11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
E. Oñate, J. Oliver and A. Huerta (Eds)

A COMPARISON OF MINIMUM CONSTRAINED WEIGHT AND FULLY STRESSED DESIGN PROBLEMS IN DISCRETE CROSS-SECTION TYPE BAR STRUCTURES

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Key Words: *Structural design, Frame optimization, truss optimization, evolutionary algorithms, fully stressed design, minimum constrained weight.*

Abstract. The fully stressed design and the minimum constrained weight problems are frequently handled in the structural engineering field. Both are highly related problems and when applied in bar structures, even under certain circumstances they can share the design solution. In the case of the fully stressed design problem (FSD), see e.g. [1], we are handling an inverse problem [2] using the maximum admissible stresses of the bars of a structure as references, and aiming to be able to obtain the structure whose maximum stresses equal those considered references. In the case of the minimum constrained weight problem (MCW), see e.g. [3,4,5], the lightest structure (the one with lower cost in terms of raw material) which also does not violate certain constraints (which guarantee the right completion of the structural mission) is searched. Here, constraints in relation with limit stresses, limit displacements and limit slenderness are taken into account. When considering real engineering design of bar structures, where normalized cross section types belonging to national codes are required, the use of discrete variables in the search is needed. To solve the FSD and MCW problems is possible through the use of evolutionary algorithms [6]. They, as well as other metaheuristics, being population based search methods allow a global optimization without stagnating in local optima and they admit without constraints the use of discrete variables. Their use in structural engineering, particularly in discrete bar structural optimization has been still widely researched in the recent years (see e.g. [7,8]). Even very recently, they also started to be considered as useful tools for extracting design knowledge in engineering problems, as in [9]. Nevertheless, the optimization process of both problems, FSD and MCW, can present noticeable differences, resulting in different number of fitness evaluations required to obtain the best solution. This difference in behaviour is shown in this work. Several test cases of different search space size bar structures are handled. An example in a truss bar structure -bar structures with articulated nodes and only loaded on nodes; only normal effort in bars is expected- is handled here. Results indicate some interesting relationship among both types of problems (FSD and MCW), and the analysis of the evolutionary search through some statistical metrics evidences the difference of problem landscape through highly divergent behaviour of the evolutionary process until achieving the best design in each case.

1 INTRODUCTION

Global optimum design of bar structures with discrete cross-section types using evolutionary algorithms (or metaheuristics) has been allowed since the first nineties of the twentieth century. Here a comparative analysis of the evolutionary behaviour in both the problems of a) minimizing the constrained structural weight and b) obtaining the fully stressed design, with the above mentioned global search methods is shown in a simple truss structure considering some statistical metrics. First, both structural handled problems are defined in section 2, and the test case in section 3. Section 4 shows results and discussion, and finally, the conclusions section ends the paper.

2 STRUCTURAL PROBLEMS

Two different optimum design problems of bar structures are handled in this chapter through one truss example.

First, we consider the problem of minimization of constrained structural weight (MCW), which is associated with the minimization of raw structural cost of the structure. It is the most frequent structural optimization problem.

$$MCW = \sum_{i=1}^{Nbars} A_i \cdot l_i \cdot \rho_i \tag{1}$$

where Ai is cross-sectional area, li is length and ρi is specific weight, all corresponding to bar i; and subjected under possible constraints of stresses, displacements and/or buckling. Here only stresses are taken into account. Constraints are treated as in [10].

Second, we consider the problem of obtaining the fully stress design (FSD) structure, handled since the beginning of the twentieth century. The FSD of a structure is defined as a design in which some location of every bar member in the structure is at its maximum allowable stress for at least one loading condition.

$$FSD = \sqrt{\sum_{i=1}^{Nbars} (\sigma_{MAX-i} - \sigma_{MAX-Ri})^2} \tag{2}$$

where $\sigma_{MAX\text{-}i}$ is the maximum stress and $\sigma_{MAX\text{-}Ri}$ the maximum allowable stress, both corresponding to bar i.

