

ASSESSING MATERIAL NONLINEARITIES IN LARGE COMPOSITE STRUCTURES BY PREDICTING ENERGY DISSIPATIONS AT THE MESOSCALE

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Abstract. Composite materials represent multi-hierarchical systems where effects at several length scales control the overall material behaviour. Nonlinearities such as damage and plasticity typically arise at smaller length scales and their consideration necessitates models of the meso- or microstructure of a given composite. Such analyses are feasible for small computational domains, however, incorporating multiple length scales into large structural analyses requires huge computational effort. The present paper aims at reducing computational cost of large scale analyses by accounting for nonlinearities in an approximate way. To this end, a simulation methodology is introduced which interprets linear elastic structural simulations by recourse to results from nonlinear simulations conducted at smaller length scales. The concept is demonstrated by the example of a U-section beam with variable cross-section in a four point bending test set-up consisting of several layers of textile fabrics.

1 INTRODUCTION

The application of fibre reinforced polymers (FRPs) has increased significantly during the past years. Especially their favourable weight specific properties, such as high specific stiffness and strength, make their usage particularly advantageous in lightweight components. In this context, composite laminates consisting of several layers of unidirectionally

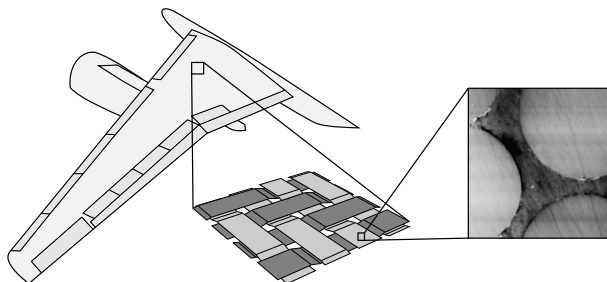


Figure 1: Different lengths scales in a component consisting of composite laminates. The structure itself represents the macroscale (left). The plies of the laminate, built from several tows, characterise the mesoscale (middle); fibres in a tow represent the microscale (right, [1]).

(UD) aligned fibres or textile prepreps are most commonly applied. Such materials represent multi-hierarchical systems with complex microstructures where effects at several length scales control the overall behaviour. The smallest length scale (microscale) to be considered in this regard is typically characterised by the dimensions of the inhomogeneities (e.g. fibre diameter) and describes the basic interaction between fibres and the surrounding matrix. The other end of the spectrum is described by the length scale of the component (macroscale), where the macroscopic boundary conditions and loads are applied. In most cases, also intermediate length scales (mesoscales) can be defined, which, for example, may be represented by the interaction of individual plies in a laminate or the textile architecture of some fabric ply, cf. Fig. 1. Within the context of this paper, the term mesoscale will always be used to denote the textile length scale. The complex microstructure and the intrinsic anisotropic behaviour of such composite textiles complicate the engineering design, especially when it comes to exploiting the full potential of these materials. In industry, the design is often conducted by applying classical lamination theory in combination with empirically or phenomenologically based failure criteria, cf. [2] and [3]. Usually, they identify stress states where first failure at ply level occurs (first ply failure, FPF) and assume linear elastic behaviour up to this point. However, FPF does not necessarily mean ultimate failure of the component but just indicates the onset of nonlinear mechanisms. Furthermore such criteria do not give any information of what happens beyond the predicted limit and, hence, give no indication of the severeness of the FPF state regarding further load increase. In order to extend the predictive capabilities, nonlinear mechanisms, such as damage and plasticity, have to be accounted for. Although these nonlinearities affect the macroscopic behaviour, they typically arise at smaller length scales. Therefore, their consideration necessitates models of the meso- or microstructure of a given composite. Analyses of this type are feasible for small computational domains (e.g. periodic unit cells), however, incorporating multiple length scales into large structural analyses, as proposed in approaches known as FE^2 (e.g. [4]), leads to vast computational demands.

