11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
E. Oñate, J. Oliver and A. Huerta (Eds)

IMPROVEMENTS ON THE NUMERICAL ANALYSIS OF VISCOPLASTIC-TYPE NON-NEWTONIAN FLUID FLOWS

A. Carmona[1], O. Lehmkuhl[1,2], C. D. Pérez-Segarra[1] and A. Oliva[1]

[1] Heat and Mass Transfer Technological Center
Technical University of Catalonia, ETSEIAT
C/Colom 11, 08222 Terrassa, Spain.
e-mail: cttc@cttc.upc.edu, web page: http://www.cttc.upc.edu

[2] Termo Fluids, S.L.
Avda. Jaquard, 97 1-E, 08222 Terrassa, Spain.
e-mail: termofluids@termofluids.com, web page: http://www.termofluids.com

Key words: Non-Newtonian Fluids, Transpose Diffusive Term, Velocity Field Discontinuity, Artificial Viscous Diffusion, Transpose Viscous Coupling

Abstract. The aim of this work is to delve into the numerical analysis of viscoplastic-type non-Newtonian fluid flows with the objective of carrying out more advanced numerical simulations for them. Specifically, improvements in the spatial discretization schemes and the temporal integration methods are proposed to overcome the numerical problems introduced by the transpose diffusive term and associated with the velocity field discontinuity, the artificial viscous diffusion and the transpose viscous coupling.

1 INTRODUCTION

CFD simulations of viscoplastic-type non-Newtonian fluid flows are of special interest due to the increment of their applications in the biomedical field, among many others. For example, cardiovascular diseases, such as the stenosis [1, 2] and the aneurysms [3, 4, 5], begin to be more often analysed by means of these computational techniques. The reason is that, on the one hand, these computational techniques allow to analyse isolatedly the fluid dynamics from others factors with which interacts in a non-linear way to jointly contribute to the disease evolution and, on the other hand, the corresponding numerical solutions can also complement the standard diagnoses of these diseases with a more accurate quantitative information.

Many researchers have used specific rheological laws together with the generalised Newtonian model to analyse non-Newtonian fluid flows which exhibit viscoplastic stresses. Brent C. Bell and Karan S. Surana [6] used a power-type rheological law to analyse the flow of some non-Newtonian fluids in different geometrical configurations. K.A. Pericleous [7] studied the fluid flow and the heat transfer in a differentially heated cavity using a power-type rheological law in order to characterise the pseudoplastic and dilatant behaviour of certain non-Newtonian fluids. M. Rudman et al. [8, 9] carried out direct numerical simulations for the turbulent flow of shear-thinning non-Newtonian fluids in a pipe to understand the turbulence flow of shear-thinning non-Newtonian fluids. M. M. Molla et al. [10] analysed the characteristics which a LES model should have in order to be conceptually consistent with the behaviour of certain non-Newtonian fluids, using a Cross rheological law to describe the behaviour of a specific non-Newtonian fluid.

Nevertheless, from our point of view, too brief numerical analyses of viscoplastic-type non-Newtonian fluid flows have been provided despite their growing presence in the field of CFD simulations.

2 MATHEMATICAL FORMULATION

The equations describing the non-Newtonian behaviour of the infinitesimal elements which form a continuum medium (from a point of view external to them), assuming an incompressible fluid, are:

$$\nabla \cdot \boldsymbol{v} = 0 \tag{1}$$

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} + \rho \nabla \cdot (\boldsymbol{v}\boldsymbol{v}) = -\nabla p + \nabla \cdot (\eta \nabla \boldsymbol{v}) + \nabla \cdot \left(\eta (\nabla \boldsymbol{v})^T\right) + \rho \boldsymbol{g} \tag{2}$$

The corresponding matrix form is:

$$M \boldsymbol{v}_f = 0 \tag{3}$$

$$\rho_c \frac{\partial \boldsymbol{v}_c}{\partial t} + C \boldsymbol{v}_c = -G p_c + D \boldsymbol{v}_c + \nabla_h \cdot \left(\eta (\nabla_h \boldsymbol{v})^T\right) + B_g \tag{4}$$

Where $c \equiv center$ and $f \equiv face$. Furthermore $\rho_c \in \mathbb{R}^{3n,3n}$, $\boldsymbol{v}_c \in \mathbb{R}^{3n}$, $p_c \in \mathbb{R}^{n}$, $M \in \mathbb{R}^{n,m}$, $G \in \mathbb{R}^{3n,n}$, $C \in \mathbb{R}^{3n,3n}$ and $D \in \mathbb{R}^{3n,3n}$. Here n and m applies for the total number of control volumes and control faces of the discretised spatial domain, respectively.