Relation among both previous problems, MCW and FSD have been established, mainly in trusses structures, where the material is allowed to work at its full potential due to the only existence of normal efforts, associated with the cross-sectional area. In this work, we show that even in the possible case that both problems (MCW and FSD) share the same optimum solution, the optimization problems still have different search topologies and characteristics, which make easier or harder to solve for the global search evolutionary algorithm.

3 TEST CASE

The purpose of this benchmark is to set a simple test case with truss bar structures based on one test case in (Murotsu et al., 1984)[11] and (Park et al. 2004)[12], and solving it with discrete cross-section types variables. The computational domain, boundary conditions and loading are shown in figure 1, with Load P = 4450 N.

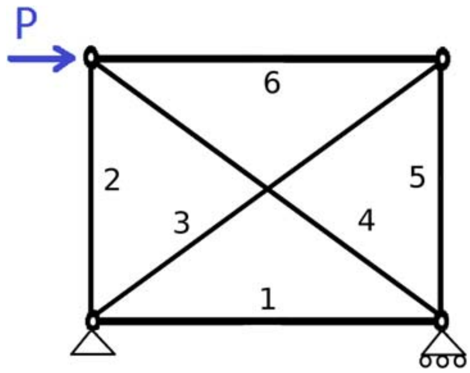

Figure 1: Truss Test Case. Computational Domain, Boundary Conditions and Loading

Each bar is associated to an independent variable. The set of cross section types and their corresponding geometric properties (area and radius of gyration) are shown in Table 1. The search space of variables is represented in table 2, where the lower and upper limit of each variable is shown. This discrete cross-section type bar test case has been also solved for simultaneous minimization of weight and maximization of the reliability index in Greiner & Hajela [8].

Table 1: Cross-section types

Order	Cross-Section	Area (cm2)	Radius of Gyration (cm)
1	C1	0,85	0,653
2	C2	0,93	0,652
3	C3	1,01	0,651
4	C4	1,09	0,650
5	C5	1,17	0,649
6	C6	1,25	0,648
7	C7	1,33	0,647
8	C8	1,41	0,646
9	C9	1,49	0,645
10	C10	1,57	0,644
11	C11	1,65	0,643
12	C12	1,73	0,642
13	C13	1,81	0,641
14	C14	1,89	0,640
15	C15	1,97	0,639
16	C16	2,05	0,638

Table 2: Search space of variables

Bar Number	Bar Variable	Cross-section type set
1	v1	From C1 to C16
2	v2	From C1 to C16
3	v3	From C1 to C16
4	v4	From C1 to C16
5	v5	From C1 to C16
6	v6	From C1 to C16

The structural evaluation has been calculated using an own implemented truss bar structure stiffness matrix calculation (articulated nodes: non-resisting moment capabilities; elastic behaviour of steel is assumed, and no buckling effect is considered here). The geometric parameters (height and width of the structure) are shown in table 3. The material properties, corresponding to those of standard construction steel, are shown in table 4.

Table 3: Geometric parameters

	Value (m.)
Height (H)	0,9144
Width (W)	1,2190

Table 4: Material properties (Steel)

Parameter	Value
Density	7850 kg/m3
Young Modulus	2.06 x 105 MPa
Maximum Stress	276 MPa

The quantities of interest are the values of the fitness function/s (minimum constrained weight and / or fully stress design) and the maximum stress of each bar in order to define the cross-section type sizing of each bar (structural design).

4 RESULTS AND DISCUSSION

4.1 Test Case Analysis

The relationship between the problem of MCW and the problem of obtaining the FSD structure in our test case is studied in this section. With that purpose, the whole search space of the previous test case has been explored, evaluating both fitness functions: a) The Constrained Weight (MCW, in kg.) and b) The square root of the sum of squared stress differences of each bar with fully stress design (FSD); as shown in equations (1) and (2).

