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COMPARISON OF VARIOUS APPROACHES FOR MODELLING INTERACTION OF DISPERSIVE MEDIA AND ELECTROMAGNETIC WAVES

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Abstract. In this paper various approaches for calculating transient response of systems containing frequency depended materials excited by an electromagnetic wave are compared. Implementation of Debye media is showed. The frequency dependence of Debye media can be expressed as a sum of single pole systems on s plane. Efficiency and accuracy of different methodologies is compared: inverse Fourier transform of a multiple frequency domain simulations and direct modification of time domain marching schemes. The latter is presented in a formalism of digital filters.

1 INTRODUCTION

Dispersive materials play an important role in modern computational electromagnetics and electromagnetic compatibility. For instance biological tissues and a large variety of organic liquids satisfy Debye relaxation model [1]. Capabilities of modern computers allow simulating complex structures, including interaction of human body and electromagnetic waves, microwave tomography etc. Rigorous analysis of problems involving living tissues requires appropriate handling of dispersive materials. Application of Debye model can be found in [2], however the results are restricted to finite difference scheme. An attempt to solve the problem in finite element time domain method has been presented in [5]. This work partially extends that work, and special attention is paid to differences between classical 2nd order time marching scheme used in finite difference simulations and Newmark scheme, used in finite element method. Another problem discussed in the paper is an inverse Fourier transformation of a multiple frequency domain simulations.

Initial-boundary value problem is described by Maxwell equations or wave equation with appropriate boundary conditions. The dispersive material can be specified by a first order Debye dispersion

$$\mathbf{D}(\omega)=\left(\varepsilon_{\infty}+\sum_i \frac{\Delta\varepsilon}{1+j\omega\tau_i}\right)\mathbf{E}(\omega) \tag{1}$$

Where **D** stands for the electric displacement vector and **E** denotes the electric field. Value ε_∞ is the hypothetical permittivity for an infinite frequency. $\Delta\varepsilon$ is related to a low frequency permittivity and τ is considered as relaxation time. In fact, the core of the problem is handling.

$$\frac{\Delta\varepsilon}{1+j\omega\tau} \tag{2}$$

Thus in the further part a single pole (i=0) problem is considered.

In the paper we discuss two basis approaches: based on inverse Fourier transformation [3] and digital filtering. Since the dispersive relation in frequency domain is straightforward, in fact no special handling of dispersive material is needed. However, in order to obtain wideband or transient response one need to solve a number of linear problems, one problem per frequency point. Solving a number of linear problems may seem an inefficient approach; however taking into account the following aspects puts a problem in a better context. Namely, in modern multicore, multiprocessor architectures this problem is perfectly scalable. Secondly, one can assume that the solution does not significantly differ between two neighboring frequency points. This fact can be utilized when iterative algorithms for linear problems are used, which significantly reduces computational effort. Note, that in the case of the time domain finite element method, one linear problem per iteration is solved. Thus the difference in efficiency of direct and indirect formulation is not significant. The indirect approach, based on inverse Fourier transform is beneficial for analysis media of complicated frequency characteristics, especially those described by a large number of Debye poles (1).

Among purely time domain algorithms, the dispersive relation is held as a digital filter. It means that the sample of the quantity of interest is a convolution of media’s pulse response and the incident field. For the Debye media the dispersive part of (1), corresponds to exponentially decaying pulse response. Such a response can be effectively modeled with a single pole digital filter of a form (keeping notation of (1)):

$$D[n\Delta t]=D\left[(n-1)\Delta t\right]+E[n\Delta t] \tag{3}$$

However, the one-to-one transformation of a pulse response to discrete time systems results in errors, related to nonlinear mapping between frequencies in continuous time and discrete time systems.

In [5] authors employed so called bilinear mapping which resulted with an accurate, but less computationally effective algorithm. The main drawback of the bilinear mapping is need of keeping in memory an additional state variable, compared to (3). In the work the authors exploit the difference between linear and bilinear mapping for 2nd order and Newmark time marching scheme.

2 PROBLEM DESCRIPTION

An initial-boundary value problem in electromagnetic consists wave equation and boundary conditions [4]. Wave equation can be given in domain V by

$$\nabla\times\left[\frac{1}{\mu_0\mu_r}\nabla\times\mathbf{E}(\mathbf{r},t)\right]+\frac{\partial^2\mathbf{D}(\mathbf{r},t)}{\partial t^2}+\sigma\frac{\partial\mathbf{E}(\mathbf{r},t)}{\partial t}=0\quad \mathbf{r}\in V \tag{4}$$

Where **E** is electric field, **D** is displacement field, μ_0 free-space permeability, μ_r relative permeability and σ electric conductivity.

