LOCAL/GLOBAL NON-INTRUSIVE PARALLEL COUPLING FOR LARGE SCALE MECHANICAL ANALYSIS

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Abstract. The permanent increase in available computing resources can achieve more and more ambitious numerical simulations (most of the time using the finite element method). When dealing with non-linear complex models on large 3D structures, the computational cost becomes prohibitive. In this paper, we present the recent developments linked to an innovative computing method: non-intrusive coupling. Such a method allows to efficiently take into account local modifications on an initial existing model in a non-intrusive way: the previously computed analysis is left unchanged. Large scale linear models can thus be easily computed, then localised non-linear complex models can be used to pinpoint the analysis where required on the structure. After a presentation of the scientific context and a description of non-intrusive coupling methods, we will present its application to crack growth simulation and parallel structure analysis.

1 CONTEXT AND INDUSTRIAL ISSUES

Every time one wants to create a new mechanical object, a conception cycle has to be respected, involving Computer Aided Design (CAD) and Finite Element Analysis (FEA). When dealing with a critical part of a mechanical product, a particular attention should be paid to the FEA. This is the point we will focus on in this paper. Indeed, in order to perform a mechanical simulation, one needs both a geometrical and a mechanical model. Some cases require a complex mechanical model, and some other a detailed geometry. The works which will be presented in this paper try to bring a solution to a recurrent FEA problem: how to compute a complex mechanical problem in a way requiring the least effort? There are two main problems which need to be addressed: geometric complexity and mechanical behaviour complexity.

1.1   Geometric modifications in Finite Element Analysis

The most direct way to perform a structural analysis via numerical simulation is to use the CAD model as the geometric model for the simulation. Indeed, the creation of a structure geometric model can be very time consuming. Moreover, for some critical parts, the mesh has to be certified before being used (aeronautical parts for instance). All in all, the fact is that we cannot afford to create a new geometric model or a new mesh each time we need to perform a simulation. Moreover, it appears that in the life cycle of a product, its geometric specifications can change (a crack can appear, some holes can be drilled). One objective of the computing method presented here is to bring a way to reuse an initial geometric model and its finite element mesh to compute structural analysis involving local details.

1.2   Localised complex mechanical behaviour

Another class of complex problems involves mechanical behaviour of the structure we analyse. Indeed, in structural analysis, two types of models have to be considered: linear and non-linear ones. Of course, the time required to complete the analysis will depend on the type of model. The use of certain complex non-linear models on very large structures fatally leads to considerable cost in terms of computer resources, often beyond what is currently available. The fact is that most of time, a non-linear model is useful only on a small part of the structure, which can be represented elsewhere with a linear model. Again it is possible to save a lot of computation time by reusing an existing linear model (and the corresponding solution) on a full structure: a non-linear model will thus be considered only on localised areas (see [1]). The objective of non-intrusive coupling is here to make us able to merge several local complex non-linear models with a global linear one, without modifying this last one. In other words, if we need to perform several analysis of a large scale structure, the linear model will be assembled on the full structure only once, whereas the localised analysis will be performed as many times as necessary.

## 2   MODEL COUPLING METHOD AND ALGORITHM

As said previously, the non-intrusive coupling algorithm (see [10] and [17]) aims to perform a structural analysis using two separate models:

- a pre-existing simple model involving the full structure, which will represent the global mechanical behaviour (linear elasticity),

- an ad-hoc complex model involving only a small part of the structure, representing the local mechanical behaviour (plasticity for example).

2.1   Coupling algorithm

In this paper, a two scale finite element method is considered. We will denote with the letter Ω the geometric domains and with the letter M the mechanical behaviour models.