Values of the 16,777,216 designs (corresponding to a 6 bar x 4 bits/bar chromosome= 24 bits; 2^{24}=16,777,216) are shown in Figures 2 and 3. Figure 2 shows the whole search space. A calculated population Pearson's correlation coefficient r gives a value r=0.71089 (where 1.0 means perfect linear relationship). In zoomed figure 3, in addition the minimum of both fitness functions (in our problem here are coincident, that is, the same design solution is the minimum of both problems simultaneously) is plotted as a black dot.

Table 5: Minimum MCW designs (cross-section types as in Table 1)

Design Order	FSD value	*MCW value*	Unconstrained Weight	Bar 1	Bar 2	Bar 3	Bar 4	Bar 5	Bar 6
1st	**11480.7**	***5.26338***	**5.26338**	**C1**	**C1**	**C3**	**C3**	**C1**	**C1**
2nd	12709.5	*5.3581*	5.3208	C1	C1	C3	C3	C2	C1
3rd	12709.5	*5.3581*	5.3208	C1	C2	C3	C3	C1	C1
4th	**11645.8**	***5.35907***	**5.35907**	**C1**	**C1**	**C4**	**C3**	**C1**	**C1**
5th	**11645.8**	***5.35907***	**5.35907**	**C1**	**C1**	**C3**	**C4**	**C1**	**C1**

Table 6: Minimum FSD designs (cross-section types as in Table 1)

Design Order	*FSD value*	MCW value	Unconstrained Weight	Bar 1	Bar 2	Bar 3	Bar 4	Bar 5	Bar 6
1st	***11480.7***	**5.26338**	**5.26338**	**C1**	**C1**	**C3**	**C3**	**C1**	**C1**
2nd	***11645.8***	**5.35907**	**5.35907**	**C1**	**C1**	**C4**	**C3**	**C1**	**C1**
3rd	***11645.8***	**5.35907**	**5.35907**	**C1**	**C1**	**C3**	**C4**	**C1**	**C1**
4th	*11649.7*	6.24373	5.16768	C1	C1	C3	C2	C1	C1
5th	*11649.7*	6.24373	5.16768	C1	C1	C2	C3	C1	C1

The detailed five best designs of each fitness function are shown in tables 5 and 6, corresponding to the MCW optimum designs and FSD optimum designs, respectively. In addition to the optimum design, which is shared by the problems of minimum constrained weight and the problem of FSD, two designs more out of this list of five are included in both sets (all three shared designs are in bold type in the tables).

