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ANALYSIS OF WOODEN FRAMED STRUCTURES WITH SEMI-RIGID CONNECTIONS

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Abstract. In wooden structures, a wide variety of structural arrangements may be built on a basis of the concept of linking parts. Structural analysis of wooden structural systems has followed the progressive understanding on the connections behavior. However, classical structural methods do not consider the deformation of the connections and their effects are not widely known. The analysis of framed structures with semi-rigid connections has two aspects: the structural model and the connection behavior model. This work presents a parametric analysis of wooden framed structures with semi-rigid connections with general nonlinear behavior, in which the connection properties are varied and their effects on different structures are observed. The results show that the structural response is affected by the connection parameters and the structure geometry too.

1 INTRODUCTION

The contribution of the deformation of connections in the behavior of structures has been an object of concern since experimental results have shown its importance. The theories on semi-rigid connections followed the development of matrix analysis of framed structures. Nowadays, with the advances in the finite element analysis commercial programs, the most natural method to account for the contribution of connection deformation is by means of connections elements. The method for the computation of the connection deformation is one of the aspects of the analysis of framed structures with semi-rigid connections. The aspect explicitly related to the material is the description of the behavior of the connections elements, accounting for the relationship between the nodal displacements and forces. For wood, previous works have been made by Jensen and Larsen [1] and others [2]. The process of design may lead to different connections with different effects in the structure, in both quantitative and qualitative senses, depending respectively on the stiffness and the geometry of the structure. It may expect that the optimum design of structures be to a large extent related to the design of the connection. In this work, a parametric analysis varying the moment resisting connection behavior to analyze the effects of the connection properties on the structure response is used.

2 THEORETICAL FOUNDATIONS

2.1 The structural model and solution method

The first order analysis of framed structures has its foundations on the Principle of Virtual Work (PVW). Nonlinear analysis including deformations of the connections is based on the same theory, except that the contribution of the connections in included in the domain, by means of connections elements (Figure 1 and 2).

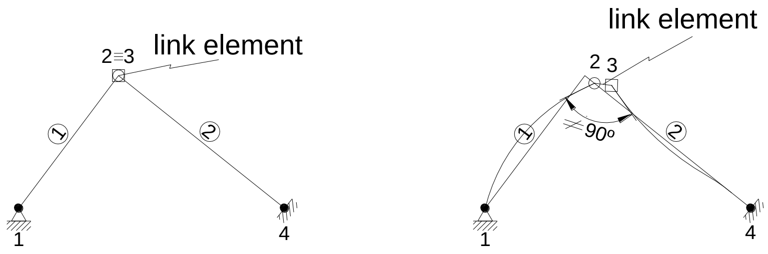

Figure 1: Connection (or link) element and nodal displacements associated with a coordinate system.

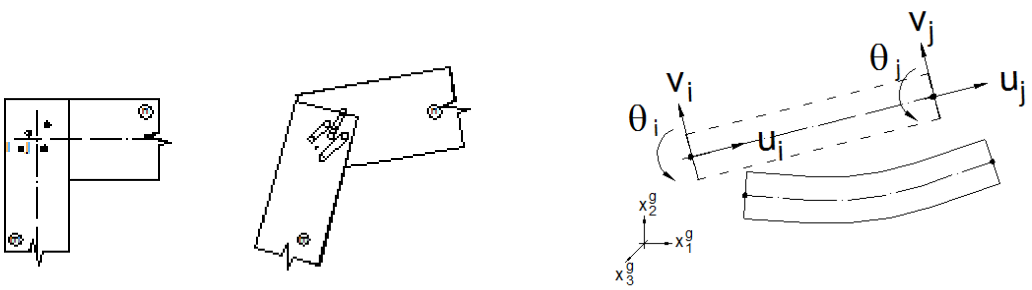

Figure 2: Connector nodal displacements associated with a coordinate system and its deformed configuration.

Internal virtual work due to the bars deformations is computed by means of the simplifications of the classical theory of bending, and the application of the PVW to a generic bar element leading to:

$$K_b^e u_b^e = a_b^e \tag{1}$$

where K_b^e is the stiffness of the bar element, and u_b^e and a_b^e are respectively the nodal displacements and the nodal forces in the bar ends [3]. Internal virtual work due to connections deformations are computed by means of simplifications in the description of the internal stresses and strains, in such a way that the application of the PVW to a generic connection element resulting in:

$$f_c^e(u_c^e) = a_c^e \tag{2}$$

where f_c^e is a vector with non-linear functions of u_c^e, and u_c^e and a_c^e are respectively the nodal displacements and the nodal forces in the connection ends.