Two coupled overlapping models are considered (see Fig. 1): a global one $M = M_1 \cup M_2$ (involving the full structure $\Omega = \Omega_1 \cup \Omega_2$, *i.e.* including a large number of nodes) and a local one $\tilde{M}_2$ (involving only $\tilde{\Omega}_2$). Thus the global model will be treated as a coarse linear one, whereas the local one will take into account the localised (potentially non-linear) behaviour. Basically, the idea is to reach the equilibrium between the global M_1 and
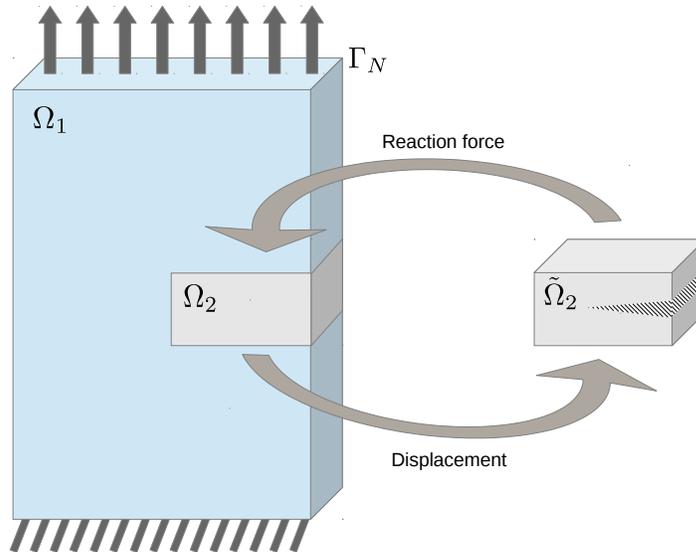
Figure 1: Non-intrusive coupling – Situation overview

the local $\tilde{M}_2$ models at the interface by the mean of an iterative algorithm, similar to those used in domain decomposition methods, *i.e.* solving alternately the Dirichlet (*resp.* Neumann) problem on the local (*resp.* global) model until convergence. Actually, we seek to replace the global solution of M_2 on Ω_2 by the one we would get with $\tilde{M}_2$. Let us consider domain decomposition in the linear case involving M_1 and $\tilde{M}_2$: we then get a monolithic coupling system (see Eq. (1)).

$$\begin{bmatrix} K_1 & 0 & C_1^T \\ 0 & \tilde{K}_2 & -\tilde{C}_2^T \\ C_1 & -\tilde{C}_2 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ \tilde{U}_2 \\ \Lambda \end{bmatrix} = \begin{bmatrix} F \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

Here K stands for the stiffness matrix, F for the load vector, U for the displacement field, C for the interface coupling matrices and Λ for the Lagrange multipliers vector (allowing for non-conforming meshes at the interface, using a mortar method). An iterative algorithm is then set up in order to dissociate the two models when solving the linear system.

Algorithm 1: Global/local domain decomposition – Iterative solver

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Data: Λ^0, ϵ

k = 0

while $\eta > \epsilon$ **do**

$\quad$ **Global problem computation (Neumann problem)**

$\qquad K_1 U_1^{k+1} = F - C_1^T \Lambda^k$

$\quad$ **Local problem computation (Dirichlet problem)**

$\qquad \begin{bmatrix} \tilde{K}_2 & -\tilde{C}_2^T \\ -\tilde{C}_2 & 0 \end{bmatrix} \begin{bmatrix} \tilde{U}_2^{k+1} \\ \Lambda^{k+1} \end{bmatrix} = \begin{bmatrix} 0 \\ -C_1 U_1^{k+1} \end{bmatrix}$

$\quad$ **Convergence test**

$\qquad \eta = \|\mathcal{F}_{\Omega_1/\tilde{\Omega}_2} + \mathcal{F}_{\tilde{\Omega}_2/\Omega_1}\|/\|F\|$

$\quad$ k = k+1

end

The convergence test used here relies on the equilibrium between the two domains Ω_1 and $\tilde{\Omega}_2$. We give below the corresponding equality in terms of finite elements objects at iteration k.

$$\mathcal{F}_{\Omega_1/\tilde{\Omega}_2} + \mathcal{F}_{\tilde{\Omega}_2/\Omega_1} = \left(K_1 U_1^k + C_1^T \Lambda^k\right)\big|_\Gamma \tag{2}$$

Here, the non-intrusiveness of the method comes from a fictitious prolongation of the solution from Ω_1 to the full global domain Ω. We then definite U so that $U|_{\Omega_1} = U_1$ and $U|_{\Omega_2} = U_2$.

