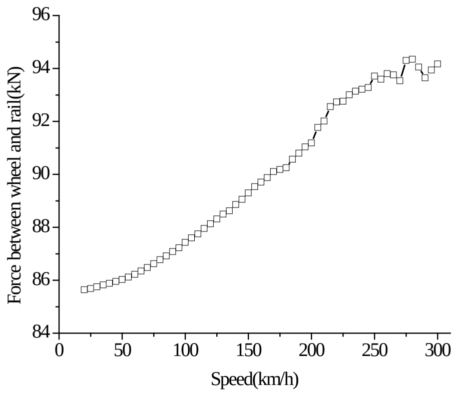

Figure 7. Increase of contact force based on vehicle-track model

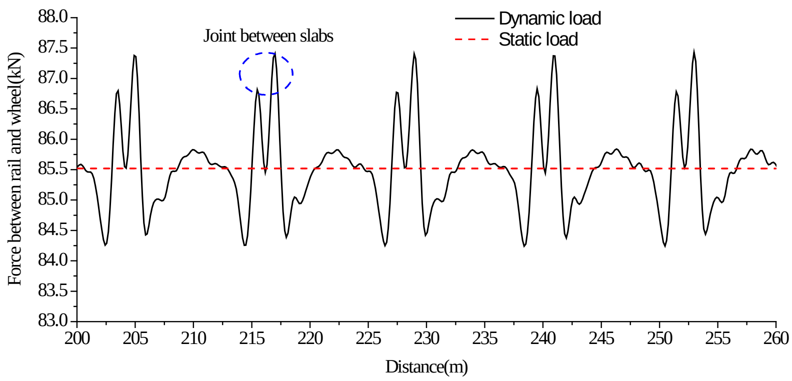

Figure 8. The time history of wheel-rail force

6 CONCLUSIONS

A vehicle-slab-track model has been established in the paper, the dynamic response of the damping pad floating slab track and vehicle, the resonance vibration are investigated. The numerical results indicate that there exist multiple resonant peaks for rail displacement of the floating slab track system, due to coincidence of the inherent frequencies of the constituting subsystems. Dynamic responses of rail at joint of slabs are greater than those of other position. Proper design and analysis are needed to reduce vibration of slab joint

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