Equations (1) and (2) may be assembled into a global equation equivalent to the equation of PVW. Prior linearization of Equation (2) may be accomplished by means of the Newton-Raphson method, and a recursive equation is obtained:

$$K_c^e(u_c^{e(k)})\Delta u_c^{e(k+1)} = a - f_c(u_c^{e(k)}) \tag{3}$$

where *k* is the number of the interactions and:

$$K_{c(ij)}^e = \frac{\partial f_{c(i)}^e}{\partial u_{c(j)}^e} \tag{4}$$

2.2 The nodal force and displacement relationship

The relationship between nodal loads and displacements for the connections elements may be developed more commonly by means of an experimental procedure or an approximate theoretical method. According to this second method, the nodal forces are assumed to be statically equivalent to the set of forces acting in each connector (*Fp*) and the nodal displacements cinematically equivalent to the slip of each connector (*Δp*). The virtual work of the forces on the virtual relative displacements of the connectors is theoretically assumed to be equivalent to the virtual work of the stresses on the virtual strains in the connection volume. The force in one connector is related to the relative displacements in the connector by means of a non-linear relationship of the type, originally proposed by Foschi [4] via:

$$F_{p,\alpha} = (C_{1,\alpha} + C_{2,\alpha}\Delta_{p,\alpha})[1 - exp(-C_{3,\alpha}\Delta_{p,\alpha} / C_{2,\alpha})] \tag{5}$$

and:

$$C_{i,\alpha} = \frac{C_{i,0}.C_{i,90}}{C_{i,0}.sin^2\alpha + C_{i,90}.cos^2\alpha} \quad (i = 1,2,3) \tag{6}$$

where $C_{i,\alpha}$ is connector stiffness, and α is the wood fiber direction, for example, $\alpha=0$ means that the parallel direction to the fibers [5].

On these bases, the relationship given by Equation (2) may be developed. The construction of the functional form of the connections behavior according to these principles was made by Jensen and Larsen [1] and also by Santana and Mascia [6].

3 PARAMETRIC ANALYSIS

3.1 On the Used Method

The method consisted of designing a framed structure with minimum and maximum typical dimensions for commercial use in Brazil, with L=8.0 and 16.0 m.

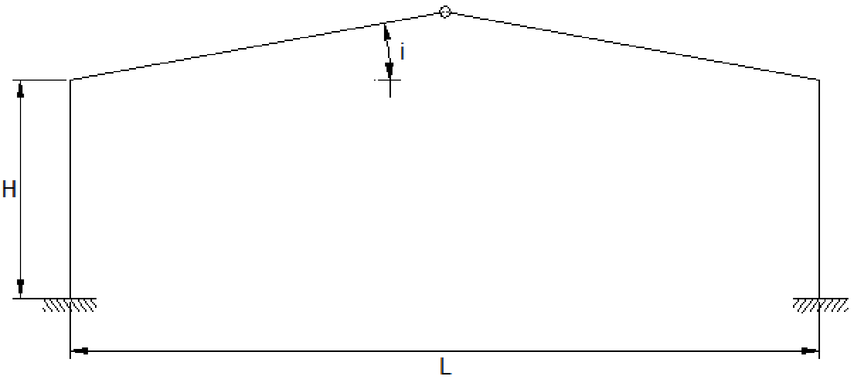

Figure 3: Framed structures and details of connectors used in the analysis

Initially, the structures were designed to characteristic forces determined according to the classical theory of framed structures. The prior results obtained in such a way, using the static scheme of the structure and a detail of the connections shown in Figures 4 and 5, are presented qualitatively in Figure 6.

