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DIFFERENT DETERMINATION PROCEDURES FOR STIFFNESS PARAMETERS OF WOVEN FABRICS AND THEIR IMPACT IN THE MEMBRANE STRUCTURE ANALYSIS

JÖRG UHLEMANN*, NATALIE STRANGHÖNER* AND KLAUS SAXE**

* Institute for Metal and Lightweight Structures
University of Duisburg-Essen
Campus Essen, 45141 Essen, Germany
e-mail: joerg.uhlemann@uni-due.de, natalie.stranghoener@uni-due.de, www.uni-due.de/iml

** Essen Laboratory for Lightweight Structures (ELLF)
University of Duisburg-Essen
Campus Essen, 45141 Essen, Germany
e-mail: klaus.saxe@uni-due.de

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Abstract. In the daily design practice, so called “fictitious” elastic constants are used for describing the tensile stiffness and Poisson’s ratio of woven fabrics applied in architectural membrane structures. Extensive investigations carried out at the Institute for Metal- and Lightweight Structures at the University of Duisburg-Essen show that for one fabric material a great variation of lower and upper limits for these constants can be determined depending on the applied different test and determination procedures. In the structural analysis, this range of fictitious elastic constants results in a wide spectrum of computed stresses and deformations for one and the same structure and load case, so that the question arises: which procedure meets the real structural behaviour best? Exemplary, comparable calculations are presented for a simple hypar consisting of a PES-PVC-material. The influence of the variety of applicable constants is demonstrated with regard to the resulting stresses and deformations.

1 INTRODUCTION

Membrane structures consist of a primary and secondary structure. Whereas the primary structure is in most cases a steel structure and only in some cases made of timber or another material, for the secondary structure, the membrane structure, predominantly woven textile fabrics are used and not so often, but more and more foils are used as well. Laying the focus on woven fabrics, it is known that fabrics show a highly nonlinear and anisotropic stress-strain behaviour under biaxial tensile loads which is considered in the daily structural analysis commonly simplified as a linear-elastic plane stress material, using the elastic constants “tensile stiffness” and “Poisson’s ratio” as more or less “fictitious” stiffness parameters.

The determination of these elastic constants from biaxial test results is found to be very complex and a rather rough approximation at the same time. The situation can become confusing for designers as several different biaxial test methods and determination procedures for elastic constants exist worldwide. The Japanese standard MSAJ/M-02-1995 “Testing Method for Elastic Constants of Membrane Materials” [1], commonly used in Europe, provides different determination procedures to calculate the stiffness parameters in the two main directions of a fabric from the results of plane biaxial tensile tests. Further procedures are established in the “European Design Guide for Tensile Surface Structures” [2] published by the TensiNet Association, in the US-code ASCE-SEI 55-10 “Tensile Membrane Structures” [3] and in the French recommendations [4].

The present paper is an addition to [5], where the different methods of MSAJ/M-02-1995 and TensiNet Design Guide have been comparatively analysed for a Glass-PTFE material. The present paper goes a step further and presents additional results for a Polyester(PES)-PVC material type III achieved on the same basis as in [5] described. Furthermore, the impact of the varying results in the analysis of membrane structures is discussed.

2 THE ORTHOTROPIC LINEAR-ELASTIC CONSTITUTIVE LAW

The description of the orthotropic linear-elastic constitutive law and its different mathematical formulations has already been explained in detail in [5]. For a better understanding the main basics of it are summarized again in the following.

In the daily design practice, it is still common that the anisotropic and highly nonlinear stress-strain-behaviour of woven fabrics is taken into account in a structural analysis as a linear-elastic orthogonal anisotropic plane-stress structure, when the membrane is modeled as a continuum and not as cable net. Although knowing that this is a rather rough and unsatisfying approximation, it is still not possible to use more sophisticated solutions because they do not exist up to now. A mathematical formulation for the load-strain-relationship is given by

$$\varepsilon_x = \frac{n_x}{E_x t} - \nu_{xy} \frac{n_y}{E_y t}, \tag{1}$$

$$\varepsilon_y = \frac{n_y}{E_y t} - \nu_{yx} \frac{n_x}{E_x t}. \tag{2}$$

Herein, ε are the strains [-] and n are the loads [kN/m], which are often called stresses in membrane structure analysis. The four elastic constants are: $E_x t$ as the tensile stiffness in warp direction [kN/m] and $E_y t$ in fill direction, respectively. The axes x and y refer to the warp and the weft (fill) yarn direction of the fabric. The transverse strains are taken into account by the Poisson’s ratio ν. ν_{xy} is the Poisson’s ratio in x-direction caused by a load in y-direction, ν_{yx} applies analogue in perpendicular direction. Transposed to the loads n and written with matrices this law becomes

$$\begin{bmatrix} n_x \\ n_y \end{bmatrix} = \frac{1}{1-\nu_{xy}\cdot\nu_{yx}} \begin{bmatrix} E_x t & \nu_{xy}\cdot E_x t \\ \nu_{yx}\cdot E_y t & E_y t \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \end{bmatrix}. \tag{3}$$