The present paper aims at reducing the computational cost of nonlinear large scale

analyses by introducing a methodology which accounts for nonlinearities in an approximate way. To this end, linear elastic structural simulations are interpreted by recourse to results from nonlinear simulations conducted at smaller length scales. First, the different steps and the operational sequence within the proposed methodology is outlined. Afterwards, each step is explained in detail followed by an example illustrating a proper application.

2 OVERVIEW OF THE PROPOSED METHODOLOGY

The proposed methodology consists of two basically decoupled steps. First, information about the nonlinear behaviour of some considered material, e.g. a textile composite, has to be gathered. Therefore, simulations of the nonlinear behaviour are conducted using the Finite Element Method (FEM). Keeping in mind the computational limitations of this kind of simulations, these analyses are done for representative volume elements of the material topology with periodic boundary conditions, i.e. periodic unit cells. In order to assess a certain stress or strain state regarding the acting nonlinear mechanisms, some specific measure describing the progression of individual mechanisms is introduced. Now, the aim is to capture the nonlinear behaviour for a representative set of loading situations so that a certain range of stress or strain states is covered. Therefore, several nonlinear simulations are conducted in order to generate a dataset with sufficient resolution. The gained information, i.e. stress and strain states and the corresponding measure for nonlinear mechanisms, is then organised in a database. This procedure has to be done once for the materials to be considered and represents the computationally costly part of the described methodology.

After these preparatory steps introduced above, linear elastic analyses of a large structures, as usual in industrial applications, are considered. Here, the basic intention is to link linear elastically computed states to nonlinear ones stored by means of the database mentioned above. Thus, every stress state of the linear elastic analysis is associated to a certain amount of the individually acting nonlinear mechanisms, e.g. damage and plasticity. Hence, additional information exceeding classical FPF criteria is gained and the severeness of the investigated loading state as well as the progression of certain mechanisms can be approximated. It shall be pointed out that this second step is done within the post-processing of some large scale analysis and, hence, is very time efficient.

2.1 Preparatory considerations

In the following, the example of a composite component consisting of several layers of textile fabrics will be used for describing the proposed methodology, however, the application to any other material is possible. Considering this component, a representative volume element of a single textile ply is chosen for describing the material topology and a periodic unit cell based homogenization approach is utilized, cf. [5]. Additionally, plane stress conditions are assumed since most laminated composite components are thin-walled.

In order to predict the nonlinear response of the textile ply, the prevailing nonlinear mechanisms at the sub-ply length scale have to be modelled by appropriate constitutive relations. For the considered textile ply, these mechanisms are fibre damage, matrix damage and the accumulation of unrecoverable strains in the matrix, i.e. plasticity type effects. The above represents the basic mesoscale model for describing the material's nonlinear response.

Now, the question arises of how certain stress or strain states can be assessed regarding quantities which identify the extent of individual nonlinear mechanisms. These shall provide measures for comparing different loading situations with respect to induced damage or plasticity. A straightforward way is represented by using the dissipated energies associated to these mechanisms since they are scalar measures and, additionally, directly accessible within FEM simulations. Furthermore, it has been shown, in the case of damage, [6], that nonlinear material behaviour due to progressing damage can be described by formulating a damage potential which supports the choice of this quantity as measure for nonlinearities. In order to generalize this approach, the dissipated energies due to damage and plasticity, W_{dam} and W_{pla} , are normalized by the total strain energy W_{tot} . The resulting energy fractions,

$$p_i(t) = \frac{W_i(t)}{W_{tot}(t)} \quad i \in \{dam, pla, \dots\} , \quad (1)$$

are then used to determine the material utilisation. The parameter t denotes the current loading state and the subscripts *dam* and *pla* indicate damage and plasticity, respectively. This characterisation can, of course, be extended to describe delamination, friction and other dissipative mechanisms.

