11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

FLATNESS DEFECTS IN SHEET ROLLING MODELISED BY ARLEQUIN AND ASYMPTOTIC NUMERICAL METHODS

K. Kpogan[1,2], H. Zahrouni[1,2], H. Potier-Ferry[1,2], H. Ben Dhia[3]

[1] Université de Lorraine, Laboratoire d'Etude des Microstructures et de Mécanique des Matériaux(LEM3), UMR CNRS 7239, Ile du Saulcy 57045 Metz, France
[2] DAMAS, Laboratory of Excellence on Design of Alloy Metals for low-mAss Structures, Université de Lorraine, France
[3] Ecole Centrale Paris, Laboratoire de Mécanique des Sols, Structures et Matériaux(MSSMat), Grande Voie des Vignes, 92295 Châtenay-Malabry Cedex, FRANCE

ABSTRACT

Rolling of thin sheets generally induces flatness defects due to thermo-elastic deformation of rolls. This leads to heterogeneous plastic deformations throughout the strip width and then to out of plane displacements to relax residual stresses.

In this work we present a new numerical technique to model the buckling phenomena under residual stresses induced by rolling process. This technique consists in coupling two finite element models: the first one consists in a three dimensional model based on 8-node tri-linear hexahedron which is used to model the three dimensional behaviour of the sheet in the roll bite; we introduce in this model, residual stresses from a full simulation of rolling (a plane-strain elastoplastic finite element model) or from an analytical profile. The second model is based on a shell formulation well adapted to large displacements and rotations; it will be used to compute buckling of the strip out of the roll bite. We propose to couple these two models by using Arlequin method.

The originality of the proposed algorithm is that in the context of Arlequin method, the coupling area varies during the rolling process. Furthermore we use the asymptotic numerical method (ANM) to perform the buckling computations taking into account geometrical nonlinearities in the shell model. This technique allows one to solve nonlinear problems using high order algorithms well adapted to problems in the presence of instabilities. The proposed algorithm is applied to some rolling cases where "edges-waves" and "center-waves" defects of the sheet are observed.

Key Words: *Rolling, Residual stresses, Buckling, Arlequin, Asymptotic Numerical Method*

Introduction

Flatness defects are among the major problems encountered in strip rolling. Their direct origin is out-of-bite stress gradients resulting in buckling in the compressive stress areas. The most important ones of flatness defects are "edge-waves" and "center-waves" buckles. These waves are the result of buckling due to self-equilibrating longitudinal residual stresses with a compressive longitudinal membrane force state in the middle of the strip ("center buckles") or in the edge zones ("edge buckles"), respectively [8]. Then depending on the stress component involved and the location of compressive areas, waves in the longitudinal, transverse or oblique directions can be found at various locations. During the production process, the buckling waves are usually suppressed by global traction. Thus, in some cases, the sheet may appear more or less flat, or even perfectly flat on the rolling line. Nevertheless we can still talk about flatness defects, insofar as there may be residual stresses in the sheet. This is why the post-bite stress profile is called “latent flatness defects”. If defects are only latent, the stress field computed beyond the bite by a 3D finite element model (FEM) should be correct. One can find in the literature three ways to model rolling, depending on the manner to account for flatness defects. The first way is based on 3D FEM that generally do not permits to capture buckling effects; they can only predict latent defects. Other models permit to account for buckling in an uncoupled approach: the 3D model yields residual stresses that are further included as the starting part of a buckling analysis. Last, few FEM are able to model rolling and buckling in a coupled manner.

In line with models where flatness defects are latent, Hacquin et al. [7] have proposed a coupled model of thermo-elastoviscoplastic strip deformation and thermo-elastic roll deformation. It is based on the 3D FEM. Its purpose is the prediction of profile defects, strain and stress maps, including residual stresses, in a rolled strip.

In the second category of rolling models, several publications have been focused on an uncoupled approach. These approaches are generally based on (semi-) analytical models. The buckling mode is assumed to be sinusoidal in the direction of rolling. Then the corresponding out-of-plane displacements are introduced into the equations of the problem. Fisher et al. [8] gave a polynomial form in transverse direction and all the geometrical parameters are determined by minimizing the elastic energy of the deformation path from flat to wavy shape.

Counhaye [6] was the first to propose a coupling model of rolling process and buckling phenomena. The model is based on a stress–relaxation algorithm applied only in the out-of-bite areas. They added an additional term to the elastic / plastic strain rate decomposition which represents the local shortening of a material segment when it becomes wavy due to buckling. Abdelkhalek et al. [5] used a different approach for coupling 3D FEM and Asymptotic Numerical Method (ANM) using a shell formulation. In this approach, a full 3D model of rolling is computed; the post-bite stress field obtained is introduced in the shell formulation where a buckling analysis under residual stresses is performed. The new distribution of the stress field obtained by buckling is considered as a new boundary condition for the 3D FEM. These computations are repeated until convergence.