The stiffness matrix has to be symmetric, what directly leads to eq. (4). Only three of the four elastic constants are independent of each other.

$$\nu_{xy} \cdot E_x t = \nu_{yx} \cdot E_{yt} \quad \Rightarrow \quad \nu_{yx} = \nu_{xy} \cdot \frac{E_x t}{E_y t}. \tag{4}$$

Due to the fact that the stiffness matrix has to be positive definite, the tensile stiffnesses and the determinante of the stiffness matrix have to be positive, too. The latter constraint leads to

$$\nu_{xy} \cdot \nu_{yx} < 1. \tag{5}$$

Another possible formulation of the constitutive law is given by

$$\begin{bmatrix} n_{11} \\ n_{22} \end{bmatrix} = \begin{bmatrix} E_{1111} & E_{1122} \\ E_{1122} & E_{2222} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \end{bmatrix}. \tag{6}$$

Herein, E_{1111} and E_{2222} are the tensile stiffnesses in warp and weft direction, respectively, and E_{1122} is the stiffness interaction between warp and weft direction. The two mathematical formulations in eq. (3) and (6) of the same constitutive law result in identical analysis results. Special attention has to be paid, as the numerical values of the elastic constants of both definitions are not equal. But the elastic constants of both definitions can easily be converted, see e.g. [5].

3 BIAXIAL TESTS AND THE DETERMINATION OF ELASTIC CONSTANTS

Several biaxial test procedures and related evaluation procedures exist today – either published in standards/recommendations or unpublished as office specific as well as project specific test procedures oftentimes used by the engineering design offices. Two common test procedures are described in the Japanese standard MSAJ/M-02-1995 [1] and in the TensiNet European Design Guide for Tensile Surface Structures [2]. Exemplary, additional test procedures are described by *Galliot* and *Luchsinger* in [6] and in the French recommendations [4]. Furthermore, to every test procedure one or more related evaluation procedures exist to determine elastic constants from the biaxial test results. This situation leads to a confusing variety of elastic constants.

The test and evaluation procedures of MSAJ/M-02-1995 and TensiNet Design Guide are already described in [5]. For this reason, it is referred to [5] for detailed information. In the frame of this contribution, both procedures have been applied for conducting experimental tests which are presented afterwards. Both recommendations have in common that they each result in only one single set of “fictitious” elastic constants based on biaxial test results. It is common practice that this one set of elastic constants is used in the structural analysis for all kinds of structural forms and all load cases.

4 RESULTS OF THE CONDUCTED BIAXIAL TESTS

The present paper shows results on three MSAJ-tests and three TensiNet Design Guide tests on a type III PES-PVC material, see the test matrix in *Table 1*. The tests were conducted in the Essen Laboratory for Lightweight Structures (ELLF) located at the University of Duisburg-Essen.

The tested material product is a B4915 of Verseidag-Indutex, which is a type III PES-PVC material with a strip tensile strength of 115/102 kN/m in warp and fill direction, respectively. The test specimens were cut from three different adjacent parts of one batch. From every one of the three parts, one MSAJ-test and one TensiNet Design Guide-test has been conducted according to the loading procedure presented in *Figures 1* and *2.* To enable a direct comparability to the MSAJ-procedure, the Design Guide-tests have been carried out with the same maximum tensile load of 25.5 kN/m as for the MSAJ-test (1/4 of the strip tensile strength in fill direction). The prestress has been chosen to be 1 kN/m in each fabric direction so that it equals the minimum load of the load-strain-paths on which the MSAJ-determination is based on. This value is fixed by the MSAJ-commentary to be 1 kN/m for PES-PVC-materials. The test matrix is summarized in *Table 1*.