Considering the explained mesoscale model in combination with some generic load case, the basic quantities describing the nonlinear behaviour of the investigated material are now represented by the homogenized stress and strain states and the corresponding dissipated energy fractions at each load increment of the simulation. The relation of

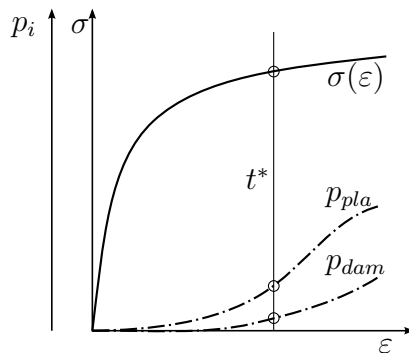


Figure 2: Sketch of a uniaxial stress-strain relation with the corresponding dissipated energy fractions p_i . t^* denotes the load increment under consideration.

these quantities is illustrated in Fig. 2 for the case of uniaxial loading. At some load increment nonlinear mechanisms are initiated and develop according to the implemented constitutive law. The considered load increment t^* relates the corresponding stress-strain state to certain amounts of dissipated energy fractions, thus, allowing for an assessment of each simulated load state $t_0 < t^* < t_{end}$. However, this assessment corresponds to one specific load state and load history, meaning that not just the current stress-strain state but also each load increment applied to reach this state has to be considered. Within this context, the curve obtained by adding up all load increments in stress space is denoted as load path.

The above example illustrates how the dissipated energy fractions, p_i , can be tracked along a specified load path. Now, the idea is to track these fractions for several load paths so that the evolutions of dissipated energies are obtained for a certain range of stress and strain states. This concept is denoted as *energy dissipation monitoring concept (EDMC)* in the following.

3 ENERGY DISSIPATION MONITORING - GENERAL CONCEPT

Some aspects of *EDMC* have already been presented in [7], [8], [9] and [10], while this work is mainly based on [11]. The goal of this methodology is to capture the nonlinear behaviour of some investigated material (e.g. textile fabric) within a defined domain in stress space. In the context of this paper the domain of interest is specified to lie within plane stress space.

To this end plane stress space is discretised by a sufficient number of plane stress states. However, this discretisation is not a unique representation of load cases since the paths connecting the unloaded state with discrete points in plane stress space have not been defined yet. Within the linear elastic regime the shape of the load path has no influence on the material response. For example, assuming that stress states are prescribed the obtained strain states will always be identical for the same stress states, independent of the path along which these states are reached. In contrast, nonlinear dissipative behaviour is load path dependent. Hence, different paths to the same stress state may result in different strain states. In order to overcome this difficulty the following assumption is introduced. Upon the onset of nonlinearities, either the stress or the strain state is assumed to increase proportionally. In other words, this assumption leads to the consideration of radial stress and strain paths, respectively. These two loading situations are expected to give some lower and upper estimates on the actual material response. Thus, a radial stress path i is defined as

$$\underline{\sigma}^i = \begin{pmatrix} \sigma_{11}^i \\ \sigma_{22}^i \\ \sigma_{12}^i \end{pmatrix} \quad \underline{\sigma}^i = s_{\max} \underline{\mathbf{d}}^i, \quad (2)$$

where $\underline{\sigma}^i$ denotes the vector of stress components, $\underline{\mathbf{d}}^i$ the directional unit vector in plane stress space and s_{\max} a scalar multiplier. By choosing s_{\max} and a distribution of $\underline{\mathbf{d}}^i$

the discretisation of plane stress space in terms of radial stress paths is defined. The corresponding radial strain paths are evaluated according to the initial (linear elastic) stiffness of the investigated material. The definition of radial strain paths follows as,

$$\underline{\underline{\varepsilon}}^i = e_{\max} \underline{\underline{\mathbf{d}}}_\varepsilon^i, \quad (3)$$

