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REVIEW OF REDUCTION METHODS BASED ON MODAL PROJECTION FOR HIGHLY DAMPED STRUCTURES

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Abstract. Constrained viscoelastic layer is a traditional way of damping the vibrations of a structure. In order to maximise the performances of a viscoelastic treatment, parametric studies or optimisation procedures are performed, which both often require the use of reduction techniques to limit their computational cost. This work aims at reviewing reduction methods based on modal projection adapted for highly damped structures. These methods are then compared in terms of precision and computational time of the approximated solution on a benchmark example.

1 INTRODUCTION

Vibration reduction in structures has been a subject of investigation for many years. Limiting resonant or nearly resonant vibrations is classically achieved through the use of constrained damping layer (CLD) treatment, consisting in the insertion of a viscoelastic layer at the core of the vibrating structure. An important tool in the design of such structures is the numerical simulation of predictive models, which makes use of the finite element approach. However, the 3D modelling of the viscoelastic layer, usually required for a good representation of the shear behaviour, leads to dynamical models with a large number of degrees of freedom, or large enough to prohibit design optimization. Model reduction can be used to find a low-order model that approximates the behaviour of the original high-order model, and thus reduce the computational cost of the analysis. This work focuses on a class of reduction methods, based on modal projection: modes superposition methods. They consist in using a small number of vibration modes to represent the dynamics of the structure with some reduced number of generalized degrees of freedom. This operation reduces the system size to be solved and can result in important computational gain. However, the classical approach is based on the assumptions that the system

is undamped or lightly damped, and that the eigenfrequencies are well separated. In the case of highly damped systems, such as sandwich structures with viscoelastic insertion, computational variants of the method have been developed. This papers aims at reviewing the different variants of the mode superposition method available in the literature for highly damped systems (section 2). A comparison is performed by application of the methods to an illustrative example (section 3).

2 REVIEW OF MODE SUPERPOSITION METHODS

2.1 Limitations of the classical mode superposition approach

The finite element discretisation of the differential equations of a problem consisting in an vibrating elastic structure results in the equations of motion of the system, which in absence of damping are typically of the form:

$$\left[\mathbf{K} - \omega^2 \mathbf{M}\right] \mathbf{U} = \mathbf{F}, \tag{1}$$

where $\mathbf{K}$ and $\mathbf{M}$ are respectively the stiffness and the mass matrices, $\mathbf{U}$ represents the solution vector, containing the unknown displacement of the structure, and $\mathbf{F}$ is the load vector. The eigenvalue problem associated with Equation (1) is:

$$\left[\mathbf{K} - \omega_k^2 \mathbf{M}\right] \mathbf{\Phi}_k = \mathbf{0}, \tag{2}$$

where ω_k and $\mathbf{\Phi}_k$ are respectively the eigenfrequency and the eigenvector, also called normal modes, associated to the mode k ($k \in [1 \dots N]$, with N the size of the system). The normal modes span the solution space so that the solution vector $\mathbf{U}$ from Equation (1) can be written as a weighted summation of vibration modes:

$$\mathbf{U} = \sum_{k=1}^{N} \mathbf{\Phi}_k \chi_k. \tag{3}$$

The modal coordinates χ_k are solutions of the equation:

$$(\omega^2 - \omega_k^2)\chi_k = \mathbf{\Phi}_k^T \mathbf{F}, \tag{4}$$

which is obtained by projection of Equation (1) on the basis of normal modes, and by application of the orthogonality conditions:

$$\begin{aligned} \mathbf{\Phi}_r^T \mathbf{M} \mathbf{\Phi}_s &= \delta_{rs}, \\ \mathbf{\Phi}_r^T \mathbf{K} \mathbf{\Phi}_s &= \omega_s^2 \delta_{rs}, \end{aligned} \tag{5}$$

where δ_{rs} denotes the Kronecker delta.