Table 1: Test matrix for the PES-PVC-material type III with a tensile strength of 115/102 kN/m

Test N^o	Test specimen N^o	Test protocol
IML5214	1	MSAJ original
IML6214	2	MSAJ original
IML7214	3	MSAJ original
IML13214	1	TensiNet Design Guide
IML14214	2	TensiNet Design Guide
IML15214	3	TensiNet Design Guide

The resulting load-strain-paths from the three MSAJ-tests are illustrated in *Figure 3*. Here, the residual strains are removed so that only the slope of the paths is illustrated, and furthermore, on the ordinate of the diagrams always the "leading load" – which is meant to be the bigger load in a load ratio n_x/n_y – is applied. It can be seen from the almost identical load-strain-paths that the statistical spread is very low. That is basically not surprising, because all test specimens came from the same batch. But the mentioned statement is even true for the zero-load paths (fill in 1:0 and warp in 0:1), which is in so far remarkable, as the commentary of the MSAJ standard states a "low repeatability of test results in the low stress range". This statement cannot be confirmed by the present tests.

The results from the three Design Guide-tests are shown in the typical illustration as time-strain-curves in *Figure 4.* Surprisingly, they show a bigger statistical spread than the MSAJ test results, see also the resulting elastic constants in *Tables 2* and *3*.

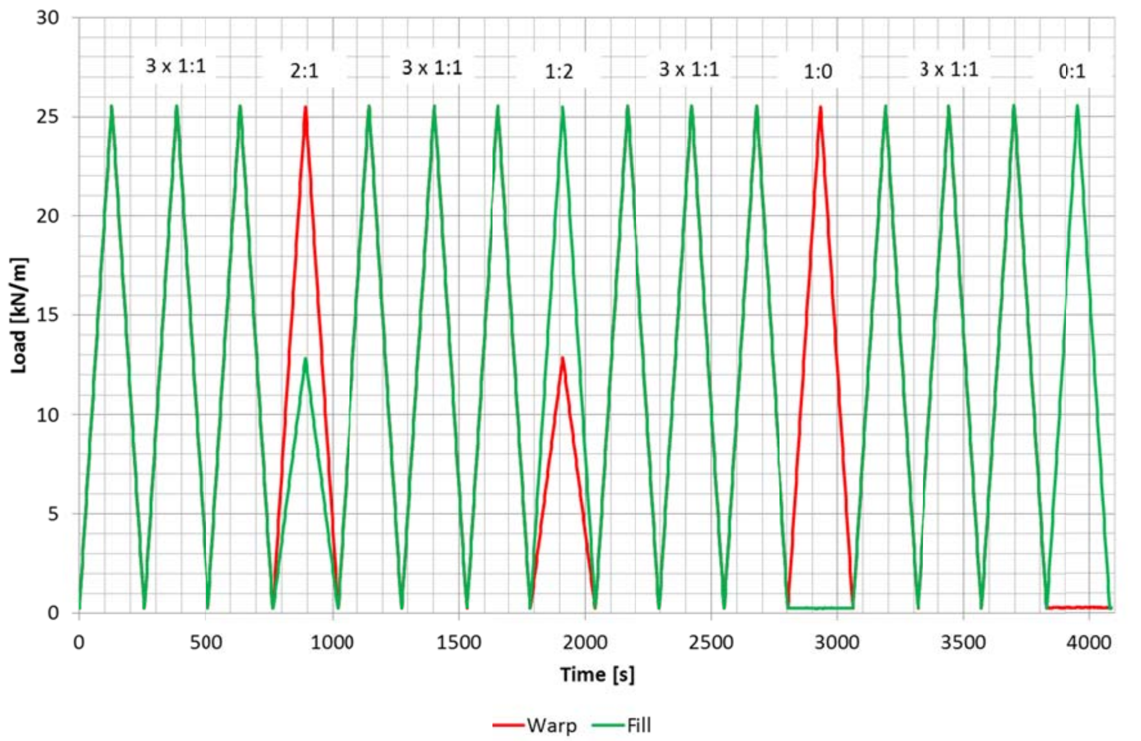

Figure 1: Used loading procedure for the biaxial tests according to MSAJ/M-02-1995