where $\underline{\underline{\mathbf{d}}}_\varepsilon^i$ is obtained by transforming the directional unit vector $\underline{\underline{\mathbf{d}}}^i$ according to linear elastic relations. The vector of strain components is represented by $\underline{\underline{\varepsilon}}^i$ and e_{\max} denotes a scalar multiplier. The relations in Eqs. (2) and (3) lead to a unique discretisation of plane stress space. By choosing a certain distribution of directions and maximum values of stresses and strains, s_{\max} and e_{\max} , the domain of interest in plane stress space and, thus, the load cases to be computed are defined. The determined $\underline{\underline{\sigma}}^i$ and $\underline{\underline{\varepsilon}}^i$ are prescribed as boundary conditions in the mesoscale model. The resulting stress-strain states and the associated dissipated energy fractions of each load increment along each load path are stored in a database. At this point the first step of the proposed methodology is finished.

Now, the generated database containing the desired information on the nonlinear material response shall be linked to results of large scale linear elastic analyses. This way, the load state of the structure with respect to locally occurring nonlinear mechanisms is assessed. However, the stress and strain states of the underlying simulations are not uniquely related. Following the assumption of proportionally increasing stress or strain states, equal stress states are linked in combination with radial stress paths. Likewise, equal strain states are linked in combination with radial strain paths which reads symbolically,

$$\begin{aligned} \underline{\underline{\sigma}}_{\text{nl}}(\underline{\underline{\varepsilon}}_{\text{nl}}) &= \underline{\underline{\sigma}}_{\text{lin}}(\underline{\underline{\varepsilon}}_{\text{lin}}) & \underline{\underline{\varepsilon}}_{\text{nl}} &\neq \underline{\underline{\varepsilon}}_{\text{lin}}, \\ \underline{\underline{\varepsilon}}_{\text{nl}}(\underline{\underline{\sigma}}_{\text{nl}}) &= \underline{\underline{\varepsilon}}_{\text{lin}}(\underline{\underline{\sigma}}_{\text{lin}}) & \underline{\underline{\sigma}}_{\text{nl}} &\neq \underline{\underline{\sigma}}_{\text{lin}}. \end{aligned} \quad (4)$$

The subscript nl denotes states obtained from nonlinear simulations, i.e. from the database, and lin represents local states from some linear elastic structural simulation. Another possibility of associating linear elastically computed states with nonlinear ones is to link those with equal strain energy density. Therefore, the relation

$$\int_0^{\underline{\underline{\varepsilon}}_{\text{nl}}} \underline{\underline{\sigma}}_{\text{nl}}^T d\underline{\underline{\varepsilon}} = \frac{1}{2} \underline{\underline{\sigma}}_{\text{lin}}^T \underline{\underline{\varepsilon}}_{\text{lin}} \quad (5)$$

has to be fulfilled. It shall be noted that the linkage of equal stress states, cf. Eq. (4), associates different values of the dissipated energy fractions p_i with the linear elastically computed state than the linkage of equal strain states, even if the response of corresponding radial stress and strain paths is identical. This effect is circumvented by the procedure in Eq. (5), which leads to identical values of p_i for identical responses. Since the large structural analysis is totally decoupled from the database containing information on nonlinear effects, macroscopic stress redistribution cannot be accounted for within the proposed methodology and, hence, an estimation of the extent of its effects is crucial

for determining the borders of applicability of the *EDMC*. To this end, the deviation of the responses of related radial stress and strain paths may be considered using the strain energy based linking procedure, cf. Eq. (5). As long as it stays small, the effect of stress redistribution can be assumed to be of minor importance.

4 ENERGY DISSIPATION MONITORING - UTILIZATION

This section is intended to exemplify the different tasks in an analysis utilizing *EDMC*. The basis of the methodology is represented by a material specific database which contains information on the nonlinear material behaviour and has to be generated once for each material to be considered. Hence, if the some composite material has been applied before this database would already be available for utilization.