In the present work, we propose a new approach to simulate rolling process taking into account buckling phenomena. A simplified model is used which consists in coupling a 3D continuum model and a shell model using Arlequin method. The buckling is due to residual stresses which are introduced in the 3D model and propagated in the shell model during the simulation. These residual stresses are given in the present work and come from a rolling computation or from analytical formulae.

Formulations

We propose an algorithm which consists in coupling 3D FEM and shell model by Arlequin method. So the idea is to combine these two models in the same finite element simulation that we solve by ANM. In the three dimensional part, we consider small strain and residual stresses which come from a full model taking into account the elastoplastic law adapted to rolling.

$$\begin{cases} \int_{\Omega} {}^{t}\sigma : \delta\varepsilon d\Omega - \lambda P_e(\delta u) = 0 \\ \sigma = D : \varepsilon + \lambda^{res}\sigma^{res} \\ \varepsilon(u) = \frac{1}{2}\left(\Delta u + {}^{t}\Delta u\right) \end{cases} . \tag{1}$$

where ε and $\delta\varepsilon$ are respectively the strain and the virtual strain tensor. P_e corresponds to the energy density due to external load, which is calibrated by a load parameter λ; σ and σ^{res} are respectively the Cauchy stress tensor and the residual stress. We use small strain in this part of the model since we assume that the sheet in upstream of the roll mill is flat. But progressively, the 3D model will transmit the residual stresses to the shell model. Thus only the shell model will buckle during the simulation.

Finally in the shell part, we consider a geometrically nonlinear model (Eq. 2). This shell model, proposed by Büchter el al. [4], avoids locking by incorporating an additional parameter describing the shell thickness variation. It is distinguished from classical shell models that are usually based on degenerated constitutive relations, since the present formulation uses the unmodified and complete three-dimensional constitutive law.

$$\begin{cases} \int_{\Omega} {}^{t}S : \delta\gamma^{u} d\Omega - \lambda P_e(\delta u) = 0 \\ \int_{\Omega_e} {}^{t}S : \delta\tilde{\gamma}\, d\Omega = 0 \\ S = D : \gamma + \lambda^{res} S^{res} \\ \gamma = \underbrace{\overbrace{\frac{1}{2}\left(\Delta u + {}^{t}\Delta u\right)}^{\gamma^{l}} + \overbrace{\frac{1}{2}\,{}^{t}\Delta u \Delta u}^{\gamma^{nl}}}_{\gamma^{u}} + \tilde{\gamma} \end{cases} \tag{2}$$

The total Green-Lagrange strain tensor is γ; it is decomposed into the compatible strain γ^{u} and an additional assumed strain term $\tilde{\gamma}$. The compatible tensor γ^{u} is in turn decomposed into a linear part γ^{l} and a nonlinear one γ^{nl}. Stress tensor is the second Piola-Kirchhoff stress S.

Arlequin coupling

The Arlequin method allows to couple two different mechanical states through reliable coupling operators as well as consistent energy distribution between the two coupling zones. The central point of the model is the coupling operator C which is selected by analogy with the deformation energy of the shell. Otherwise the coupling area S_c varies during the rolling process. In this paper, we limit ourselves to the H^1 coupling, and hence

$$C(\mu, u) = \int_{S_c} \{\mu . u + \varepsilon(\mu) : D : \varepsilon(u)\} d\Omega \tag{3}$$

where u and μ are respectively the displacement and the Lagrange multiplier field. D is the elasticity tensor.

The total energy of the coupling elements is given by:

$$\Pi^{total}\left(u_{3D},u_{shell},\mu\right)=\Pi^{1}(u_{3D})+\Pi^{2}(u_{shell})+C(\mu,u_{3D}-u_{shell}) \tag{4}$$

where Π^1 and Π^2 are respectively the potential energy of the 3D FEM model and the shell model. In addition we introduce into these potential energies, the weighting functions α_i and β_i in order in to share the energy between the two models in the coupling area. These weighting functions are used respectively for strain energy and external force work, and they satisfy the following relations:

$$\alpha_{3D}+\alpha_{shell}=\beta_{3D}+\beta_{shell}=1 \tag{5}$$

Then the Arlequin method seeks the stationary points of the functional Π^{total}. In the following, let us denote $(.)_{3D}\equiv(.)_1$ and $(.)_{shell}\equiv(.)_2$. Then we have:

$$\Pi^{1}(\delta u_1)=\int_{\Omega_1}\alpha_1\,{}^{t}\sigma:\delta\varepsilon d\Omega-\lambda\underbrace{\left(\int_{\Omega_1}\beta_1 f_{vol}^{1}\delta u_1 d\Omega+\int_{\partial\Omega_1}\beta_1 f_{surf}^{1}\delta u_1 d\Gamma\right)}_{P_e(\delta u_1)} \tag{6}$$

$$\Pi^{2}(\delta u_2)=\int_{\Omega_2}\alpha_2\,{}^{t}S:\delta\gamma^{u}d\Omega-\lambda\left(\int_{\Omega_2}\beta_2 f_{vol}^{2}\delta u_2 d\Omega+\int_{\partial\Omega_2}\beta_2 f_{surf}^{2}\delta u_2 d\Gamma\right) \tag{7}$$

f_{vol}^{i} and f_{surf}^{i} are the volume force applied on the body Ω and the surface force acting on the body boundary $\partial\Omega$.

From Eq. 4, Eq. 6 and Eq. 7, we obtain the variational form for the problem:

$$\begin{cases}\Pi^{i}(\delta u_i)+(-1)^{i+1}C(\mu,\delta u_i)=0\\ C(\delta\mu,u_1-u_2)=0\end{cases}\forall\delta u_i,\delta\mu\quad(i=1,\ 2) \tag{8}$$

Asymptotic Numerical Method

ANM is a technique for solving nonlinear equations based on the Taylor expansion to higher order. It has been proven to be an efficient method to deal with nonlinear problems in solid mechanics [1,3]. The technique consists in transforming a given nonlinear problem into a sequence of linear ones to be solved successively, leading to a numerical representation of the solution in the form of power series truncated at relatively high orders. Once the series are fully determined, an accurate approximation of the solution path is provided inside a determined validity range. Compared to iterative methods, ANM allows significant reduction of computation time since only one decomposition of the stiffness matrix is used to describe a large part of the solution branch.

First, the procedure allows developing the unknown variables of the problem in the form of power series with respect to a path parameter "*a*" and truncated at order *N*

$$\begin{Bmatrix} u_i \\ \mu \\ \tilde{\gamma} \\ \sigma \\ S \end{Bmatrix} = \begin{Bmatrix} u_i^0 \\ \mu^0 \\ \tilde{\gamma}^0 \\ \sigma^0 \\ S^0 \end{Bmatrix} + a \begin{Bmatrix} u_i^1 \\ \mu^1 \\ \tilde{\gamma}^1 \\ \sigma^1 \\ S^1 \end{Bmatrix} + \ \dots \ a^N \begin{Bmatrix} u_i^N \\ \mu^N \\ \tilde{\gamma}^N \\ \sigma^N \\ S^N \end{Bmatrix} \tag{9}$$

The series thus formed is composed of N sequences of the unknown variables and with the initial state of the problem given.

For simplicity, we assume that there are no boundary forces applied on the boundary of the coupling domain. We denote that $f_{vol}^i \equiv f$. We obtain then the linear problem for order 1:

$$\begin{cases} \int_{\Omega_1} \alpha_1 \, {}^t\sigma^1 : \varepsilon(\delta u_1) d\Omega + \int_{S_c} \mu \delta u_1 + \varepsilon(\mu) : D : \varepsilon(\delta u_1) d\Omega = \lambda_1 \int_{\Omega_1} \beta_1 f_1 \delta u_1 d\Omega \\ \int_{\Omega_2} \alpha_2 \left\{ {}^tS^1 : \left[\gamma^l(\delta u_2) + 2\gamma^{nl}(u_2^0, \delta u_2) \right] + {}^tS^0 : 2\gamma^{nl}(u_2^1, \delta u_2) \right\} d\Omega \\ \qquad - \int_{S_c} \left\{ \mu \delta u_2 + \varepsilon(\mu) : D : \varepsilon(\delta u_2) \right\} d\Omega = \lambda_1 \int_{\Omega_2} \beta_2 f_2 \delta u_2 d\Omega \\ \int_{S_c} \left\{ \delta\mu u_1 + \varepsilon(\delta\mu) : D : \varepsilon(u_1) \right\} d\Omega - \int_{S_c} \left\{ \delta\mu u_2 + \varepsilon(\delta\mu) : D : \varepsilon(u_2) \right\} d\Omega \quad = 0 \\ \int_{\Omega_2} {}^tS^1 : \delta\tilde{\gamma} \; d\Omega = 0 \\ S^1 = D : \left[\gamma^l(u_2^1) + 2\gamma^{nl}(u_2^1, u_2^0) + \tilde{\gamma}^1 \right] + \lambda_1^{res} S^{res} \end{cases} \tag{10}$$