$$KU = K_1 U_1 + K_2 U_2 \tag{3}$$

Using this equality at the global computation step gives us the expression of the equation standing for the prolonged global model at each iteration k.

$$KU^{k+1} = F - C_1^T \Lambda^k + K_2 U_2^k \tag{4}$$

Actually, the iterative algorithm tends to replace the global stiffness on Ω_2 by the local one, through an additional right hand side load vector. The coupled equations system in the non-intrusive case then rewrites into the following form.

$$KU^{k+1} = F - C_1^T \Lambda^k + K_2 U_2^k \tag{5}$$
$$\begin{bmatrix} \tilde{K}_2 & -\tilde{C}_2^T \\ \tilde{C}_2 & 0 \end{bmatrix} \begin{bmatrix} \tilde{U}_2^{k+1} \\ \Lambda^{k+1} \end{bmatrix} = \begin{bmatrix} 0 \\ -C_1 U_1^{k+1} \end{bmatrix}$$

In a few words, the global and the local models are coupled via displacement and effort swap at the interface. The computational cost involved by the algorithm can be significantly smaller than the one involved by a full scale nonlinear complex computation. Still, it may be noted that, as such, the performance of the method is dependent on the stiffness gap between the two models M_2 and $\tilde{M}_2$. Indeed, the more the stiffness gap is important, the more the algorithm will require a large number of iterations to converge.

This is very inconvenient for crack propagation simulation: the stiffness gap increases as the crack growths. Hopefully, a Quasi-Newton correction (see [10]) allows to get rid of that problem: thanks to the Sherman-Morrison-Woodbury formula, it is possible to modify the tangent stiffness of the global model in a non-intrusive way (see [1]). First of all, let us remark the following equalities arising from the coupling formulation:

$$KU^k = K_1U_1^k + K_2U_2^k \tag{6}$$
$$\left.(K_1U_1^k)\right|_{\Gamma_N} = F \tag{7}$$

It is possible to reformulate Eq. (5) into a Newton algorithm (*i.e.* in an incremental formulation) by adding $-KU^k$ both on the left and right sides of the full domain equation at iteration k.

$$K\left(U^{k+1} - U^k\right) = F - C_1^T\Lambda^k + K_2U_2^k - KU^k \tag{8}$$

Indeed, making use of Eq. (6) and (7) into Eq. (8), one can give the following formulation:

$$U^{k+1} = U^k - K^{-1}f(U^k) \tag{9}$$

where f is the finite element operator computing the forces equilibrium residual on Γ between Ω_1 and $\tilde{\Omega}_2$ given the full displacement U^k at iteration k.

$$f(U^k) = \left.(K_1U_1^k + C_1^T\Lambda^k)\right|_{\Gamma} = \mathcal{F}_{\Omega_1/\tilde{\Omega}_2} + \mathcal{F}_{\tilde{\Omega}_2/\Omega_1} \tag{10}$$

In practice matrix K is a bad approximation of the true gradient ∇f. Thus, the Newton scheme given at Eq. (9) cannot be used as such, as it would lead to tremendous number of iterations when the stiffness gap between the local and the global model is important. Instead, one can rely on Quasi-Newton methods to update the matrix K. The Symmetric Rank One (SR1) update is an easy-to-implement and efficient way to build a sequence of matrices K_k convergent toward ∇f (we will assume $K_0 = K$ when initialising the algorithm).
Let us define $d_k = U^{k+1} - U^k$ and $y_k = f(U^{k+1}) - f(U^k)$. At iteration k, we seek to update K_k into K_{k+1} with the SR1 formula, *i.e.* with an update rank-one symmetric update, while verifying the secant equation for each iteration $k > 1$:

$$K_{k+1} = K_k + \frac{(y_k - K_kd_k)(y_k - K_kd_k)^T}{d_k^T(y_k - K_kd_k)} \tag{11}$$

Note that in the context of the SR1 update, Eq. (9) rewrites $K_kd_k = -f_k$, so that Eq. (11) can be given in a simplified form (where $f_k = f(U^k)$).