In order to assess the loading state of some component, a linear elastic analysis is assumed to be sufficient. The computed stress and strain states are forwarded to an *EDMC* post processing routine where the following steps are automated. For the sake of simplicity, these steps are described for one specific stress state. First, load paths enclosing the forwarded stress state from the linear elastic analysis have to be identified in the database. This is done for radial stress and radial strain paths. After that, the linking procedures according to Eqs. (4) and (5) are applied and the sought states are interpolated accordingly. Now, values for the dissipated energy fractions p_i have been determined and are assigned to the considered linear elastically computed state. Applying this procedure to every local state of the linear elastic analysis, the structure can be assessed regarding the extent of occurring nonlinear mechanisms. This way, the severeness of the loading state of a structural component can be evaluated according to the amount of locally dissipated energies. Hence, in comparison to typical FPF criteria, information on the behaviour beyond the linear elastic limit is revealed. The evolution of the dissipated energies (i.e. the slope) may be considered for estimating the effects of further load increase. Hence, states where slowly increasing dissipated energies are predicted may be considered less critical than others with fast increasing dissipated energies. By identifying critical levels of dissipated energies, predictions on the strength reserve of the component may be conducted. Since the described procedure is just comprised of post processing steps (besides the database generation) the loading of different components can be assessed very time efficiently.

5 APPLICATION EXAMPLE

The procedure for generating a database containing information on the nonlinear material behaviour and the utilization of the *EDMC* is now demonstrated for a structural component. Therefore, a U-section beam with variable flange height in a four point bending test set-up consisting of four layers of biaxial $\pm 30^\circ$ braidings is considered. All FEM simulations are carried out using the commercial solver Abaqus/Standard v6.12 (Dassault Systemes Simulia Corp., Providence, RI, USA).

Table 1: Initial engineering elastic constants, nominal strength values and specific fracture energies of the tows and elastic constants of the matrix pockets. The subscript l denotes the fibre direction while q and r represent the transverse in-plane and out of plane directions. Taken from [10].

tow elastic	E_l 203.165 GPa	$E_q = E_r$ 11.988 GPa	$\nu_{lq} = \nu_{lr}$ 0.22	ν_{qr} 0.6785	
constants	$G_{lq} = G_{lr}$ 5.611 GPa	G_{qr} 3.571 GPa			
tow nominal	X^T 2791 MPa	X^C 1400 MPa	Y^T 33 MPa	Y^C 175 MPa	S 76.4 MPa
tow fract.	$G^{(ft)}$ 89.8 N/mm	$G^{(fc)}$ 78.3 N/mm	$G^{(mt)}$ 0.2 N/mm	$G^{(mc)}$ 0.76 N/mm	$G^{(ps)}$ 1.0 N/mm
matrix	E 2890 MPa	ν 0.38			

Table 2: Predicted effective elastic material data of four layers of the $\pm 30^\circ$ braiding stacked in-phase. The transverse shear moduli (*) are estimated. Coordinate 1 corresponds to the braid axis, while 2 and 3 denote the transverse in-plane and out of plane directions, respectively.

E_1	E_2	ν_{12}	G_{12}	G_{13}^*	G_{23}^*
42569 MPa	8717.8 MPa	1.512	24140 MPa	24140 MPa	2890 MPa

5.1 Mesoscale model - database generation

The mesoscale structure of a single layer of the $\pm 30^\circ$ twill braiding is modelled using a periodic unit cell approach in combination with the finite element method. Details on the used modelling strategy can be found in [5] and [10]. The tow undulation and matrix pockets of the fabric are explicitly modelled. In order to capture the nonlinear behaviour, an elasto-plasto-damage model, cf. [12], is chosen to simulate the tow behaviour while the matrix pockets are modelled by linear elastic constitutive relations. The essential material properties are given in Tab.1. Additional input parameters needed for the elasto-plasto-damage model can be found in [1]. Now, several hundred simulations are conducted in order to compute a distribution of load paths with sufficient resolution in plane stress space, cf. Eqs. (2) and (3). The mesoscale model is also used to determine the effective elastic material properties used in the structural simulation. Therefore, the initial stiffness of four layers of the $\pm 30^\circ$ braiding stacked in-phase is evaluated, cf. Tab. 2.