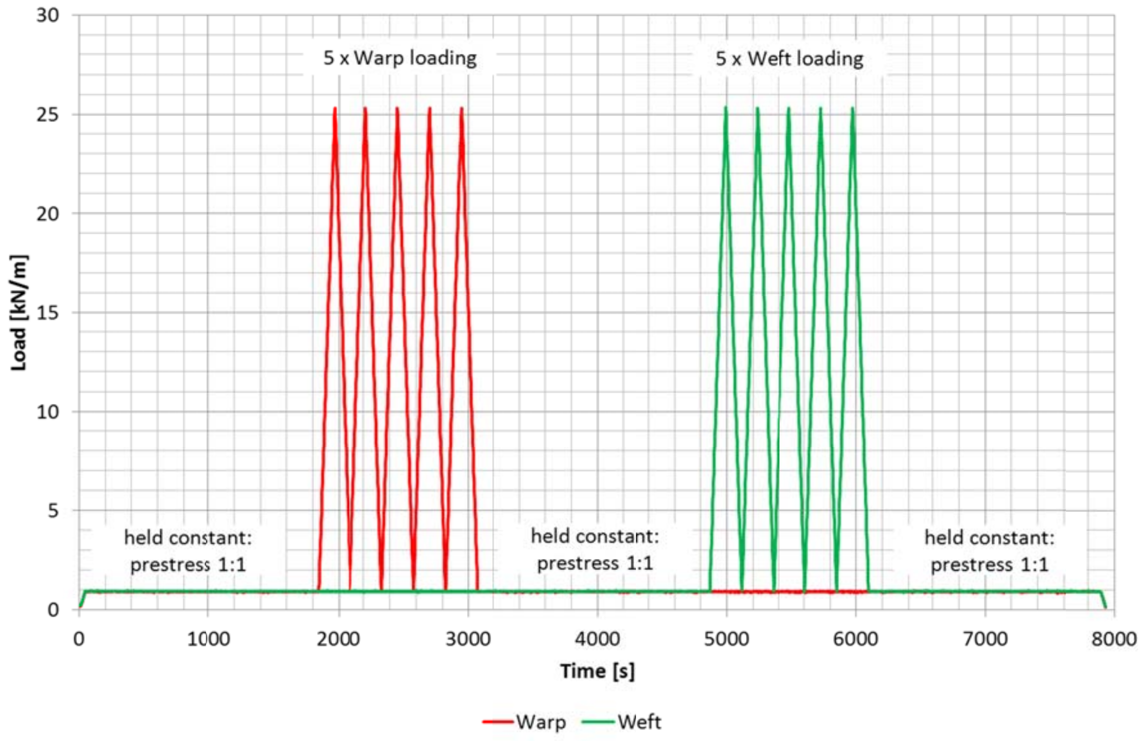

Figure 2: Used loading procedure for the biaxial tests according to TensiNet Design Guide