$$K_{k+1} = K_k + \frac{f_{k+1}f_{k+1}^T}{d_k^Tf_{k+1}} \tag{12}$$

Nevertheless, one has to keep in mind the non-intrusiveness constraint of the coupling algorithm, *i.e.* do not modify the full stiffness matrix K. This can be achieved using the

Sherman-Morrison formula on Eq. (12), leaving us with the following relation which can be used in order to compute $K_k^{-1}f_k$ in a iterative manner based upon the knowledge of $K_0^{-1}f_k$.

$$K_{k+1}^{-1} = K_k^{-1} - K_k^{-1}f_{k+1}\frac{f_{k+1}^T K_k^{-1}}{f_{k+1}(d_k + K_k^{-1}f_{k+1})} \tag{13}$$

2.2 Connections with standard methods

For several years, model coupling is used in nearly all the engineering departments. One can cite two main classes of method:

- First, when dealing with large mechanical structures, one often needs a very precise analysis on localised small parts. As said before, we cannot afford to use a fine mesh and a precise model on the whole structure. Instead, structural zooming is often a solution (see [8]). It consists on computing a precise solution on a small area of a structure, using the pre-computed coarse solution on the full structure as boundary conditions. No iteration is applied in such methods.

- Then, one can cite domain decomposition method. These methods allow to efficiently couple models and meshes; nevertheless it requires significant efforts in order to make interconnections between the models (see *e.g.* [2], [4] and [9]). Moreover, no pre-computed solution can be reused, resulting on very large computing resource needs.

In fact, the non-intrusive coupling algorithm provides a generic method which allows coupling several models with the least effort, while preserving the inherent advantages of the methods presented above:

- The global pre-existing model is unmodified.

- Incompatible meshes can be interconnected via a mortar-like method, for example.

- Parallel resolution can reduce the computation time in case of multiple local models.

3 APPLICATION TO CRACK GROWTH SIMULATION

The main application we focus on in this paper is crack growth simulation. For a lot of engineers, sustainability in construction (aeronautical, naval) is a priority. Indeed, during the life cycle of a mechanical structure (steel, concrete), cracks can appear, endangering the integrity of the structure. Thus, forecasting the propagation path of such cracks is a major issue for engineers. Nevertheless cracks locations cannot be known a priori when designing a structure or setting up a finite element mesh for initial structural analysis. Using the common FEA tools, when one needs to simulate crack propagation, two main solutions are available:

- Set up a crack conforming mesh (see [3] and [7]) at each step of the propagation. When combined to an adapted mesh refinement at crack tip, it leads to accurate results. The main drawback of conforming meshing is the substantial computational resulting cost. For that reason, direct crack meshing is rarely used as such, unless a very fast and efficient remeshing algorithm is set up.

- Use X-FEM method (see [13]) on an existing mesh. Theoretically, such a method allows the mesh to be not conforming to the crack faces. Nevertheless, most of time, remeshing is necessary at crack tip if one wants to get an accurate enough solution.

All in all, common methods do not allow reuse of existing meshes without (at least local) modifications, resulting in an extra computational cost. Instead, another possibility is to consider two different models, standing for different scales: a global linear elastic model, representing the full structure (healthy structure) and a local (potentially nonlinear and/or XFEM) model for the cracked domain (see Fig. 2). Using the non-intrusive algorithm for
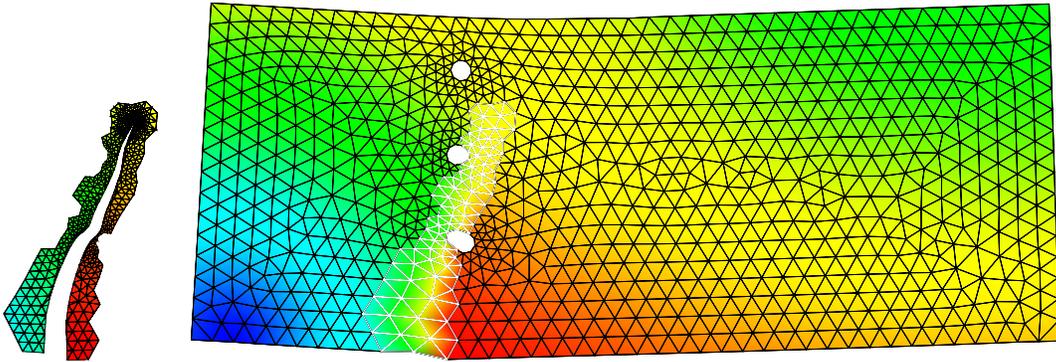
Figure 2: Non-intrusive FEM/X-FEM coupling

crack propagation simulation (see [12] and [15]) will provide the following advantages:

- Remeshing will be necessary only on the local model.