Often used boundary conditions are given on surface *S* by

$$\hat{n}\times(\nabla\times\mathbf{E})=0\quad \mathbf{r}\in S \tag{5}$$

Behavioral of electric field on absorbing boundary conditions is given by

$$\vec{n}\times\left[\frac{1}{\mu_0\mu_r}\nabla\times\mathbf{E}(\mathbf{r},t)\right]+Y_0\frac{\partial}{\partial t}\left[\vec{n}\times\vec{n}\times\mathbf{E}(\mathbf{r},t)\right]=0 \tag{6}$$

In equation (4) are two independent variables. Relationship between them is given in frequency domain by (1) for the Debye model. In following chapter we describe useful techniques for solving this kind of initial-boundary value problem including dispersive model.

3 APPROACHES

Problem define by (4) - (6) can be solved by various techniques [7]. Lots of them were developed for finite difference time domain method. We use method based on digital filtering and method based on inverse Fourier transformation for finite element time domain method.

3.1 Digital Filter approach

As the time domain simulation can be considered as a discrete time system, a digital filter framework is a natural formalism for implementation frequency dependent material. In consideration presented in this paragraph we stick to discussion of the relation between electric field *E* and displacement field *D*, since it is the only place where the digital filtering has to be used. The convolution, which appear in the relation corresponds to the fact that current *D* sample is not only depended on current *E* sample, but also its previous values. It leads to the necessity of introduction of one or more state variables, keeping previous values of *D*.

The pulse response of a single pole Debye medium is a decaying exponential function of a form

$$h(t)=Ae^{-t/\tau} \tag{7}$$

The convolution of an input signal representing electric field samples with such a function leads to a straightforward formula for the newest D sample, taken at time instant $n\Delta t$, namely

$$D[n\Delta t]=e^{-\Delta t/\tau}\cdot D[(n-1)\Delta t]+E[n\Delta t] \tag{8}$$

Although simple, this formula applied to the Newmark scheme leads to significant inaccuracies. The origin of this behavior lays in aliasing of the pulse response in a frequency domain. In result, a discrete convolution with a pulse response obtained by direct sampling of its continuous prototype leads significant distortion of the filter's characteristics, compared to its continuous analog.

As an alternative, so called bilinear mapping can be used. Bilinear mapping uses the following formula to transform s plane into Z plane:

$$S = \frac{2}{\Delta t}\frac{1-z^{-1}}{1+z^{-1}} \tag{9}$$

This formula transforms imaginary axis of an s plane into unit circle and negative real part into interior of the circle. In the discussed case of relation between dielectric field and displacement, the formula leads to much more complicated equation for displacement update, namely

$$D\left[n\Delta t\right] = -C_1 \cdot D\left[\left(n-1\right)\Delta t\right] + C_2 \cdot E\left[n\Delta t\right] + C_2 \cdot E\left[\left(n-1\right)\Delta t\right] \tag{10}$$

Where

$$\begin{aligned} C_1 &= {\left(1-\frac{2\tau}{\Delta t}\right)}\Big/{\left(1+\frac{2\tau}{\Delta t}\right)} \\ C_2 &= {\Delta\varepsilon}\Big/{\left(1+\frac{2\tau}{\Delta t}\right)} \end{aligned} \tag{11}$$

Notice, that one more operation is involved, and not only displacement value but also electric field sample must be memorized. Fortunately, in the discussed 2nd order scheme obtained by discretization of a wave equation, it does not yield additional storage requirements. A detailed application of handling this dispersive scheme in a wave equation is presented in [5]

3.2 Inverse Fourier transformation

Method based on Inverse Fourier transformation has been successfully implemented by finite difference method. This approach has lots of advantages [3]. Implementation by finite element time domain method brings useful method. We compare this method with method mention in previous paragraph.

The relationship between electric field and displacement field can by defined as

$$D = \varepsilon_0 \varepsilon_\infty E + P \tag{12}$$

Where P is polarization.

The description of Debye model is given by the differential equation which has form

$$\tau \frac{\partial p}{\partial t} + p = \varepsilon_0\left(\varepsilon_s - \varepsilon_\infty\right)e \tag{13}$$

After spatial discretization of (1) and implementation of (12) we obtain equation which has now form

$$\mathbf{T}\frac{\partial^2 e}{\partial t^2}+\left(\mathbf{Q}+\mathbf{B}\right)\frac{\partial e}{\partial t}+\mathbf{S}e+\mathbf{T}^p\frac{\partial^2 p}{\partial t^2}+\mathbf{g}=0 \tag{14}$$

Where **T Q B T g** are matrices and force vector [4].