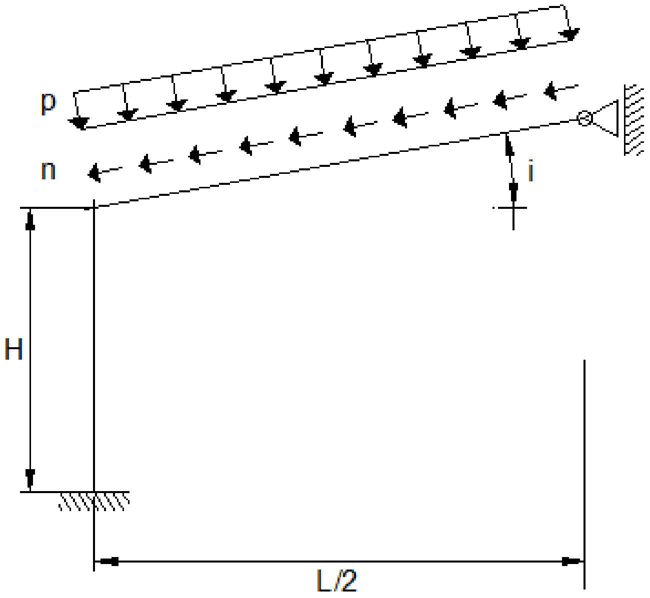

Figure 4: Static scheme of framed structure used in the analysis

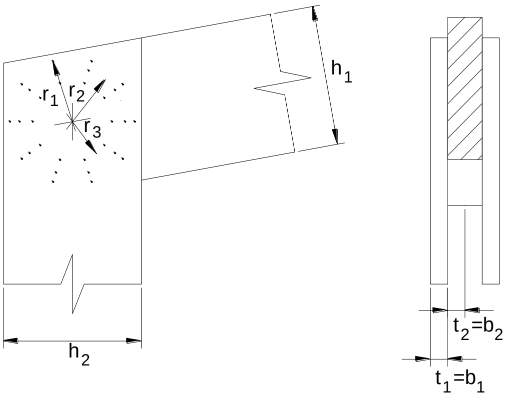

Figure 5: Details of connectors used in the analysis

The initial stiffness of the connections and the parameters that define its behavior were determined. Then, the analysis of each structure was repeated according to the model described previously. The parametric analysis involved the assignment of different connections for the same demand of strength, and the consideration of non-linear and approximate linear behavior for the connections in addition to this.

Figure 6 presents the typical diagrams of bending moment, normal and shear forces for the structure shown in Figure 4.

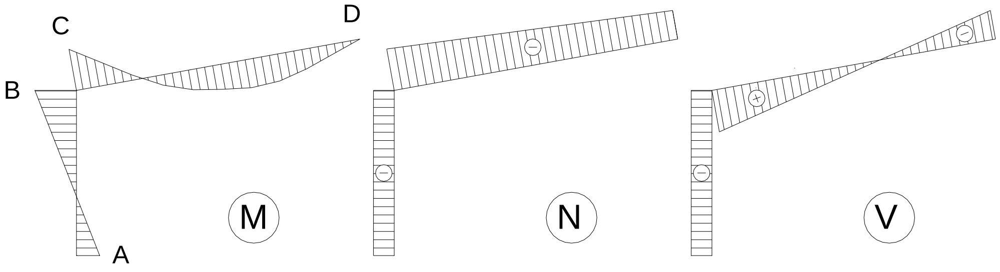

Figure 6: Typical internal bending moment, normal force and shear force diagrams

3.2 Design governing parameters

Usual design parameters are the initial stiffness and the resistance of the connections. Technical codes are founded in linear behavior of connections ([7], [8]). For the purpose of analysis, the connections are considered, which could be actually non linear before the limit value of resistance, according to the equations (5) and (6). Thus, the design parameters are $C_{1,0,d}$, $C_{2,0,d}$, $C_{3,0,d}$, $C_{1,90,d}$, $C_{2,90,d}$ and $C_{3,90,d}$, which may be obtained from tests of an individual connector in both parallel and perpendicular directions in relation to fibers [9]. The only parameter considered in linear analysis is the stiffness of the connectors, taken as the initial stiffness of the corresponding non-linear behavior.

4 RESULTS AND DISCUSSIONS

In this work two examples based on Figure 2, 3,4 and 5 were considered. Table 1 presents the geometrical characteristics of the structures and the applied loads:

Table 1: Geometrical characteristics of the structures (in m) and the applied loads (in KN/m)

L	H	h_1	h_2	b_1	b_2	p	n
8	4	0.5	0.5	0.06	0.08	0.024	0.006
16	6.4	0.7	0.7	0.12	0.16	0.028	0.007

Some of the results obtained for internal, moments and displacements forces are listed in Tables 2 and 3.It should be noted that, for example, M_A represents the bending moment at point A, shown in Figure 6. Also the first number of each line results from non-linear analysis, and the second, from linear analysis. Numbers in parentheses indicate the deviation from rigid connections analysis.