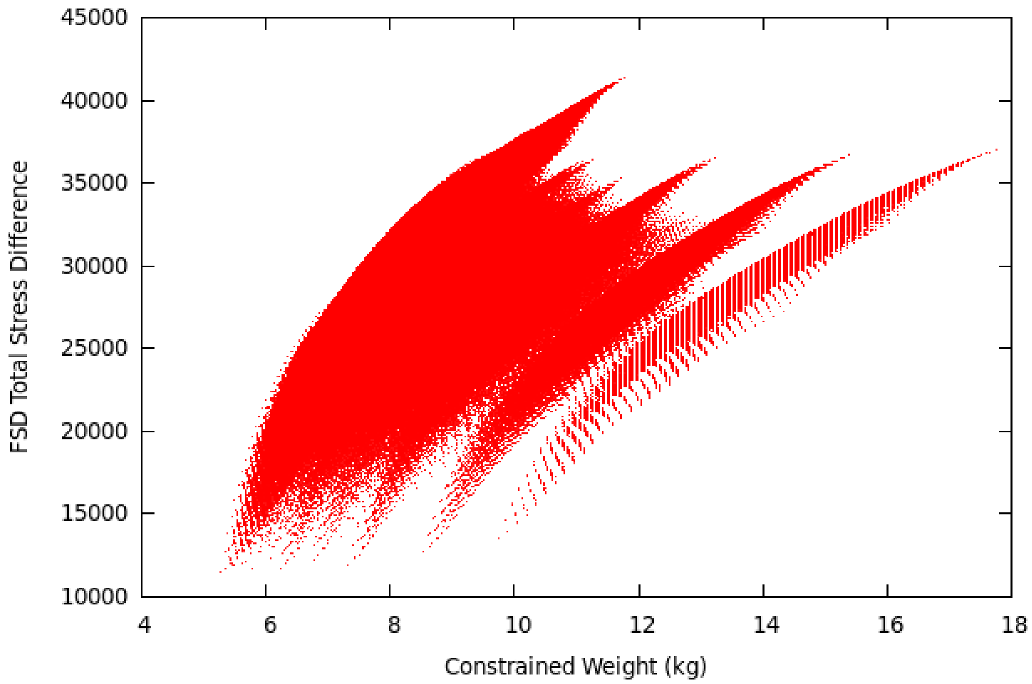

Figure 2: Whole search space designs of Test Case cR00

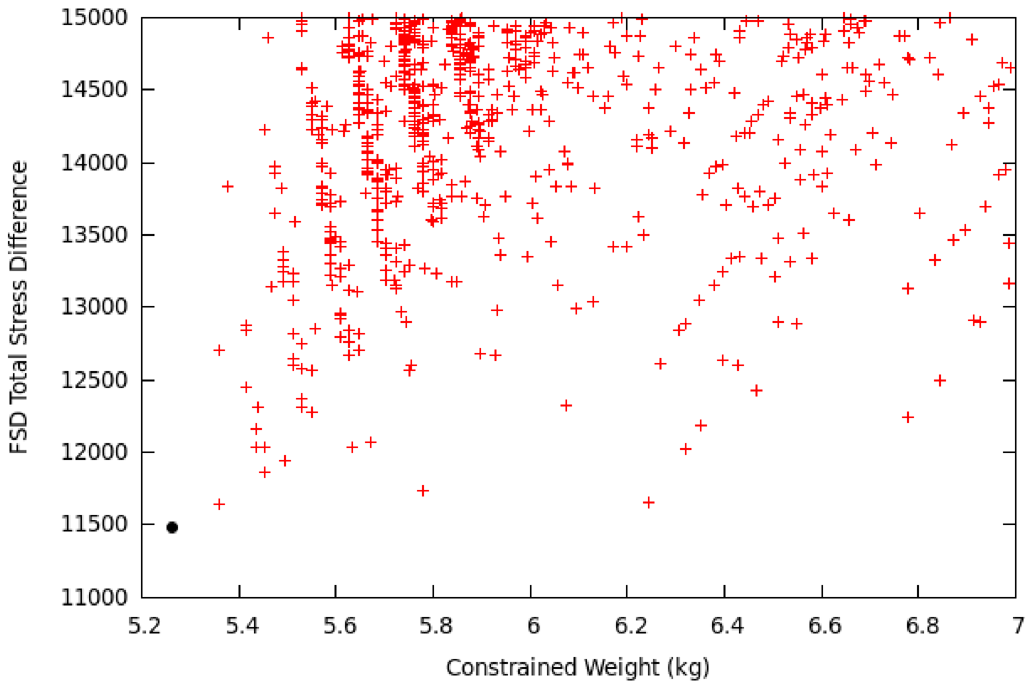

Figure 3: Zommed vision over Figure XX1 (search space designs of Test Case cR00)

4.2 Optimization

Both problems, MCW and FSD are optimized here. An evolutionary algorithm with population size 100 individuals, mutation rate 3%, uniform crossover, and gray codification (as in [10]) has been executed in 100 independent runs with 25,000 evaluations as stopping criterion. Results are shown in terms of average and best value (figures 4 and 6) and standard deviation (figures 5 and 7) of the fitness function.

a) Solving the problem of constrained weight optimum MCW design (figures 4 and 5).