The term with second derivation of *p* can be replaced by equation (13) after appropriate modification.

The time Discretization of (14) which has small error is given by

$$p^n\left(1+\frac{1}{2}\frac{\Delta t}{\tau}\right)=\left[1-\frac{1}{2}\frac{\Delta t}{\tau}\right]p^{n-1}+\varepsilon_0\left(\varepsilon_s-\varepsilon_\infty\right)\frac{\Delta t}{\tau}\left(\frac{1}{2}e^n+\frac{1}{2}e^{n-1}\right) \tag{15}$$

Final equation of inverse Fourier transformation for finite element method is

$$\begin{aligned}\left(\frac{1}{\Delta t^2}\mathbf{T}+\frac{1}{2\Delta t}\left(\mathbf{Q}+\mathbf{B}+\mathbf{T}^e\right)+\frac{1}{4}\left(\mathbf{S}+\mathbf{T}^m\right)\right)e^{n+1}&=-\left(\frac{-2}{\Delta t^2}\mathbf{T}+\frac{1}{2}\left(\mathbf{S}+\mathbf{T}^m\right)\right)e^n\\&-\left(\frac{1}{\Delta t^2}\mathbf{T}-\frac{1}{2\Delta t}\left(\mathbf{Q}+\mathbf{B}+\mathbf{T}^e\right)+\frac{1}{4}\left(\mathbf{S}+\mathbf{T}^m\right)\right)e^{n-1}-\mathbf{T}^k p^n-\mathbf{g}^n\end{aligned} \tag{16}$$

This equation is solved in each time step. For solving p^n is used equation (15).

4 RESULTS

In this paragraph we show an example of simulations using two above mentioned models from the paragraph 3.1. We chose a very simple structure. In a WR90 waveguide of a length equal to 60mm, a dielectric slab was put. The thickness of the slab was as large as 6mm, and the following parameters of the Debye model was assumed: $\varepsilon_\infty = 7.87$, $\Delta\varepsilon = 20.14$, $\tau = 1.62$ and $\sigma = 0.71$. These parameters was chosen according to measurements of a living tissues obtained by [1]. A little modification was done, however, in order to obtain pseudo-resonant characteristics of the analyzed configuration. The purpose of this is the fact, that errors at resonant characteristics are better visible and can better show potential problems.

The waveguide was discretized into 200 cells along direction of propagation and the eigen-function expansion technique [2] was used to transform 3D problem into 1D one. As the boundary conditions a low order ABC described in [6].

The s_11 parameter calculated using two above described models is presented in Figure 1. As the reference a results from frequency domain analyses was used. Since handling dispersive material in frequency domain approach is straightforward this simulation can be considered as reliable as reference.

As seen the time domain simulation where the bilinear model for handling the Debye model was used gives almost identical results. Some differences originate from the fact that the absorbing algorithm used in time domain yielded reflection error of level -50dB, while in the frequency domain the reflection error was below -70dB.

What is visible, the simpler model, based on direct sampling of the pulse response resulted in significant error. The s_11 characteristics obtained in this simulation only generally follows the correct plot. However, also a reflection minimum of similar depth is observed.