The idea behind mode superposition methods, introduced by Rayleigh, is to look for the solution in a solution subspace of reduced dimension n ($n < N$), by truncating the series in Equation (3):

$$\mathbf{U} = \sum_{k=1}^{n} \mathbf{\Phi}_k \chi_k + \underbrace{\sum_{k=n+1}^{N} \mathbf{\Phi}_k \chi_k}_{\text{truncated}}. \tag{6}$$

The approximated solution vector $\mathbf{U}_r$ is then described by n modes ($n < N$), associated with the lowest eigenfrequencies:

$$\mathbf{U}_r = \sum_{k=1}^{n} \mathbf{\Phi}_k \chi_k. \tag{7}$$

The effects of the truncated modes on the low frequency dynamic of the structure can be taken into account by a static correction, which usually improves the solution:

$$\mathbf{U}_r = \sum_{k=1}^{n} \mathbf{\Phi}_k \chi_k + \chi_{\text{static}} \mathbf{K}^{-1}\mathbf{F}. \tag{8}$$

The approximated solution is then computed by projecting Equation (1) on the following reduction basis:

$$\mathbf{T} = \left[\mathbf{K}^{-1}\mathbf{F}, \mathbf{\Phi}_1, \ldots, \mathbf{\Phi}_n\right]. \tag{9}$$

However, in the case of damped structures, such as structures with CLD treatments, the equations of motion are of the following form:

$$\left(\mathbf{K}_e + K^*(\omega)\mathbf{K}_v^s + G^*(\omega)\mathbf{K}_v^d - \omega^2\mathbf{M}\right)\mathbf{U} = \mathbf{F}, \tag{10}$$

where $\mathbf{K}_v^s$ and $\mathbf{K}_v^d$ are respectively the spheric and the deviatoric part of the stiffness matrix associated with the viscoelastic layer, and $\mathbf{K}_e$ is the stiffness matrix associated with the elastic structure. $K^*(\omega)$ and $G^*(\omega)$ are the frequency dependent complex bulk and shear modulus of the viscoelastic material, which can be described by a fractional derivative model [9]:

$$K^*(\omega) = \frac{K_0 + K_\infty(\mathrm{i}\omega\tau_K)^{\alpha_K}}{1 + (\mathrm{i}\omega\tau_K)^{\alpha_K}} \qquad G^*(\omega) = \frac{G_0 + G_\infty(\mathrm{i}\omega\tau_G)^{\alpha_G}}{1 + (\mathrm{i}\omega\tau_G)^{\alpha_G}}. \tag{11}$$

Due to the frequency dependence of viscoelastic properties, the eigenproblem associated with Equation (10) becomes nonlinear:

$$\left[\mathbf{K}^*(\lambda_k^*) - \lambda_k^{*2}\mathbf{M}\right]\mathbf{\Phi}_k^* = \mathbf{0}, \tag{12}$$

where the eigenfrequency $\lambda_k^{*2} = \omega_k^2(1 + \mathrm{i}\eta_k)$ and the normal modes $\mathbf{\Phi}_k^*$ are complex. Several methods exist in the literature to solve nonlinear problems, such as the asymptotic numerical method [1]. The latter gives an accurate estimation of the normal modes, which can be used for projection. However, this method can be computationally expensive and is out of the scope of this paper. Other methods, avoiding the resolution of a nonlinear eigenvalue problem, are reviewed in the next sections.

2.2 Modal strain energy method (MSE)

The modal strain energy method, introduced by Johnson and Kienholz [2], supposes that the normal modes of the undamped system are representative of the normal modes of the damped system. The projection basis is then defined as:

$$\mathbf{T}_{\text{MSE}} = \left[\mathbf{K}_0^{-1}\mathbf{F}, \boldsymbol{\Phi}_1(0), \ldots, \boldsymbol{\Phi}_n(0)\right], \tag{13}$$

where $\boldsymbol{\Phi}_k(0)$, referred to as pseudo-normal modes, are real and solution of:

$$\left[\mathbf{K}_0 - \lambda_k^2\mathbf{M}\right]\boldsymbol{\Phi}_k = \mathbf{0}, \tag{14}$$

and $\mathbf{K}_0 = \mathbf{K}^*(\omega = 0)$ is the static stiffness matrix. This method gives good results when the structure is lightly damped, but may lead to significant errors when it is highly damped. Different approaches can be then adopted in order to improve the accuracy of the approximated solution:

- extend the modal strain energy, by iteratively seeking a better approximation of the complex modes solution of Equation (12) (section 2.3),
- combine several modal basis using the multi-model approach (section 2.4),
- enrich the modal basis of Equation (13) by addition of corrective terms (section 2.5).

2.3 Extensions of the modal strain energy

Iterative modal strain energy method (IMSE) While the modal strain energy method uses a constant stiffness matrix for the calculation of the pseudo-normal modes, the iterative modal strain energy method [3] considers the effects of a variation in the complex moduli by computing the real undamped pseudo-normal modes of an iteratively updated eigenproblem (Algorithm 1). The projection basis then consists of a static correction and pseudo-normal modes, obtained for the converged values of eigenfrequencies of the following eigenproblem:

$$\left[\Re\left(\mathbf{K}^*(\omega_p)\right) - \lambda_k^2(\omega_p)\mathbf{M}\right]\boldsymbol{\Phi}_k(\omega_p) = \mathbf{0}, \tag{15}$$

where $\lambda_k^2(\omega_p)$ and $\boldsymbol{\Phi}_k(\omega_p)$ are respectively the eigenfrequency and the pseudo-normal mode of the k^{th} mode, computed for a stiffness matrix evaluated at ω_p. Convergence is achieved when:

$$\frac{|\omega_p - \lambda_k(\omega_p)|}{\lambda_k(\omega_p)} < \epsilon_{\text{tol}}, \tag{16}$$

with ϵ_{tol}, chosen convergence criterion. This method supposes that the complex part of the stiffness matrix does not affect the dynamic of the structure.

Algorithm 1 Iterative modal strain energy algorithm.

1. Compute the n first couples $(\lambda_k(0), \mathbf{\Phi}_k(0))$, solutions of Equation (14).
2. Initialise the projection basis: $\mathbf{T}_{\text{IMSE}} = [\mathbf{K}^{-1}\mathbf{F}]$.
3. **for** $j = 1$ **to** n **do**
4. Initialise the frequency of evaluation of the complex moduli $\omega_p = \lambda_j(0)$ and the error $\epsilon = 1$.
5. **while** $\epsilon > \epsilon_{\text{tol}}$ **do**
6. Compute the j first couples $(\lambda_k(\omega_p), \mathbf{\Phi}_k(\omega_p))$, solutions of Equation (15).
7. Compute the error: $\epsilon = \dfrac{|\omega_p - \lambda_j(\omega_p)|}{\lambda_j(\omega_p)}$.
8. Update the frequency of evaluation of the complex moduli $\omega_p = \lambda_j(\omega_p)$.
9. **end while**
10. Update the projection basis $\mathbf{T}_{\text{IMSE}} = [\mathbf{T}_{\text{IMSE}}, \mathbf{\Phi}_j(\omega_p)]$.
11. **end for**

Iterative complex eigensolution method (ICE) The iterative complex eigensolution method [4] is similar to the IMSE method in the sense that the same iterative algorithm is used (Algorithm 2). However, unlike the IMSE method which computes real pseudo-normal modes, the projection basis of the ICE method is composed of complex pseudo-normal modes, solutions of:

$$\left[\mathbf{K}^*(\omega_p) - \lambda_k^{*2}(\omega_p)\mathbf{M}\right] \mathbf{\Phi}_k^*(\omega_p) = \mathbf{0}. \tag{17}$$

In the case of highly damped structures, the use of complex pseudo-normal modes in the projection basis improves the accuracy of the approximated solution, but usually increases the computational cost as well. Indeed, numerical solver are generally less efficient when the eigenproblem to solve is complex.