Similarly, we obtain the linear problem for order k ($1 < k \leq N$)

$$\begin{cases} \int_{\Omega_1} \alpha_1 \, {}^t\sigma^k : \varepsilon(\delta u_1) d\Omega + \int_{S_c} \left\{ \mu \delta u_1 + \varepsilon(\mu) : D : \varepsilon(\delta u_1) \right\} d\Omega \quad = \lambda_k \int_{\Omega_1} \beta_1 f_1 \delta u_1 d\Omega \\ \int_{\Omega_2} \alpha_2 \left\{ {}^tS^k : \left[\gamma^l(\delta u_2) + 2\gamma^{nl}(u_2^0, \delta u_2) \right] + {}^tS^0 : 2\gamma^{nl}(u_2^k, \delta u_2) \right\} d\Omega \\ \quad - \int_{S_c} \left\{ \mu \delta u_2 + \varepsilon(\mu) : D : \varepsilon(\delta u_2) \right\} d\Omega = \lambda_k \int_{\Omega_2} \beta_2 f_2 \delta u_2 d\Omega - \int_{\Omega_2} \sum_{j=1}^{k-1} {}^tS^j : 2\gamma^{nl}(u_2^{k-j}, \delta u_2) \; d\Omega \\ \int_{S_c} \left\{ \delta\mu u_1 + \varepsilon(\delta\mu) : D : \varepsilon(u_1) \right\} d\Omega - \int_{S_c} \left\{ \delta\mu u_2 + \varepsilon(\delta\mu) : D : \varepsilon(u_2) \right\} d\Omega \; = 0 \\ \int_{\Omega_2} {}^tS^k : \delta\tilde{\gamma} \; d\Omega = 0 \\ S^k = D : \left[\gamma^l(u_2^k) + 2\gamma^{nl}(u_2^k, u_2^0) + \sum_{j=1}^{k-1} \gamma^{nl}(u_2^{k-j}, u_2^j) \; + \tilde{\gamma}^k \right] + \lambda_k^{res} S^{res} \end{cases} \tag{11}$$

Finite element discretization

We propose for the shell model a discretization using isoparametric quadrilateral element with eight nodes and reduced integration. The three dimensional model is based on 8-node tri-linear hexahedron elements. We note that the Lagrange multiplier field μ is discretized as the shell model. The displacement field in the two coupling domains, its gradients and the Lagrange multiplier are discretized in the following form:

$$\begin{cases} \{u_i\} = [N_i]\{q_i\}^e \\ \{\theta(u_i)\} = [G_i]\{q_i\}^e \\ \{\mu\} = [N_i]\{\mu\}^e \end{cases} \tag{12}$$

where $[N_i]$, $[G_i]$ are respectively the shape functions associated to the displacement and gradient matrix. Nodal displacements and Lagrange multiplier are respectively contained in the vectors $\{q_i\}^e$ and $\{\mu\}^e$.

By using Eq. 10, Eq. 11 and Eq. 12, we obtain the linear systems of equations for asymptotic order *k*:

$$\underbrace{\begin{bmatrix} K_1 & 0 & C_1 \\ 0 & K_2 & -C_2 \\ {}^tC_1 & -{}^tC_2 & 0 \end{bmatrix}}_{[K]} \underbrace{\begin{Bmatrix} q_1 \\ q_2 \\ \mu \end{Bmatrix}_k}_{\{U\}_k} = \lambda_k \underbrace{\begin{Bmatrix} f_1 \\ f_2 \\ 0 \end{Bmatrix}}_{f} + \lambda_k^{res}\{S^{res}\} + \{F^{nl}\}_k \tag{13}$$

In the matrix $[K]$, we note that $[K_1]$ is the stiffness matrix for the three dimensional model and $[K_2]$ for the shell model; $[C_1]$ and $[C_2]$ are the coupling matrix derived from the coupling operator *C*. Finally, $\{F^{nl}\}_k$ is a nonlinear force vector which depends on solutions of previous orders, and is zero for the order 1 (see Eq. 10, Eq. 11).

To solve Eq. 13, an additional equation is needed. In this work, we introduce a condition similar to the arc-length type continuation condition:

$$a = \langle \{u\} - \{u_0\}, \{u_1\} \rangle \tag{14}$$

where $\langle . , . \rangle$ is the dot product for two vectors.