Moreover, the equations generating the non-Newtonian behaviour of the infinitesimal elements which form a continuous medium (from a point of view internal to them) are:

$$\boxed{\boldsymbol{T} = -p\boldsymbol{\delta} + \boldsymbol{S}} \quad \text{where} \quad \boldsymbol{S} = 2\eta\left(\dot{\gamma}\right)\boldsymbol{D} \tag{5}$$

In the specialised literature this is called the *Generalized Newtonian Model*.

3 NUMERICAL ANALYSIS

Improvements on the spatial discretization schemes and the temporal integration methods are proposed to overcome the numerical problems introduced by the transpose diffusive term and associated with the discontinuous velocity fields, the artificial viscous dissipation and the transpose viscous coupling.

Concerning the velocity field discontinuity, the discretised transpose diffusive term should be composed of contiguous values of the collocated discrete variable in order to reproduce faithfully the non-Newtonian behaviour. Consequently, an expression for the transpose diffusive term based on the divergence theorem and with the staggered velocity gradient approached by an arithmetic mean (AM) is proposed:

$$\begin{aligned} \nabla_h \cdot (\eta (\nabla_h \boldsymbol{v})^T) &= \frac{1}{\Omega_P} \left(\frac{1}{2\,\Omega_P} \sum_{PF} \boldsymbol{A}_{PF} \frac{\boldsymbol{v}_F}{2} \right) \cdot \sum_f \eta_f \boldsymbol{A}_f \\ &+ \frac{1}{\Omega_P} \sum_f \left(\frac{\eta_f}{2\,\Omega_F} \sum_{F2F} \boldsymbol{A}_{F2F} \frac{\boldsymbol{v}_{2F}}{2} \right) \cdot \boldsymbol{A}_f \end{aligned} \tag{6}$$

Moreover, a unique definition for the velocity at the control face is proposed to eliminate the artificial viscous diffusion in the staggered discrete operator.

As for the artificial viscous dissipation, the aforementioned term should be cancelled when the non-Newtonian viscosity takes the value of the Newtonian viscosity under the hypothesis of incompressible fluid (see Figure 1), this time with the objective of reproducing accurately the Newtonian behaviour. Accordingly, another expression for the transpose diffusive term based on the Green's first identity is proposed:

$$\nabla_h \cdot (\eta (\nabla_h \boldsymbol{v})^T) = \frac{1}{\Omega_P} \left[\left(\frac{1}{\Omega_P} \sum_{PF} \boldsymbol{A}_{PF} \frac{\boldsymbol{v}_F}{2} \right) \cdot \left(\sum_f \eta_f \, \boldsymbol{A}_f \right) \right] \tag{7}$$

Furthermore, a specific evaluation for the non-Newtonian viscosity at the control face is proposed to avoid the velocity field discontinuity in the collocated discrete operator.

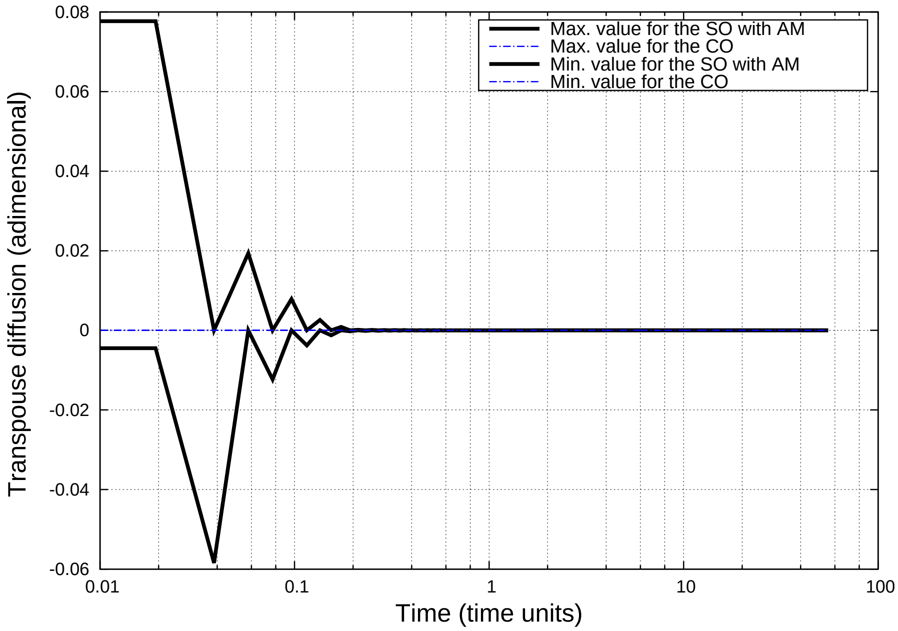

Figure 1: Transpose diffusion

Regarding the transpose viscous coupling, the transpose diffusive term has been analysed in the field of the temporal integration methods in order to deal with the numerical difficulty that entail the participation of the transversal velocity components in the momentum equation through this term.