Modified modal strain energy method (MMSE) The modified modal strain energy method [5] constitutes a compromise between the IMSE and the ICE methods: it aims at taking into account the complex part of the stiffness matrix without having to solve a complex eigenproblem (Algorithm 3). This is done through an empirical coefficient:

$$\left[\Re(\mathbf{K}^*(\omega_p)) + \beta(\omega_p)\Im(\mathbf{K}^*(\omega_p)) - \lambda_k^2(\omega_p)\mathbf{M}\right] \mathbf{\Phi}_k(\omega_p) = \mathbf{0}, \tag{18}$$

where the coefficient $\beta(\omega_p)$ is computed as follows:

$$\beta(\omega_p) = \frac{\text{Tr}\left(\Im(\mathbf{K}^*(\omega_p))\right)}{\text{Tr}\left(\Re(\mathbf{K}^*(\omega_p))\right)}. \tag{19}$$

Algorithm 2 Iterative complex eigensolution algorithm.

1. Compute the n first couples $(\lambda_k(0), \mathbf{\Phi}_k(0))$, solutions of Equation (14).
2. Initialise the projection basis: $\mathbf{T}_{\text{ICE}} = [\mathbf{K}^{-1}\mathbf{F}]$.
3. **for** $j = 1$ **to** n **do**
4. Initialise the frequency of evaluation of the complex moduli $\omega_p = \lambda_j(0)$ and the error $\epsilon = 1$.
5. **while** $\epsilon > \epsilon_{\text{tol}}$ **do**
6. Compute the j first couples $(\lambda_k^*(\omega_p), \mathbf{\Phi}_k^*(\omega_p))$, solutions of Equation (17).
7. Compute the error: $\epsilon = \dfrac{|\omega_p - \sqrt{\Re(\lambda_j^{*2}(\omega_p))}|}{\sqrt{\Re(\lambda_j^{*2}(\omega_p))}}$.
8. Update the frequency of evaluation of the complex moduli $\omega_p = \sqrt{\Re(\lambda_j^{*2}(\omega_p))}$.
9. **end while**
10. Update the projection basis $\mathbf{T}_{\text{ICE}} = [\mathbf{T}_{\text{ICE}}, \mathbf{\Phi}_j^*(\omega_p)]$.
11. **end for**

Algorithm 3 Modified modal strain energy algorithm.

1. Compute the n first couples $(\lambda_k(0), \mathbf{\Phi}_k(0))$, solutions of Equation (14).
2. Initialise the projection basis: $\mathbf{T}_{\text{MMSE}} = [\mathbf{K}^{-1}\mathbf{F}]$.
3. **for** $j = 1$ **to** n **do**
4. Initialise the frequency of evaluation of the complex moduli $\omega_p = \lambda_j(0)$ and the error $\epsilon = 1$.
5. **while** $\epsilon > \epsilon_{\text{tol}}$ **do**
6. Compute the coefficient $\beta(\omega_p)$, using Equation (19).
7. Compute the j first couples $(\lambda_k(\omega_p), \mathbf{\Phi}_k(\omega_p))$, solutions of Equation (15).
8. Compute the error: $\epsilon = \dfrac{|\omega_p - \lambda_j(\omega_p)|}{\lambda_j(\omega_p)}$.
9. Update the frequency of evaluation of the complex moduli $\omega_p = \lambda_j(\omega_p)$.
10. **end while**
11. Update the projection basis $\mathbf{T}_{\text{MMSE}} = [\mathbf{T}_{\text{MMSE}}, \mathbf{\Phi}_j(\omega_p)]$.
12. **end for**