Table 2: Bending moment, shear force and displacements according to stiffness parameters for L=8.0 m

	$K_{M,ini}$ (KNcm/rad)	M_A (kNcm)	M_C (kNcm)	V_C (kN)	θ_{AB} (rad)	u_C (cm)
L=8.0m	∞	982	-1,213	7.96	0	-0.1344
Non-lin. Linear	441,780	1,303 **(33%)**	-1,090 **(-10%)**	7.76 **(-3%)**	1.8496	-0.2940 **(119%)**
		1,444 **(47%)**	-1,128 **(-7%)**	7.67 **(-4%)**	1.5264	-0.2659 **(98%)**
	625,422	1,213 **(24%)**	-1,113 **(-8%)**	7.81 **(-2%)**	1.0999	-0.2455 **(83%)**
	(42%>previous)	1,289 **(31%)**	-1,152 **(-5%)**	7.77 **(-2%)**	1.2848	-0.2294 **(71%)**

Table 3: Bending moment, shear force and displacements according to stiffness parameters for L=16.0 m.

	$K_{M,ini}$ (KNcm/rad)	M_A (kNcm)	M_C (kNcm)	V_C (kN)	θ_{AB} (rad)	u_C (cm)
L=16.0m	∞	4,423	-5,070	17.88	0	-0.2488
Non-lin. Linear	2,126,140	5,780 **(31%)**	-4,468 **(-12%)**	17.38 **(-3%)**	1.2371	-0.4994 **(101%)**
		6,347 **(43%)**	-4,645 **(-8%)**	17.17 **(-4%)**	1.6620	-0.4354 **(75%)**
Non-lin. Linear	2,366,820	5,652 **(28%)**	-4,550 **(-10%)**	17.42 **(-3%)**	1.3651	-0.4704 **(89%)**
	(11%>previous)	6,083 **(38%)**	-4,685 **(-8%)**	17.27 **(-3%)**	1.4684	-0.4546 **(83%)**

From these results, it can be observed that the linear analysis leads to results very close to those obtained with non-linear analysis, with major deviations for the displacements. The results are dependent on the connector parameters and the resultant global initial stiffness of the connection ($K_{M,ini}$). By comparing the rigid and the semi-rigid results, the displacements are significantly affected, as well as the bending moment distribution. On the other hand, forces are not affected to a great extent.

5 CONCLUSIONS

The consideration of structures with semi-rigid connections may be accomplished with a matrix method, which is widely available inside finite element analysis commercial programs [10]. Linear analysis may be reasonably used for the design of semi-rigid connections, according to a linear or bilinear behavior of connections. In spite of that, non-linear analysis applies to the modeling of the real behavior of the connections. In any case, the effect of connections deformation is significant, especially for displacements and moment distribution. The re-design and the optimization of the structure depend primarily on a balance between the strength and stiffness of the connection founded in its effect on the bar elements.

REFERENCES

[1] Jensen, J. L., Larsen, H. J. Modelling of semi-rigid joints and their influence on the behaviour of structures. In: World Conference on Timber Engineering, 5. *Proceedings*. Montreux, Switzerland: Presses polythecniques et universitaires romandes, 1998. v. 1, p. 305-312.

[2] Santana, C. L. O. *Analysis of framed structures with semirigid connections*. Campinas: State University of Campinas, 2002. 214 p. (PhD Thesis).

[3] Gere, J. M., Weaver JR., W. *Analysis of Frame Structures.* Princeton: Van Nostrand, 1965.

[4] Foschi, R. Load-Slip Characteristics of Nails. *Wood Science.* v. 7, n. 1, p. 69-76, 1974.

[5] Bodig, J., Jayne, B. A. Mechanics of wood and wood composites. New York: Van Nostrand Reinhold, 1982. 711 p.

[6] Santana, C. L. O., Mascia, N. T. Modeling of semi-rigid nailed joints for application in structural analysis. In: International Rylem Symposium. Stuttgart, Germany, 2001. *Joints in timber structures; Proceedings*... Cachan, France. p. 111-120.

[7] European Committee for Standardization. prEN 1995-1-1. *EUROCODE 5*: Design of timber structures – part 1-1: General rules and rules for buildings. Brussels, Belgium; 2003.

[8] ABNT/NBR 7190 - Projeto de Estruturas de Madeira (*Timber structures design*). Rio de Janeiro, Brazil. 1997.

[9] Santana, C. L. DE O.; Mascia, N. T. . Wooden framed structures with semi-rigid connections: quantitative approach focused on design needs. *Structural Engineering and Mechanics,* v. 31, p. 315-331, 2009.

[10] ANSYS INC. “*ANSYS* User´s manual; Revision 5.2”, Houston:1995, Student version.

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