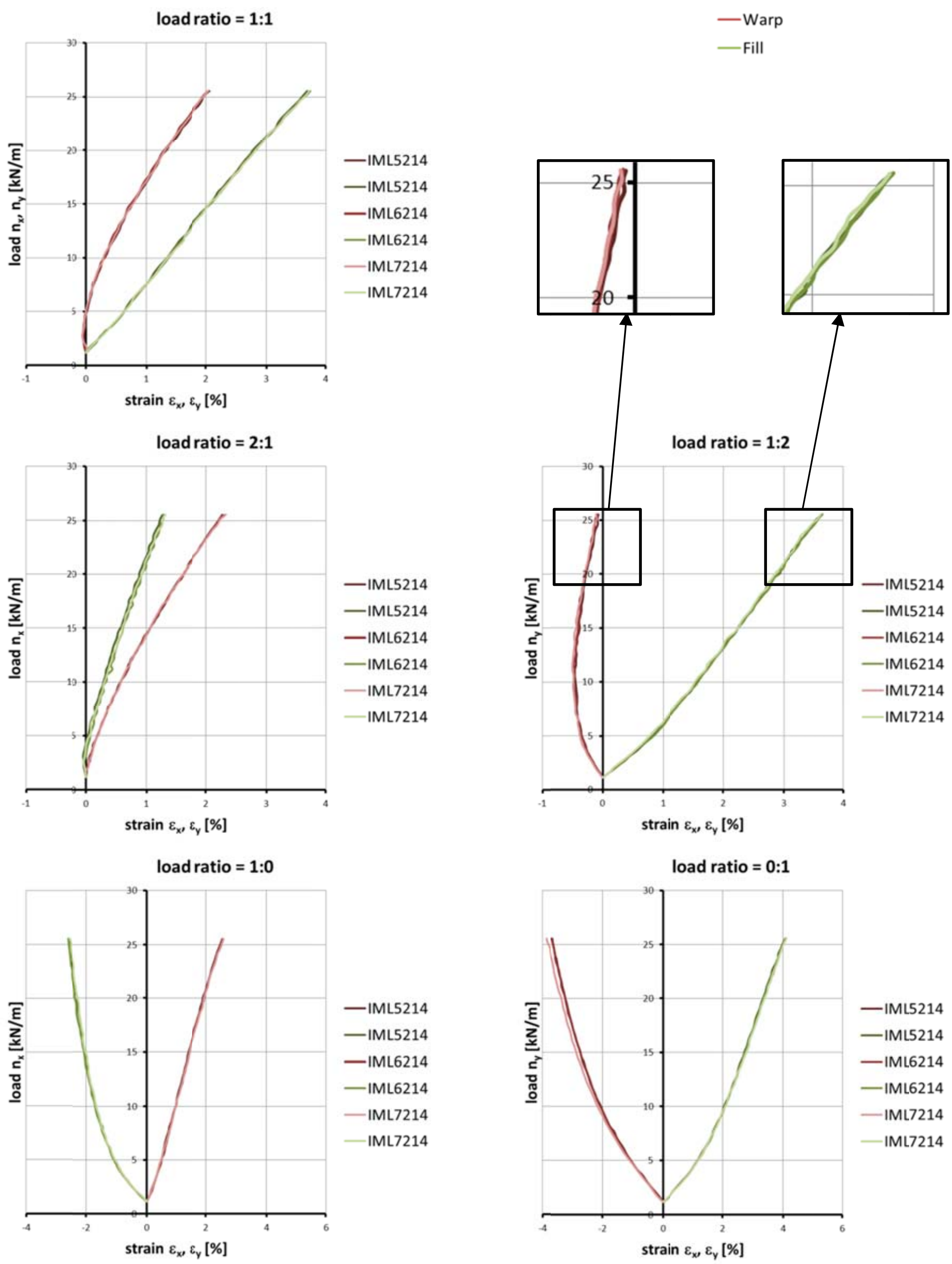

Figure 3: Load-strain-paths resulting from the MSAJ-test procedure for three biaxial tests on a PES-PVC type III with a tensile strength of 115/102 kN/m, all taken from the same batch

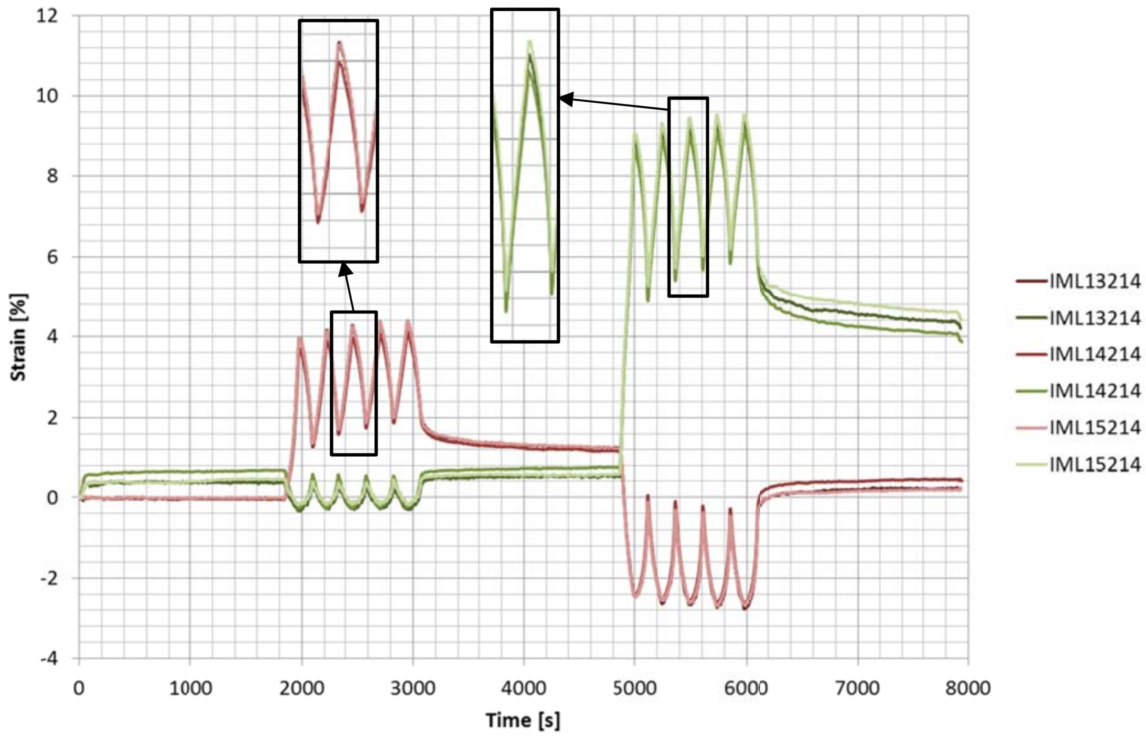

Figure 4: Strain-time-paths resulting from the TensiNet Design Guide-test procedure for three biaxial tests on a PES-PVC type III with a tensile strength of 115/102 kN/m, all taken from the same batch