- Non-linear behaviour will be used only on the local model.

- The global linear model will be assembled once and the stiffness matrix will be factorised only once too.

Altogether, the non-intrusive coupling algorithm allows reusing a pre-existing mesh and linear elastic model (*i.e.* stiffness matrix) in order to perform computationally cheap crack growth simulation. The results presented in this paper have been computed using Code_Aster, a structural engineering software developed by Électricité de France. Both global and local models have been computed as a black box using this software, whereas

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the coupling (*i.e.* interface data exchange) has been done using a Python code developed for that purpose.

## 4 PARALLEL COMPUTING, DISTRIBUTED MICRO MODELS

The two main features of the non-intrusive coupling algorithm are: "non-intrusive" which is the possibility to locally modify an initial model, and "coupling" which is the possibility to use different models to compute a single structure. This last feature will be developed now more in details. Indeed, it is possible to consider several non-overlapping local models (for example if we want to represent several cracks on the same structure). Each local model will be completely independent from the others, allowing for an efficient parallel solver to be set up, using for instance MPI communications. When dealing with non-linear local models, parallelism is often the only possibility making FEA possible. We want to draw the readers attention here on an important detail: unlike domain decomposition methods (*e.g.* FETI method, see [9]), each local analysis can be carried out independently from the others. An application of this property is assemblies analysis. Indeed, for very large structures, simulation of assemblies is computationally very expensive. Moreover, if the model contains too much contact areas, it becomes difficult to make the analysis possible because of the high complexity of the non-linear behaviour. Thanks to non-intrusive coupling, it is possible to compute each junction assembly separately: common contact algorithms will be able to perform the computation easily. We present here another application of parallel computing applied to non-intrusive coupling: a multi-cracked plate (see Fig. 3). We consider here, as an academic test case, three disjoint cracks. From a global coarse mesh, we generate three local refined patches on which an X-FEM model is applied. For a given iteration of the coupling algorithm, the three local models are computed in parallel, thanks to the MPI communication. The important point here is that there is no direct communication between the local patches. Every interface data exchange (displacement and effort) takes place between the global and one local model, through a coupling engine. It may also be noted that, in that example, no load is applied to the global model; we constrained only three degrees of freedom in order to disable rigid body motions. The only load applied is a hydrostatic pressure on the crack lips. Then, the local loading spreads to the global model only through the additional global right hand side load vector. Moreover, the stiffness gap between the global and the local models is very important here, as we considered three cracks. Still, the Quasi-Newton method allows for an important speed-up (see Fig. 4).