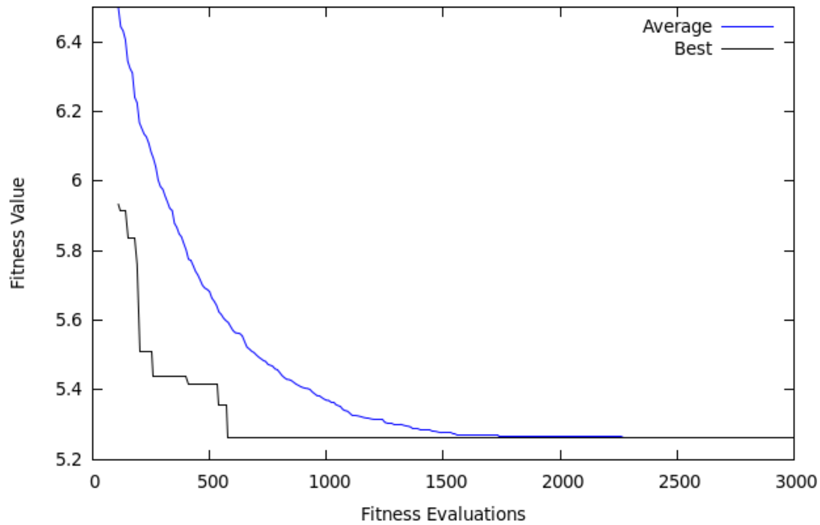

Figure 4: Constrained Minimum Weight evolution over 100 independent runs in cR00 test case

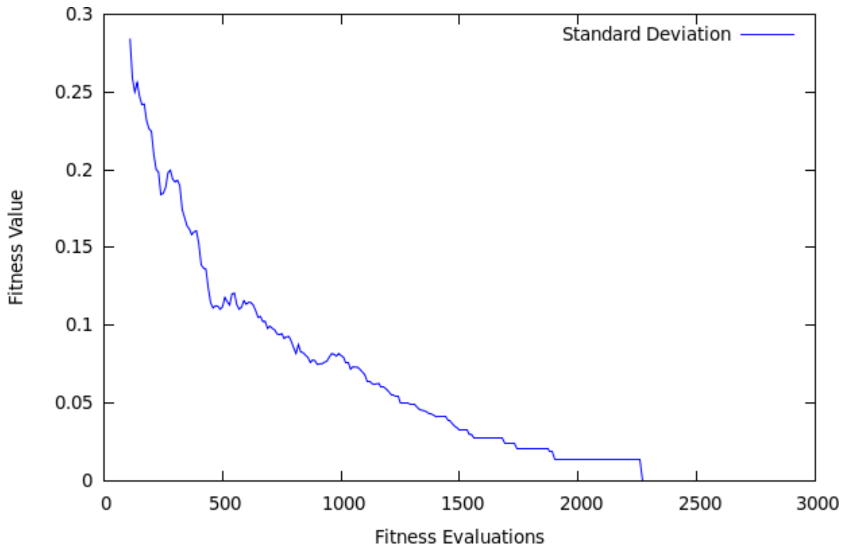

Figure 5: Constrained Minimum Weight evolution over 100 independent runs in cR00 test case

b) Solving the problem of fully stressed design FSD optimum design (figures 6 and 7).