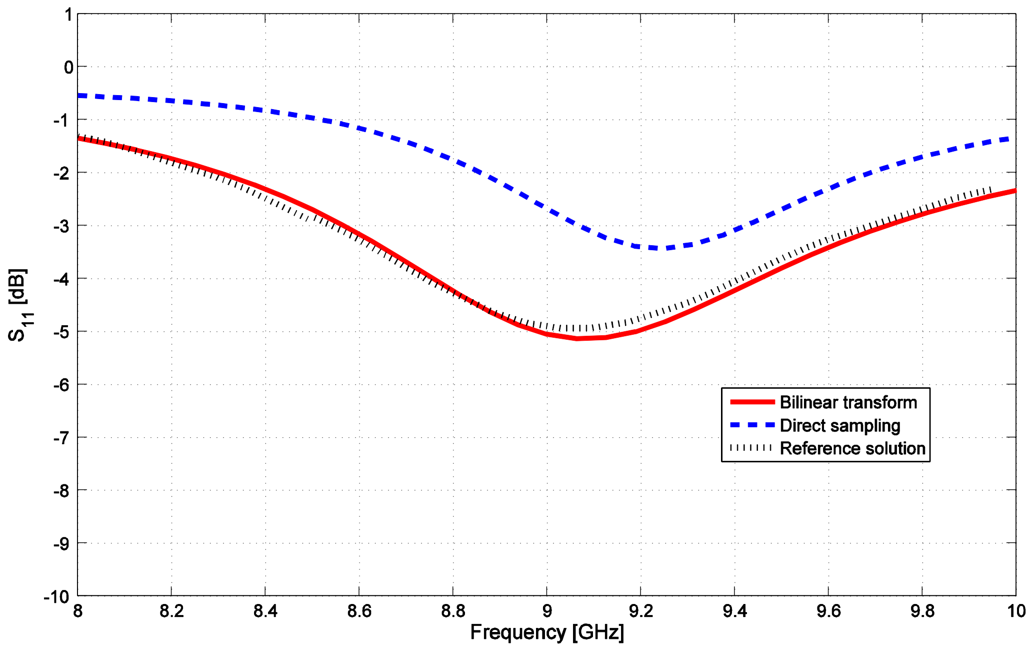

Figure 1: S_ 11 parameter

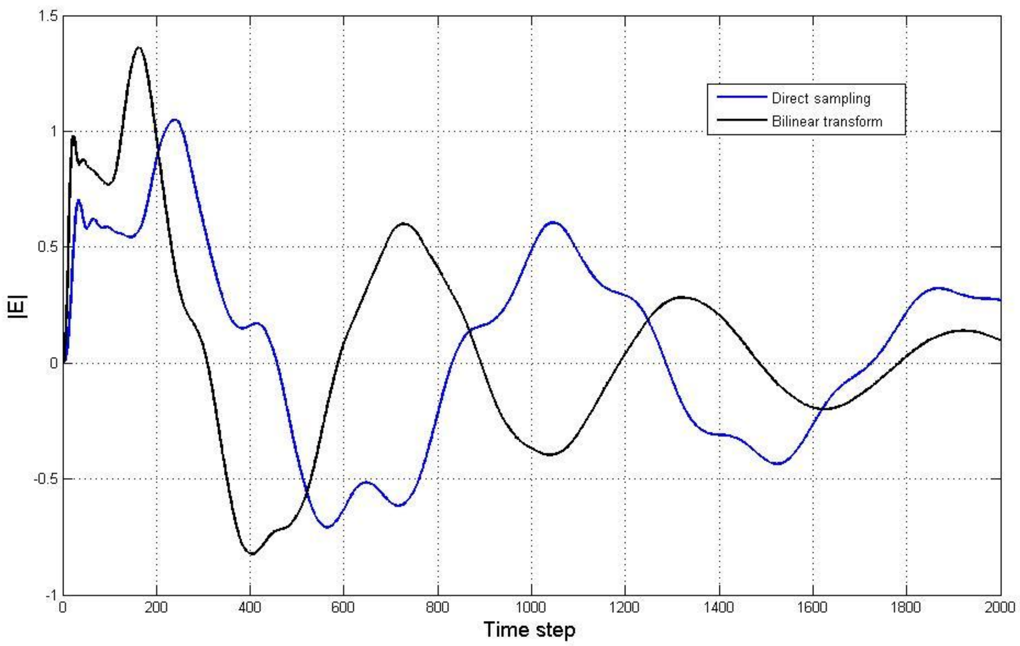

Figure 2: pulse response

In order to emphasize the difference in these models, in Figure 2, pulse responses of the system are compared. The waveguide with a load was excited by the Kronecker delta inside the dispersive dielectric. A response 2 samples away from the excitation was recorded. As seen, although there are some common elements of these 2 plots, like harmonic decaying shape, the pulse responses are absolutely different.

Other example compares method which uses bilinear transformation with inverse Fourier method. We chose a test case from [7]. In free space is an obstacle which is describe by Debye model. $\varepsilon_\infty = 1$, $\varepsilon_s = 78.2$ and $\tau = 8.1\times10^{-12}$. As an excitation pulse is used sine wave with carrier frequency 10 GHz with 12 cycles.

In Figure 3 is the solution of pulse response inside dispersive media for both methods. In Figure 4 is detail of Figure 3. From Figures is seen excellent agreement of both methods.

5 CONCLUSION

The accuracy and efficiency of handling dispersive material in time domain algorithms is shown. Approach based on direct and bilinear transform of a pulse response of Debye medium is compared. Also indirect approach, namely obtaining the time depended signal as an inverse Fourier transform of results of multiple frequency domain simulations is discussed.

Two methods of handling dispersive material based on digital filter approach have been presented. The models ware applied to Newmark time marching scheme. We show that only response obtained by bilinear transform from s to Z characteristics lead accurate and reliable results.

Comparison results by bilinear transformation and inverse Fourier transform verify accuracy of first method.