2.4 Multi-model approach (MM)

The multi-model approach is inspired by the Takani-Sugeno fuzzy model which is often used to represent nonlinear dynamic systems by interpolating locally linear models obtained from a nonlinear system. It has been applied by Balmès [6] to build a projection basis representative of the complex nonlinear eigenvalue of Equation (12). This basis is formed by a combination of several modal bases $\mathbf{T}_{p_j}$ and the static correction:

$$\mathbf{T}_{MM} = [\mathbf{K}^{-1}\mathbf{F}, \mathbf{T}_{p_1} \dots \mathbf{T}_{p_m}]. \tag{20}$$

Each modal basis $\mathbf{T}_{p_j}$ contains the pseudo-normal modes solutions of the following eigenvalue problem:

$$\left[\mathbf{K}^*(\omega_{p_j}) - \lambda_k^{*2}(\omega_{p_j})\mathbf{M}\right] \mathbf{\Phi}_k^*(\omega_{p_j}) = \mathbf{0}, \tag{21}$$

where ω_{p_j} is imposed a priori. A strong colinearity may arise between the pseudo-normal modes of several modal bases, which justifies the use of a Gramm-Schmidt orthonormalisation algorithm. In the literature, the projection basis composed of a static correction and two modal bases evaluated at the minimum and the maximum frequency of the frequency range investigated lead to good approximation of the dynamic response of highly damped structures.

2.5 Enrichment of the projection basis

First-order correction (MSE+C) The resolution of a real eigenproblem usually implies a computational cost much lower than that of a complex eigenproblem, which supports the common practice of neglecting the imaginary part of the stiffness matrix when computing the pseudo-normal modes. However in case of highly damped structures, this approximation is not justified and may lead to significant errors. Plouin and Balmès [7] then proposed to complement the projection basis of Equation (13) by the static response to the load generated by the imaginary part of the stiffness when exciting a given pseudo-normal mode:

$$\mathbf{T}_{\text{MSE+C}} = [\mathbf{T}_{\text{MSE}}, \mathbf{K}_0^{-1}\Im(\mathbf{K}^*(\lambda_1))\mathbf{\Phi}_1, \dots, \mathbf{K}_0^{-1}\Im(\mathbf{K}^*(\lambda_N))\mathbf{\Phi}_N]. \tag{22}$$

where λ_k and $\mathbf{\Phi}_k$ are the eigenfrequencies and pseudo-normal modes from Equation (14). As in the multi-model approach, a orthonormalisation procedure may be applied.

Displacement residuals (MSE+R) The concept of this method, introduced by Balmès and Bobillot [8], is to increase the accuracy of the approximation by enriching the projection basis of Equation (13) by the static response of a load residual:

$$\mathbf{T}_{\text{MSE+R}} = [\mathbf{T}_{\text{MSE}}, \mathbf{R}_d]. \tag{23}$$

where the residual $\mathbf{R}_d$ is computed as follows:

$$\mathbf{R}_d(\omega) = \mathbf{K}_0^{-1}\left(\left[\mathbf{K}^*(\omega) - \omega^2\mathbf{M}\right]\mathbf{U}_r(\omega) - \mathbf{F}\right). \tag{24}$$

The residuals calculated at the eigenfrequencies λ_k of the undamped eigenproblem (Equation (14)) are added to the projection basis, until a satisfying precision of the solution is obtained, as indicated in Algorithm 4:

$$\epsilon_R = \frac{\left|\left|\mathbf{R}_d^T \mathbf{K}_0 \mathbf{R}_d\right|\right|_2}{||\mathbf{U}_r^T \mathbf{K}_0 \mathbf{U}_r||_2} < \epsilon_{\text{tol}}, \tag{25}$$

where ϵ_R is a error estimate of the strain energy and ϵ_{tol} is a chosen criterion.

Algorithm 4 Procedure of enrichment of the projection basis by residuals.