5.2 Structural application

The model of the U-section beam with variable flange height in a four point bending test set-up is illustrated in Fig. 3. The connection of the half cylinder, the plate and the beam is modelled via a surface to surface contact formulation in order to simulate a

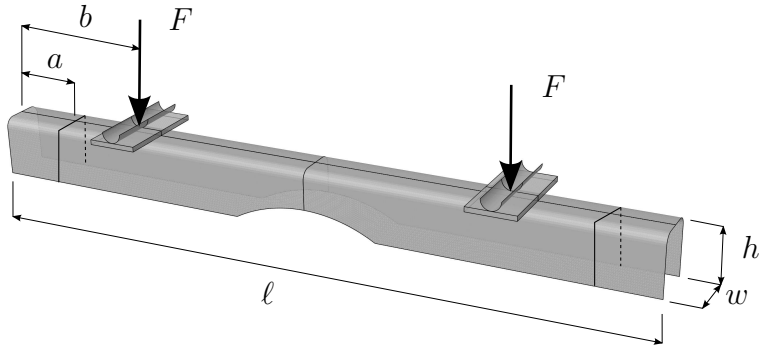


Figure 3: Isometric view of the investigated four point bending beam with the applied load F . $\ell = 700\text{mm}$, $w = 50\text{mm}$, $h = 60\text{mm}$, $a = 50\text{ mm}$, $b = 150\text{ mm}$. The marked cross sections indicate the locations of supports.

realistic test set-up. The plates between the steel cylinders and the beam are assumed to consist of aluminium and ensure that the concentrated load is distributed at the region of load application. The beam is simply supported at two cross sections, as indicated in Fig. 3. The applied load is $F = 200\text{N}$. It consists of four layers of $\pm 30^\circ$ braidings, where the braid axis is aligned in parallel with the beam axis. The effective material properties are listed in Tab. 2. The geometry of the beam is meshed using four noded, fully integrated shell elements with an average edge length of $1/24$ of the beam height h . Furthermore, symmetries at $\ell/2$ and $w/2$ are utilized.

5.3 Results - Assessment of nonlinearities

After a linear elastic simulation of the beam model described above has been conducted the *EDMC* post processing routine is applied. In order to get an overview of the critical mechanisms and the locations of their occurrence, a contour plot of the reserve factor regarding damage initiation and plasticity onset at the nominal load is shown in Fig. 4. As can be seen, plasticity is already induced at the lower edge at $\ell/2$, indicated by a reserve factor of about 2. The most critical spot regarding damage is also found to be in the region of $\ell/2$ but somewhere in the middle of the cross-section. The corresponding reserve factor of 0.6 reveals that damage is not predicted to occur at the investigated load. These considerations can be interpreted in the same way as some FPF criterion. The advantage of *EDMC* is now that the severeness of the current state can be investigated regarding the dissipated energies and, furthermore, the effect of further (proportional) load increase can be estimated. Therefore, the evolutions of dissipated energy fractions for the two critical sites are depicted in Fig. 5. The superscripts σ , ε and w indicate the linking procedures, cf. Eqs. (4) and (5), which deliver some upper and lower estimates on the dissipated energy fractions p_i . Considering Fig. 5 left, the dissipated energy due to plasticity stays quite low from the onset at a load factor of 0.4 up to a load factor of 0.6. Hence, it may be deduced that plasticity is not critical within this range. If just the linear elastic limit would be considered in this case, the load carrying capability