5 SPECTRUM OF FICTITIOUS DESIGN STIFFNESS PARAMETERS

Recently a discussion on the determination of elastic constants had been started and modifications of existing evaluation procedures have been proposed, resulting in a great variety of values for the "fictitious" elastic constants. In this paragraph, some selected sets of elastic constants are presented, which can be obtained for one and the same exemplary material. On the basis of the aforementioned tests with PES-PVC-material, different sets of elastic constants have been determined at the Institute for Metal and Lightweight Structures (IML) at the University of Duisburg-Essen. The results are given in *Tables 2* and *3*, starting with the set of elastic constant obtained from the original MSAJ-test and determination procedure, see determination option (DO) 1 in *Table 2*.

Bridgens&Gosling [8] have emphasized, that the zero-load-paths of the load ratios 1:0 and 0:1 – which are omitted in the MSAJ determination procedure – are highly relevant for the critical design case of anticlastic membrane structures. Due to mathematical reasons, considering these two load paths, the tensile stiffnesses decrease and the Poisson's ratios increase compared to the original MSAJ procedure, see DO 2 in *Table 2*.

As for the load ratios 1:0 and 0:1 a good correlation between measured load-strain-paths and calculated straight lines can only be obtained with rather big values for the Poisson's ratio while for other load ratios (1:1, 2:1) considerable smaller values are required [9], it is impossible to model all load-strain-paths with only one single set of elastic constants. This problem can be solved if the elastic constants are determined particularly for a specific structure and a specific load case [7]. This means, e.g. for an anticlastic membrane structure with predominant warp stressing under one load case, that the evaluation of the load ratios 2:1 and 1:0 might be reasonable. In this case, the load ratios 1:2 and 0:1 have to be picked out for opposite loading. For plane and synclastic structures as well as anticlastic structures with very small curvature, the load ratios 1:1 and 2:1 fit best. For the exemplary analysed structures in paragraph 6 this proposal leads to the set of elastic constants shown under DO 3 for a plane structure and DO 4 and 5 for anticlastic structures.

From *Table 2* it can be generally seen, that the very low statistical spread of the MSAJ biaxial results is confirmed in the elastic constants of the three validated tests IML5214, IML6214 and IML7214. The Poisson's ratios are much smaller than for Glass-PTFE-materials [5]. The maximum value of the product $\nu_{xy}\nu_{yx}$ is 0.37 in DO5 and with 0.33 almost as high in DO2, but is always clearly smaller than 1, i.e. all the determined sets of fictitious elastic constants are in principle usable in structural analyses.

To determine elastic constants according to the TensiNet Design Guide, three tests have been conducted in the Essen Laboratory for Lightweight Structures. The test conditions have been already described in chapter 4. Elastic constants have been determined with the third loading cycle. The results are shown in *Table 3* for both mathematical formulations: as defined in eq. (6) and for a better comparability as defined in eq. (3) as well.

The determined elastic constants show the above mentioned bigger statistical spread and, furthermore, much bigger Poisson's ratios than those obtained by the original MSAJ procedure. But – in contrast to the behavior of Glass-PTFE [5] – the product $\nu_{xy}\nu_{yx}$ is for all sets of constants clearly smaller than 1, which means, that they are all feasible for a structural analysis, see explanations above.

The enormous spectrum of elastic constants shown in *Tables 2* and *3* could be used by design engineers for one and the same material. Consequently, the question arises whether this rather wide spectrum of elastic constants has a significant influence on the stresses and deformations obtained from the structural analysis or whether the influence is negligible. This question shall be answered in the following paragraph.