$$\frac{(\rho \boldsymbol{v})_c^{n*+1} - (\rho \boldsymbol{v})_c^{n}}{\Delta t} = -C(\boldsymbol{v}_f^{n*+1})\boldsymbol{v}_c^{n*+1} + D\boldsymbol{v}_c^{n*+1} + D^T \boldsymbol{v}_c^{n*+1} - Gp_c^n \tag{8}$$

For the viscoplastic-type non-Newtonian fluid flows with a permanent regime and a steady state, the implicit projection method with a special treatment for the pressure term (IPM2) has turned out to be robust and accurate while presents an efficiency that appears to be much closer to the optimum than in the others analysed methods, see Figures 2 and 3. The temporal discretization scheme of the pressure term seems to be the key aspect for the aforementioned method.

$$\frac{(\rho \widetilde{\boldsymbol{v}})_c^{n*+1} - (\rho \boldsymbol{v})_c^{n*+1}}{\Delta t} = \frac{1}{2} G p_c^n \qquad \frac{(\rho \boldsymbol{v})_c^{n+1} - (\rho \widetilde{\boldsymbol{v}})_c^{n*+1}}{\Delta t} = -\frac{1}{2} G p_c^{n+1} \tag{9}$$

$$\frac{(\rho \boldsymbol{v})_c^{n+1} - (\rho \boldsymbol{v})_c^{n*+1}}{\Delta t} = -\frac{1}{2}\Delta t G \left(\frac{p_c^{n+1} - p_c^n}{\Delta t} \right) \tag{10}$$