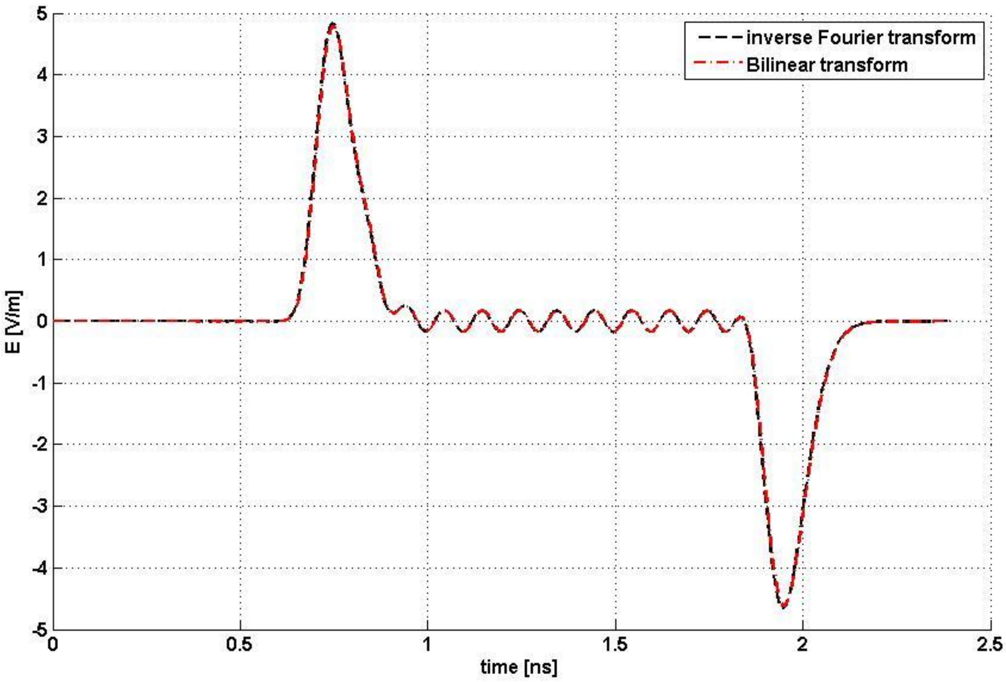

Figure 3: Pulse response of second example

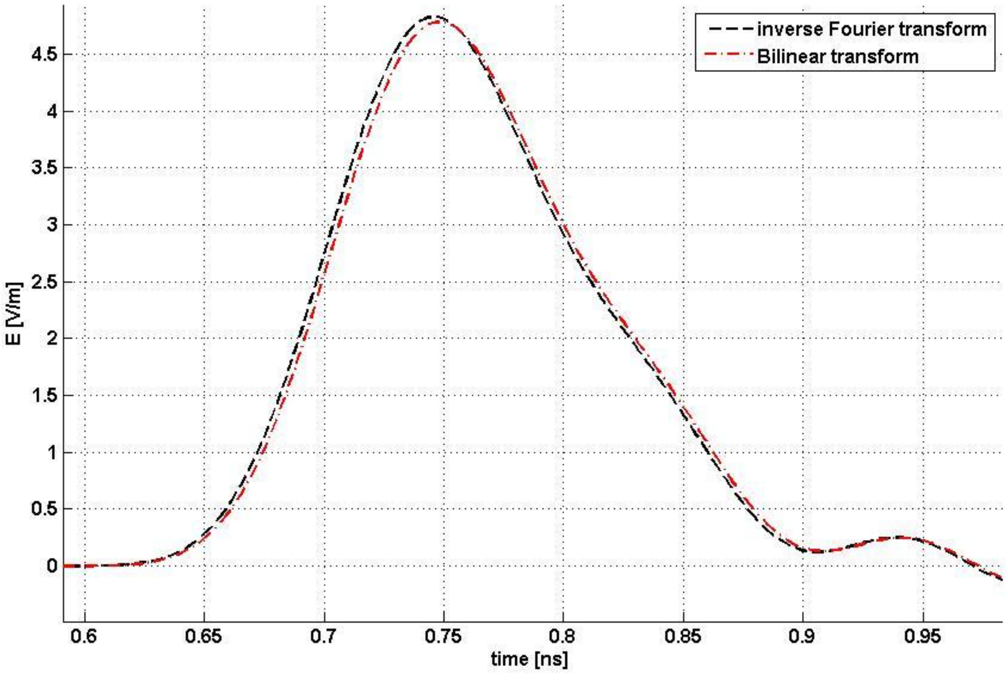

Figure 4: Pulse response - detail of second example

6 ACKNOWLEGMENTS

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