Table 2: Different sets of elastic constants obtained by different determination options from one and the same set of MSAJ-test data for a PES-PVC-material type III with a tensile strength of 115/102 kN/m

Determination option (DO)		Test data IML	Tensile stiffness [kN/m]		Poisson's ratio [-]		$\nu_{xy}\nu_{yx}$ [-]
			$E_x t$	$E_y t$	ν_{xy}	ν_{yx}	
1	Original MSAJ-determination: 8 load-strain-paths evaluated (zero-load-paths omitted) [1]	5214	978	650	0.10	0.15	0.02
		6214	974	648	0.10	0.15	0.02
		7214	972	646	0.10	0.15	0.02
2	MSAJ modified: All ten load-strain-paths evaluated (Bridgens&Gosling [8])	5214	746	522	0.47	0.67	0.31
		6214	738	516	0.47	0.67	0.31
		7214	730	514	0.48	0.68	0.33
3	Particular for plane structures: MSAJ-load-ratios 1:1, 2:1, 4 load-strain-paths (Univ. of Duisburg-Essen [7])	5214	890	592	0.14	0.21	0.03
		6214	882	566	0.16	0.25	0.04
		7214	876	562	0.16	0.25	0.04
4	Particular for anticlastic structures, load case with warp stressing: MSAJ-load-ratios 1:0, 2:1, 4 load-strain-paths (Univ. of Duisburg-Essen [7])	5214	826	300	0.33	0.91	0.30
		6214	822	292	0.33	0.93	0.31
		7214	824	296	0.33	0.92	0.30
5	Particular for anticlastic structures, load case with fill stressing: MSAJ-load-ratios 0:1, 1:2, 4 load-strain-paths (Univ. of Duisburg-Essen [7])	5214	300	554	0.82	0.44	0.36
		6214	300	554	0.82	0.44	0.36
		7214	282	552	0.85	0.43	0.37
min/max			282/ 978	292/ 650	0.10/ 0.85	0.15/ 0.93	0.02/ 0.37

Table 3: Three sets of elastic constants obtained by the TensiNet Design Guide tests for three test specimens of a PES/PVC-material type III with a tensile strength of 115/102 kN/m

Test N° IML	Elastic constants according to eq. (6) [kN/m]			Elastic constants according to eq. (3)				
	E_{1111}	E_{2222}	E_{1122}	Tensile stiffness [kN/m]		Poisson's ratio [-]		$\nu_{xy}\nu_{yx}$ [-]
				$E_x t$	$E_y t$	ν_{xy}	ν_{yx}	
13214	1189	866	482	921	671	0.41	0.56	0.23 < 1
14214	1507	1004	699	1020	680	0.46	0.70	0.32 < 1
15214	1220	832	465	960	655	0.38	0.56	0.21 < 1
min/max	1189/ 1507	832/ 1004	465/ 699	921/ 1020	655/ 680	0.38/ 0.41	0.56/ 0.70	0.21/ 0.32

6 INFLUENCE OF THE FICTITIOUS STIFFNESS DESIGN PARAMETERS ON THE STRUCTURAL ANALYSIS RESULTS

Exemplary, comparable calculations are presented for a simple hypar consisting of a PES-PVC-material in order to demonstrate the quantitative influence of the spectrum of fictitious elastic constants obtained in paragraph 5. The same example has been chosen as already considered in [5]. The 10x10 m square hypar has two high points and two low points with fixed edges, see *Figure 5*. The prestress has been chosen to be isotropic with p = 3.0 kN/m in the main anisotropic fabric directions. The shear modulus is supposed to be G = 50 kN/m. The structural analysis has been performed using the finite element software package SOFiSTiK 2012 [10] applying a third order analysis. The structure is vertical loaded downwards with q = 0.60 kN/m^2.

Three different curvatures are analysed: h = 0 m (plane structure), h = 2 m and h = 4 m. The warp direction is running between the high points, so that for the curved variations of the structure the warp direction is stressed for a downward load while in the weft direction the prestress decreases. Load ratios of approximately 4:1 and greater occur in the center of the structure. This lies between the MSAJ load ratios 1:0 and 2:1, thus the elastic constants of DO 4 in *Table 2* are used here. The plane variation of the structure is characterized by load ratios between 1:1 and 2:1. Correspondent to that, elastic constants of DO 3 are used.

Table 4 gives the results of the comparative structural analyses: The maximum membrane stress in warp direction n_w and the maximum deflection f_z, both in the middle of the hypar. For the purpose of the structural analyses, the elastic constants from those tests are used here, that show “average” values: for the MSAJ sets of elastic constants the values of test IML6214 are used, for the Design Guide sets it is test IML15214.

The results in *Table 4* show differences up to 21% between the respective maximum stresses. Regarding the respective deflection, a maximum difference of 40% results. The maximum stresses for the plane structure come from DO2 (10 strain paths), for the curved structures from DO4 (special DO for anticlastic structures).