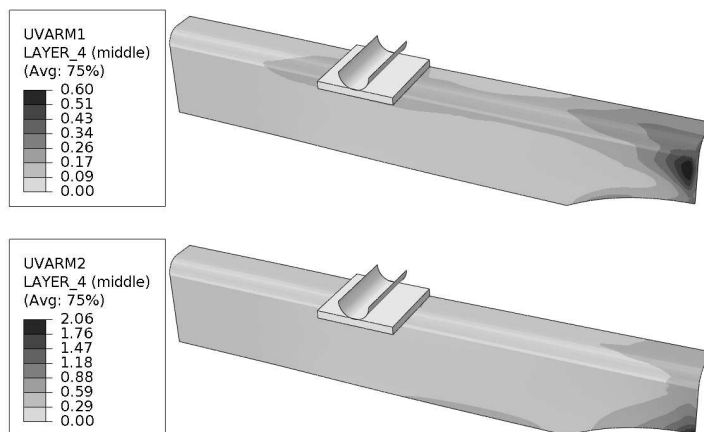


Figure 4: Contour plots of reserve factors for the most critical layer at $F = 200\text{N}$. Top: Damage initiation. Bottom: Plasticity onset.

would probably be underestimated. At the nominal load the dissipated energy fraction for plasticity reaches from values of about 2.5% up to 4%, depending if equal stress or strain states are linked. The energy fractions corresponding to the equal strain energy linking procedure, $p_{pla}^{W(\sigma)}$ and $p_{pla}^{W(\varepsilon)}$, are coincident at this state indicating that the effect of stress redistributions is still negligible. Considering further load increase, dissipated energy fractions due to plasticity, p_{pla} , of approximately 16% may occur at a load factor of 2 at the identified critical spot. At this stage, $p_{pla}^{W(\sigma)}$ and $p_{pla}^{W(\varepsilon)}$ already deviate by a certain amount and stress redistributional effects may be of non-negligible influence. Here, the limitations of the *EDMC* should be kept in mind. Figure 5 right shows that the dissipated energy fraction due to damage, p_{dam} , stays low up to a load factor of 2, thus revealing

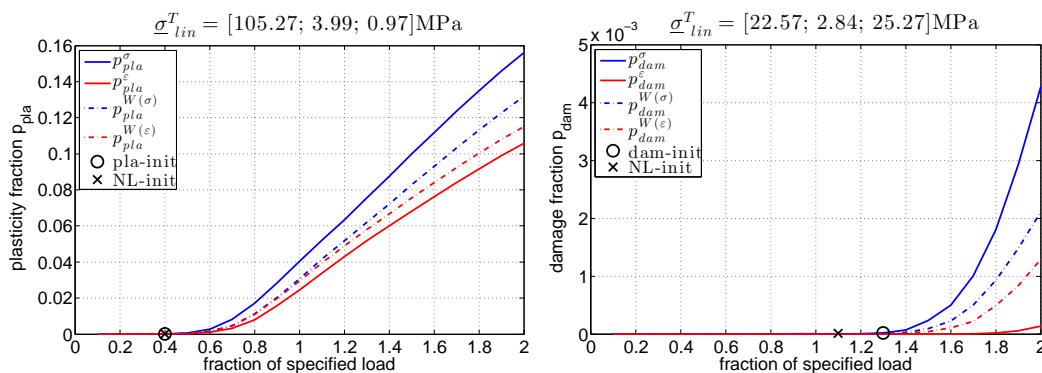


Figure 5: Left: Evolution of the dissipated energy fraction due to plasticity, p_{pla} , in terms of proportional load increase at the critical location illustrated in Fig. 4 bottom. Right: Evolution of the dissipated energy fraction due to damage, p_{dam} , at the critical location according to Fig. 4 top. The specified stress state $\underline{\sigma}_{lin}$ denotes the stress components according to Voigt notation. The circle marks the onset of the depicted mechanism, while the cross marks the initiation of the not shown one.

that damage is not predicted to be a critical mechanism in this case. The situation at other locations of the beam can be assessed by the same considerations.