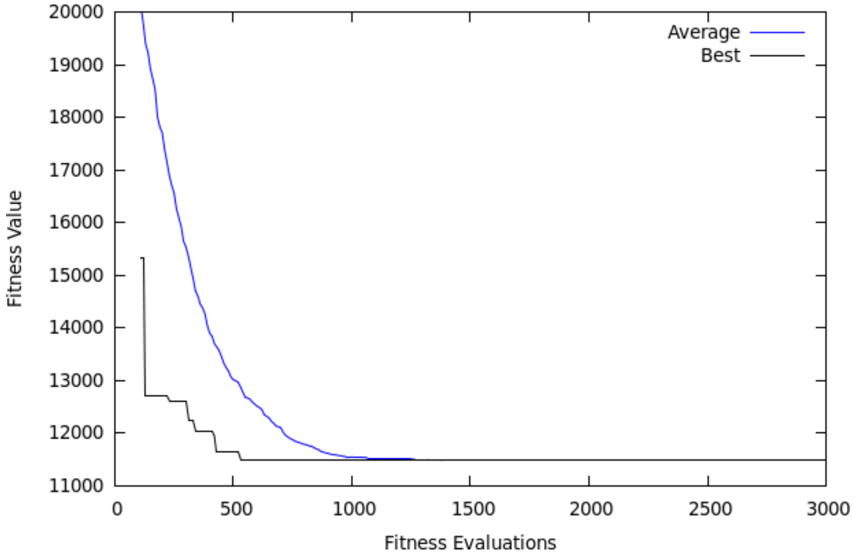

Figure 6: FSD evolution over 100 independent runs in cR00 test case

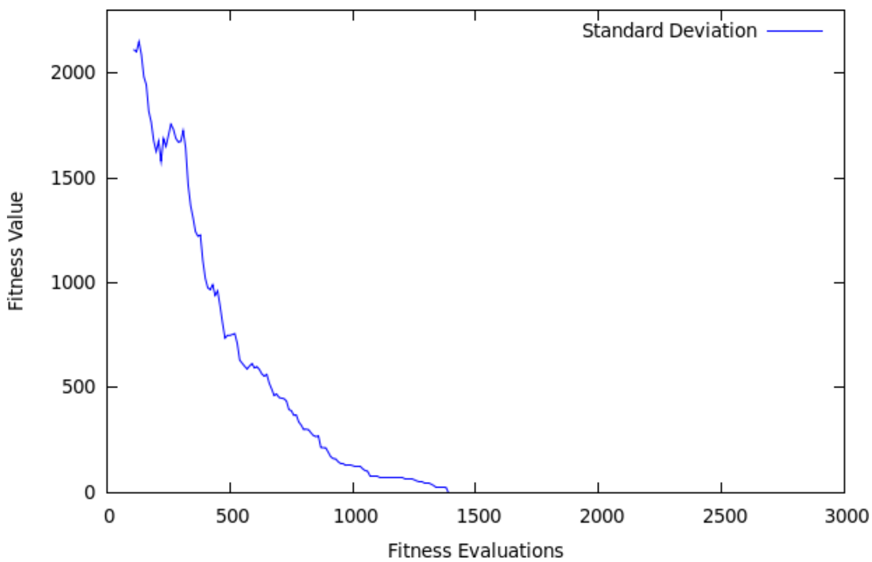

Figure 7: FSD evolution over 100 independent runs in cR00 test case

4.3 Discussion

Both problems, MCW and FSD have been optimized in this section with evolutionary algorithms. When comparing both convergence evolutions in terms of i) average results, ii) best number of evaluations required (both shown in previous figures 4 and 6) and also, iii) in terms of standard deviation (shown in figures 5 and 7), the FSD problem is easiest to solve for the evolutionary algorithm, as lower number of fitness function evaluations is required.

5 CONCLUSIONS

In discrete cross-section types sizing optimization of bar structures, the evolutionary behaviour through the global search process has been compared showing differences in a simple truss test case where both problems share the optimum design solution. The fully stressed design (FSD) problem has shown an easier topology for the evolutionary algorithm optimization requiring less number of fitness function evaluations to achieve the best design.

As future research, we propose to investigate the possibility to take advantage in terms of diversity enhancement of the population search of evolutionary algorithms, of coevolving the fitness functions of this two problems to improve the search, applying it to a greater number of test cases.

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