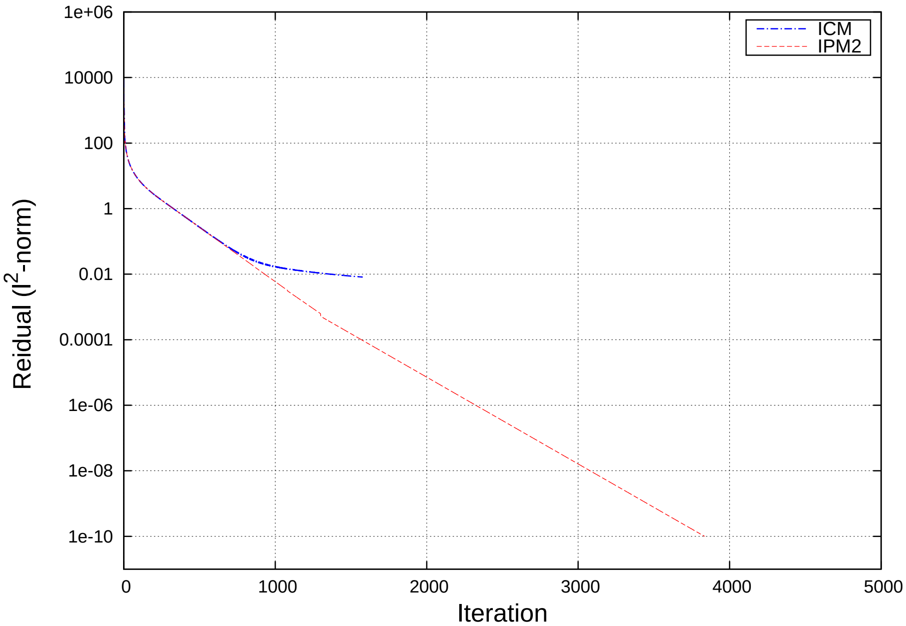

Figure 2: Convergence history; Correction vs Projection Step

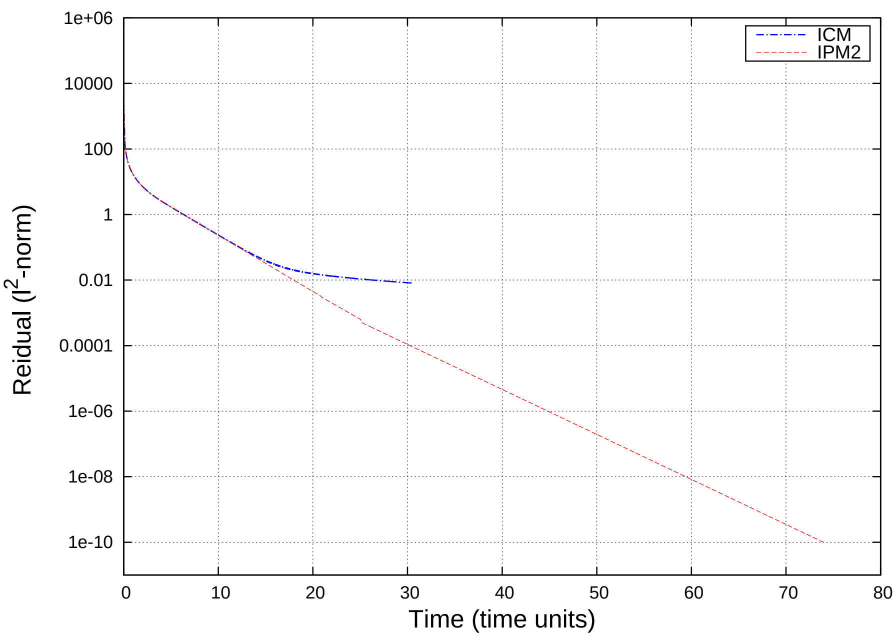

Figure 3: Temporal evolution; Correction vs Projection Step

4 CONCLUSIONS

In order to overcome the numerical problems that seem to remain in the spatial discrete operators, a solution for each scheme has been proposed in this work. Concretely, a unique definition for the velocity at the control face to eliminate the artificial viscous diffusion in the staggered discrete operator (SO) and a specific evaluation for the non-Newtonian viscosity at the control face to avoid the velocity field discontinuity in the collocated discrete operator (CO). Regarding the transpose viscous coupling, for the viscoplastic-type non-Newtonian fluid flows with a permanent regime and a steady state, the implicit projection method with a special treatment for the pressure (IPM2) has turned out to be robust and accurate while presents an efficiency that appears to be much closer to the optimum than in the others analysed methods.

To confirm and extend the current results are part of our current and future research plans.

ACKNOWLEDGMENTS

This work has been financially supported by the Ministerio de Ciencia e Innovación of Spain (Contract/Grant No. ENE2010-17801) and Termo Fluids S.L.. The authors thankfully acknowledge these institutions.

REFERENCES

[1] M. Rosenfeld. Validation of numerical simulation of incompressible pulsatile flow in a constricted channel. *Computers and Fluids*, 22:139–156, 1993.

[2] M. Rosenfeld. A numerical study of pulsating flow behind a constriction. *J. of Fluid Mechanics*, 30:203–223, 1995.

[3] S. C. M. Yu, W. K. Chan, B. T. H. Ng, and L. P. Chua. A numerical investigation on the steady and pulsatile flow characteristics in axi-symmetric abdominal aortic aneurysm models with some experimental evaluation. *J. of Medical Engineering & Technology*, 23:228–239, 1999.

[4] S. C. M. Yu. Steady and pulsatile flow studies in abdominal aortic aneurysm models using particle image velocimetry. *Int. J. of Heat and Fluid Flow*, 21:74–83, 2000.

[5] T.H. Yip and S.C.M. Yu. Oscillatory flows in straight tubes with an axisymmetric bulge. *Experimental Thermal and Fluid Science*, 26:947–961, 2002.

[6] B.C. Bell and K.S. Surana. p-version least squares finite element formulation for two-dimensional, incompressible, non-Newtonian isothermal and non-isothermal fluid flow. *Int. J. for Numerical Methods in Fluids*, 18:127–162, January 1994.

[7] K.A. Pericleous. Heat transfer in differentially heated non-newtonian cavities. *Int. J. of Numerical Methods for Heat & Fluid Flow*, 4:229–248, 1994.

[8] M. Rudman, H.M. Blackburn, L.J.W. Graham, and L. Pullum. Turbulent pipe flow of shear-thinning fluids. *J. Non-Newtonian Fluid Mechanics*, 118:33–48, 2004.

[9] H.M. Blackburn M. Rudman. Direct numerical simulation of turbulent non-newtonian flow using a spectral element method. *Applied Mathematical Modelling*, 30:1229–1248, 2006.

[10] M. M. Molla, A. Hossain, B.-C. Wang, and D. C. S. Kuhn. Large-eddy simulation of pulsatile non-newtonian flow in a constricted channel. *Progress in Computational Fluid Dynamics*, 12:231–242, 2012.