6 SUMMARY

The proposed simulation methodology combines nonlinear periodic unit cell analyses with large scale linear elastic analyses. This combination approximates the effects of locally acting nonlinear mechanisms and reduces the computational effort compared to their direct consideration. Thus, the assessment of large structures regarding occurring nonlinearities becomes feasible. A major advantage of the energy dissipation monitoring concept is that the material specific database containing the necessary information on the nonlinear behaviour has to be generated just once. Hence, if this database is already available the assessment of large components can be conducted very time efficiently since just linear elastic analyses and post processing need to be done. The severeness of certain load states regarding further load increase can be assessed based on the *EDMC*, thus revealing additional information compared to classical FPF criteria. Furthermore, critical levels of energy dissipations may be defined for individual mechanisms in order to estimate the load carrying capability of a component. As the combination of linear elastic simulations and the database is meant to be entirely decoupled, effects like stress redistribution cannot be accounted for at the macroscopic length scale. Hence, *EDMC* is intended to be used up to moderately nonlinear regimes, however, at regions of pronounced nonlinearities it has to be applied with special care. Nevertheless, the influence of non-proportional loading may be estimated by comparing the responses for radial stress and strain paths. Finally, it shall be noted that the presented methodology is not just restricted to composites but may be applied to any material featuring dissipative behaviour.

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REFERENCES

- [1] Thomas Flatscher. *A Constitutive Model for the Elasto-Plasto-Damage Ply Behaviour in Laminated FRP Composites: Its Development, Implementation and Application in FEM Simulations*. PhD thesis, Institute of Lightweight Design and Structural Biomechanics, Vienna University of Technology, 2010.
- [2] Alfred Puck and Helmut Schürmann. Failure analysis of FRP laminates by means of physically based phenomenological models. *Computers Science and Technology*, 62:1633–1662, 2002.

- [3] Zvi Hashin. Failure criteria for unidirectional fiber composites. *Journal of Applied Mechanics*, 47:329–334, 1980.
- [4] M. Geers, V. Kouznetsova, and W. Brekelmans. Multi-scale computational homogenization: Trends and challenges. *Journal of Computational and Applied Mechanics*, 234:2175–2182, 2010.
- [5] J. Gager and H.E. Pettermann. Numerical homogenization of textile composites based on shell element discretization. *Composites Science and Technology*, 72:806–812, 2012.
- [6] Antonio DeSimone, Jean-Jacques Marigo, and Luciano Teresi. A damage mechanics approach to stress softening and its application to rubber. *Eur. J. Mech. A/Solids*, 20:873–892, 2001.
- [7] J. Gager and H. E. Pettermann. Assessing the operation load range of nonlinear textile composites by means of stress envelopes. In *Proceedings of the 15th European Conference on Composite Materials ECCM15, Venice, Italy*, 2012.
- [8] J. Gager, M. Meindlhumer, M. Schwab, and H. E. Pettermann. Numerically predicted damage and failure envelopes of composites featuring nonlinear material behaviour. In *The 19th International Conference on Composite Materials*, 2013.
- [9] M. Meindlhumer. Schädigungs- und Versagensvorhersage von Gewebelaminaten durch mechanismenbasierte Hüllflächen. Master’s thesis, Institute of Lightweight Design and Structural Biomechanics, Vienna University of Technology, 2013.
- [10] J. Gager. *Modeling and Simulation Concepts for Advanced Braided Composites*. PhD thesis, Vienna University of Technology, Institute of Lightweight Design and Structural Biomechanics, 2013.
- [11] Martin Schwab. Prediction of energy dissipation due to material nonlinearities and its application to textile composites. Master’s thesis, Institute of Lightweight Design and Structural Biomechanics, Vienna University of Technology, 2013.
- [12] T. Flatscher and H. E. Pettermann. A constitutive model for fiber-reinforced polymer plies accounting for plasticity and brittle damage including softening - Implementation for implicit FEM. *Composite Structures*, 93:2241–2249, 2011.