1. Initialise the projection basis $\mathbf{T}_{\text{MSE+R}} = \mathbf{T}_{\text{MSE}}$.
2. **for** $k = 1$ **to** N **do**
3. Compute residual $\mathbf{R}_d(\lambda_k)$ according to Equation (24).
4. Compute the error estimate ϵ_R from Equation(25).
5. **while** $\epsilon_R > \epsilon_{\text{tol}}$ **do**
6. Update and orthonormalise the projection basis : $\mathbf{T}_{\text{MSE+R}} = [\mathbf{T}_{\text{MSE+R}}, \mathbf{R}_{d_k}(\lambda_k)]$.
7. Compute the residual $\mathbf{R}_d(\lambda_k)$ from Equation (24).
8. Compute the error estimate from Equation (25).
9. **end while**
10. **end for**

3 ILLUSTRATIVE EXAMPLE

The reduction methods described in the previous section can be combined in order to improve the accuracy of the approximated solution. However, for the comparative purpose of this work, each method is applied to a benchmark example and their performances, in terms of precision and computational time, are assessed individually [9]. The sandwich cantilever beam depicted in Figure 1(a) is considered for this study. The elastic faces of the beam are made of steel and the core layer is made of Deltane 350, whose properties have been measured by DMA (Figure 2). A fractional derivative model is used to represent the frequency-dependent shear modulus (Figure 2):

$$G_0 = 1.36 \text{ MPa}, \qquad G_\infty = 0.64 \text{ GPa}, \qquad \tau = 0.34 \ \mu\text{s}, \qquad \alpha = 0.58, \tag{26}$$

and the bulk modulus is assumed constant: $K^*(\omega) = 2.22$ GPa. The three layers are meshed using 20-nodes hexahedral elements, leading to a finite element model of the form of Equation (10), with $N = 13317$ degrees of freedom (see Figure 1(b)). The frequency response of the damped structure is calculated on the frequency range [0–800] Hz by applying each of the reduction methods presented in the previous section, and compared with the solution computed by a direct method. A convergence tolerance of 0.005 is considered for the iterative variants of the modal strain energy, and the convergence tolerance in the procedure of enrichment of the modal basis is set to 0.001. The dimensions

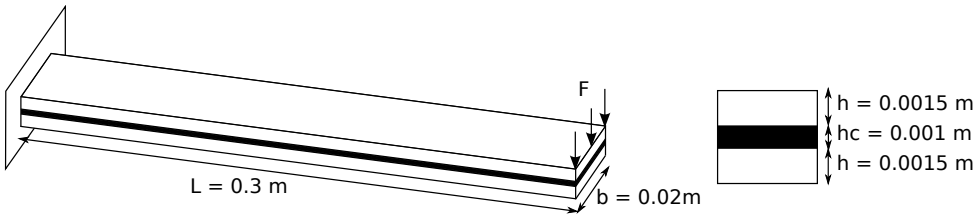
of the beam have been chosen in order to introduce important damping in the structure, as evidenced by the strongly attenuated resonance peaks in Figure 3. For each reduction method, several criteria of selection of the pseudo-normal modes are taken: $f < f_{\max}$, $f < 2f_{\max}$, $f < 3f_{\max}$ and $f < 4f_{\max}$ (Figure 4). Figure 3 compares the frequency responses computed by each method when choosing $f < 2f_{\max}$ as the selection criterion, which is the most commonly used in the literature. In order to compare the precision of the approximated solution, two error estimators are calculated:

- the error between the approximated solution $\mathbf{U}_r$ and the solution $\mathbf{U}$ computed by a direct method:
$$\epsilon_U(\omega) = \frac{||\mathbf{U}_r - \mathbf{U}||_2}{||\mathbf{U}||_2}, \tag{27}$$
- the square of the error estimator of the strain energy, defined in Equation (25).