This exemplary structural analysis demonstrates the immense range of stresses and deflections due to a great variety of fictitious elastic constants that could be used by design engineers for one and the same material product. Nevertheless, it has to be kept in mind, that although the relative deviations are high, the absolute stresses are still far below the allowable stresses – for this more or less simple structure. This may be quite different for more complex structures. None of the underlying determination options is validated by static load tests on curved structural components, which means that the real stresses and deflections are left unknown to the engineer.

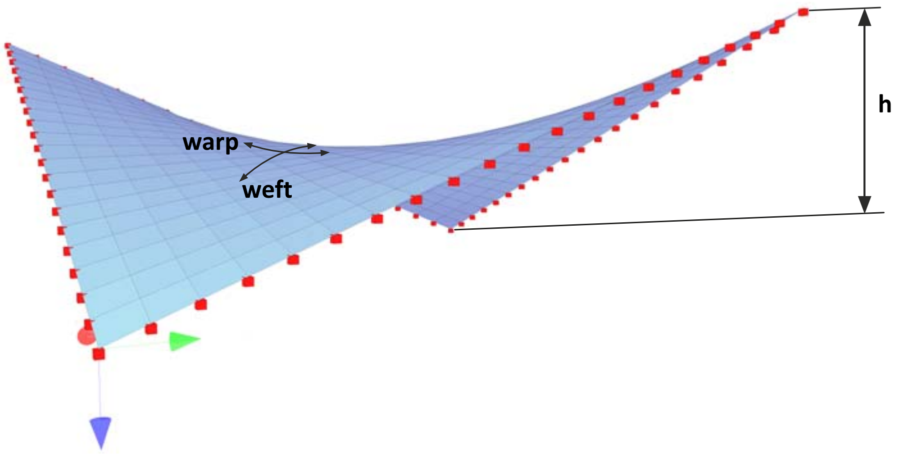

Figure 5: Simple hypar with 2 low points and 2 high points and fixed edges for the exemplary analyses

Table 4: Structural analysis results due to different sets of elastic constants: Maximum membrane stress in warp direction n_w and the maximum deflection f_z, both in the middle of the hypar

Elastic constants fom	max n_w [kN/m]			max f_z [cm]		
	h=0 m	h=2.0 m	h=4.0 m	h=0 m	h=2.0 m	h=4.0 m
MSAJ DO 1	9.3	11.4	8.0	56	28	10
MSAJ DO 2	10.0	11.4	8.8	50	32	14
MSAJ DO 3*	8.4	-	-	56	-	-
MSAJ DO 4**	-	12.0	9.7	-	29	14
MSAJ DO 5	inappropriate for warp stressing due to the exemplary gravitation load					
Design Guide	9.4	11.8	8.7	49	28	11
min/max	8.4/10.0	11.4/12.0	8.0/9.7	49/56	28/32	10/14
Difference [%]	19	5	21	14	14	40

*used for the plane version of the structure
**used for the curved version of the structure (because the gravitation load leads to warp stressing)

7 CONCLUSIONS

Woven textile fabrics are predominantly used for membrane structures. The most used fabrics are Glass-PTFE and PES-PVC material. Both materials is common that they show an anisotropic highly nonlinear stress-strain-behaviour which is – due to a lack of sophisticated solutions – considered in the structural analysis in a very simplified way by means of “fictitious” elastic constants. In the design practice oftentimes only one single set of fictitious elastic constants is determined – like it is proposed in the MSAJ- and the TensiNet Design Guide procedures. This one set is assumed to be valid for all load cases and structural forms and herewith to be independent from them.

In the frame of this contribution, a PES-PVC material has been investigated by experimental biaxial testing according to MSAJ/M-02-1995 and TensiNet European Design Guide. A variation of elastic constants has been evaluated considering different approaches.

As already shown by the authors in [5] for Glass-PTFE material, it could be demonstrated in this contribution that also for PES-PVS material a wide range of elastic constants can be evaluated which results in the end in different stresses and deformations in the structural design whereby the deviations are not negligible. The main question – which set of elastic constant meets the real structural behaviour best – cannot be answered up to now, because no comparable experimental tests exist so far. But what can be concluded from the authors point of view is that a generalized set of constants evaluated on the basis of all load cases does not cover the structural behaviour. The structural designer has to be aware of this sensitivity.

The development of a European design standard for membrane structures as well as a European standard for biaxial testing – in which the authors are involved – is currently under way. This, together with the related research, hopefully leads to a better understanding for the determination of elastic constants and a more unified approach in the structural analysis.

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