5 INTEGRATION OF RESEARCH CODES INTO COMMERCIAL SOFT-WARE

Finally, the last interest of non-intrusive coupling we will develop is the possibility to easily merge research codes and commercial softwares. Indeed, as presented previously, the only data exchange occurring between the global and the local models is interface
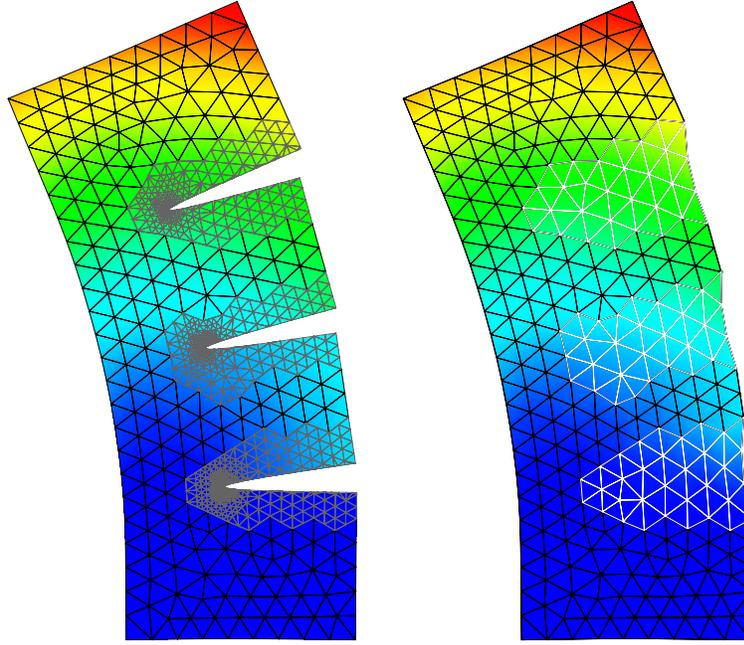
Figure 3: Multi-crack distributed patches
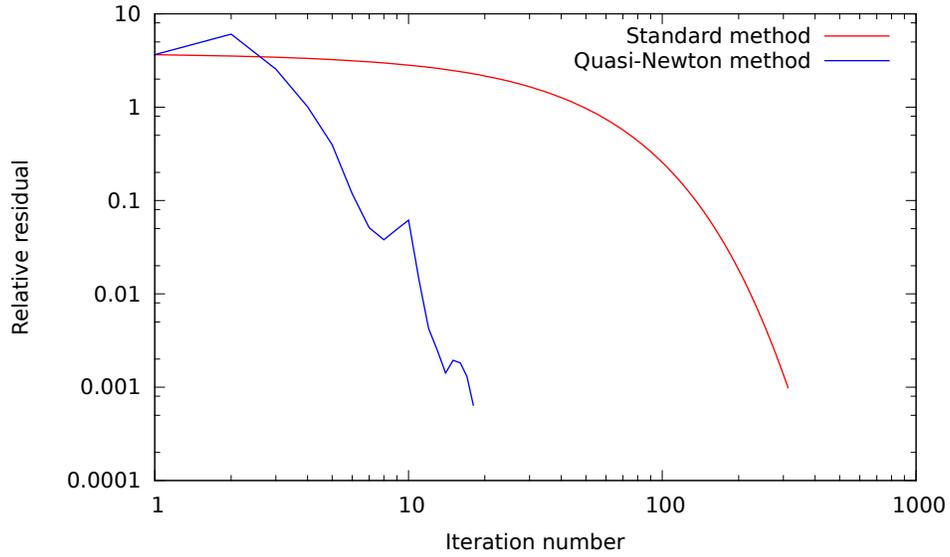
Figure 4: Convergence of the algorithm

displacements and forces. Thus it is possible to compute the global solution from a commercial software using the existing models and solvers, and compute the local solution

using an ad-hoc model developed with any code. Using a MPI communication between the two codes (*i.e.* between the two models, global and local) makes the communication straightforward: we only have to focus about the data we need to exchange, as MPI will provide his own standard for language compatibility. For instance we can cite [10] which compute an aeronautical structure analysis with localised plasticity within a linear model from Abaqus/Standard. If we focus our interest on crack growth simulation, we can cite [5] and [14] which propose a special treatment for crack tip displacement singular field (analytical solution, adapted radiating mesh) within an elastic linear model.

6 CONCLUSION

In this paper, a non-intrusive coupling approach has been presented. This method allows to take into account local features in existing FEA models without actually modifying it. The main purpose of the method is to make FEA easier, as finite element models preparation can sometimes be more time consuming than the computation itself. Thanks to this algorithm, we have been able to set up a two and three dimensional crack growth simulation, using Code_Aster for the mechanical computations and MPI based communications for the interface coupling. All of the process is wrapped into a Python API. The distributed implementation of the algorithm we proposed here allows for high performance multi-patch parallel computations. In a near future, we seek to make the method even more flexible by extending it to non-coincident patches (see [16]). It may be noted that the algorithm can also be used to couple different mechanical representations (e.g. 2D/3D coupling, see [11]) or to couple different analysis methods (deterministic/stochastic models, see [6]) in a non-intrusive way.

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