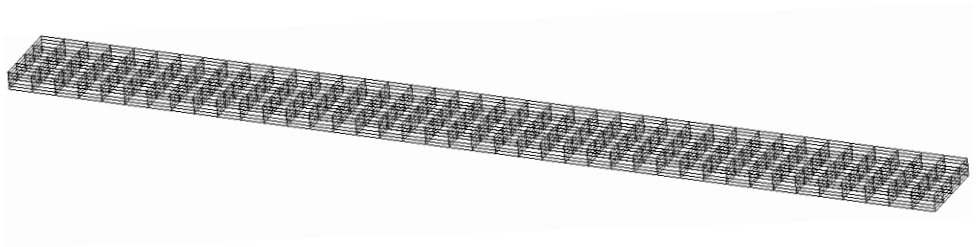
Figure 3 clearly evidences the limitations of the modal strain energy method. It also indicates the method with the modal basis enriched by displacement residuals as the most precise. Figure 4 confronts the precision of the solution (defined as the mean of the displacement error ϵ_U) with its relative computational time (defined as the ratio between the computational time of the approximated solution to that of the solution calculated by the direct method). From this comparison, four groups of methods can be identified. The first one consists of the modal strain energy method, which allows a quick but poorly accurate approximation of the solution. Conversely, the second group, consisting of the iterative method of enrichment by displacement residuals, gives a very accurate approximation of the solution but at the cost of a more important computational time. This is due to the online calculation of the error estimate. The third group includes the multi-model method and the modal strain energy method with first-order corrective terms. They both represent a good compromise between precision and computational time of the approximated solution. The iterative variants of the modal strain energy method constitute the fourth and last group. These method do not represent a good compromise between precision and computational time of the solution, but can be used instead for a direct calculation of the modal parameters.

4 CONCLUSIONS

This paper reviews several reduction methods based on modal projection adapted for highly damped structures, such as structures with viscoelastic treatments. The different methods are tested and compared on a benchmark example. The multi-model method and the modal strain energy method with first-order corrective terms are identified as the methods presenting the best compromise between precision and computational time of the approximated solution. However, the results of this analysis should only be used as recommendations since they are dependent on the level of damping introduced in the structure and the implementation of the reduction techniques.



(a)



(b)

Figure 1: Geometry (a) and mesh (b) of the sandwich cantilever beam studied.

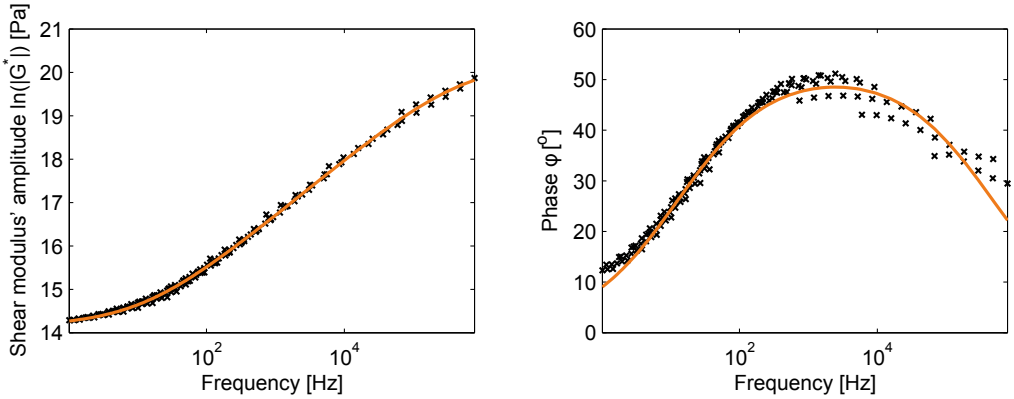


Figure 2: Frequency-dependent shear modulus of Deltane 350, measured by DMA (crosses) and modelled by a fractional derivative model (solid line).

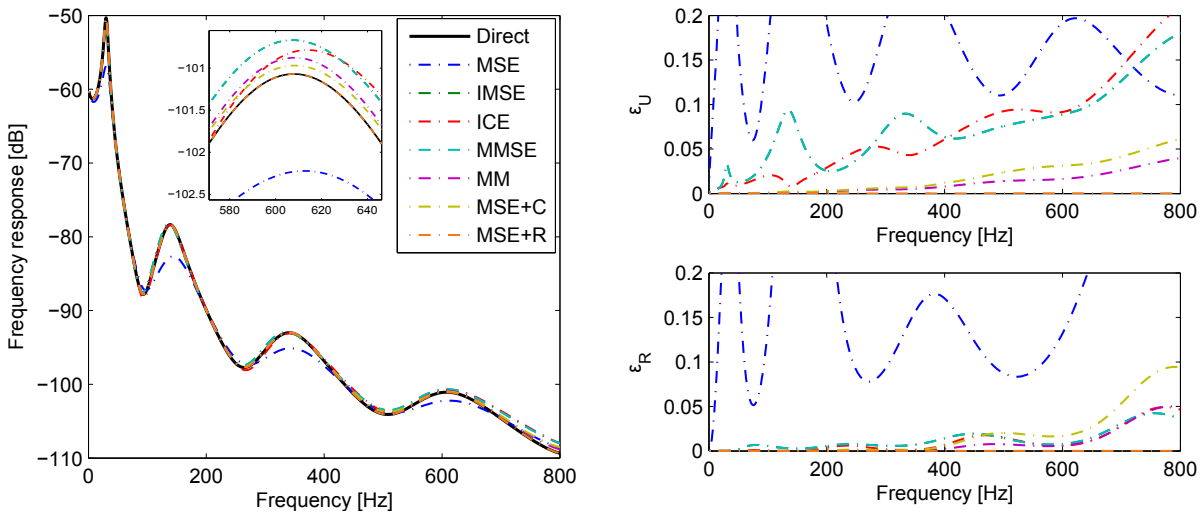


Figure 3: Frequency response and error estimators computed by the direct method, the modal strain energy method (MSE), the iterative modal strain energy method (IMSE), the complex eigenvalue method (ICE), the modified modal strain energy method (MMSE), the multi-model method (MM), the modal strain energy method corrected with first-order terms (MSE+C) or residuals (MSE+R).

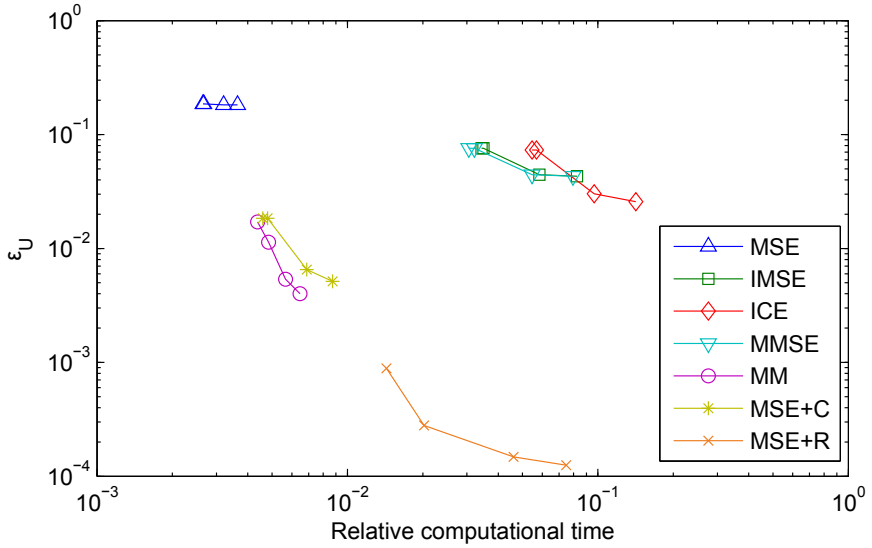


Figure 4: Mean displacement error as a function of the relative computational time for the modal strain energy method (MSE), the iterative modal strain energy method (IMSE), the complex eigenvalue method (ICE), the modified modal strain energy method (MMSE), the multi-model method (MM), the modal strain energy method corrected with first-order terms (MSE+C) or residuals (MSE+R).

The reduction methods reviewed in this paper can be used in the frame of component mode synthesis [10] or combined with Padé approximants for a further reduction of the computational time [11].

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