The resolution is performed using two steps. In the first step, an interstand tension is applied and in the second step the residual stresses are introduced.

Results

To simulate the advance of the sheet in the process, the residual stress field is translated along the sheet. The position of the roll and the coupling area are located by a parameter x_0 which varies along the length of the sheet. Meanwhile, we use the Cumulative Distribution Function $CDM(x, x_0)$, centered at x_0. This function, progressively, distribute the residual stresses in the sheet at the upstream area (the 3D FEM model) to the downstream area of the roll mill (the shell model). Thus *CDM* is zero in the 3D zone except the coupling area $\Omega_1 \setminus S_c$, between 0 and 1 in the coupling area S_c, and is equal to 1 in the shell zone except the coupling area $\Omega_2 \setminus S_c$ (see Fig. 1).

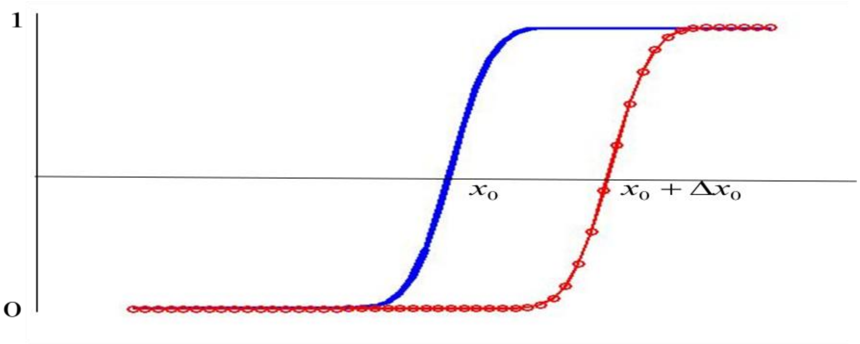

Fig. 1 Cumulative Distribution Function $CDM(x, x_0)$

We consider a thin sheet of width B=1000mm, thickness h=0.25mm and length L=2000mm. The elastic constants of the material are E=210MPa (the Young's modulus) and ν=0.3 (the Poisson's ratio). The sheet is loaded by a self-equilibrating residual stresses $S^{res}(x, y)$ and a constant global traction N_0. To validate the model we have developed a model of shell without coupling. And to be closer to the coupling model that we have developed, we have imposed additional boundary conditions (see Fig. 2).

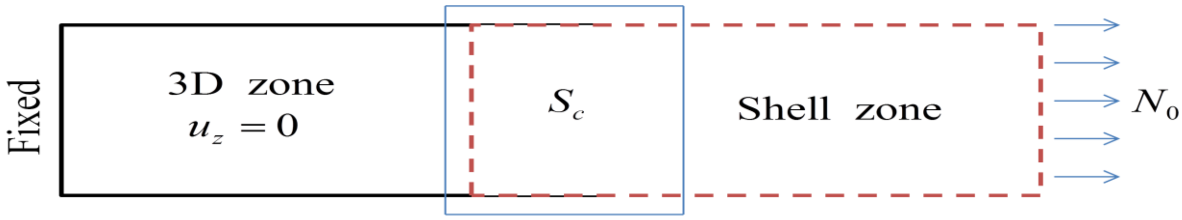

Fig. 2 Boundary conditions at each step Δx_0

We consider $S^{res}(x, y)$ with an analytical form as follows:

$$\begin{cases} S^{res}(x,y) = N^{res}(y) CDM(x, x_0) \\ N^{res}(y) = 60\left(-5\left(\frac{2y}{B} - 1\right)^4 + 1\right) \end{cases} \tag{15}$$

This profile of residual stress allows having "edges-waves" buckling on the shell model (see Fig 3 and Fig. 4). The propagation of the "edge-waves" buckling is presented in the Fig. 4.

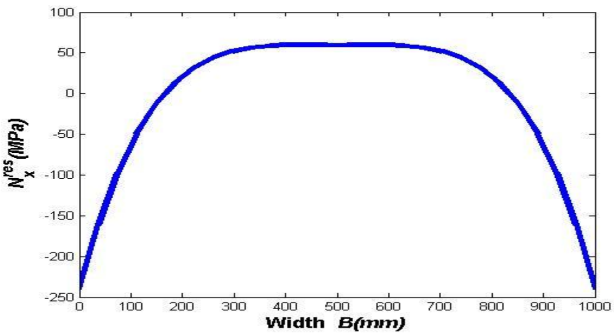

Fig. 4 Residual stress on the shell model when CDM is equal to 1; then $S^{res}(x, y) = N^{res}(y) \quad \forall x$

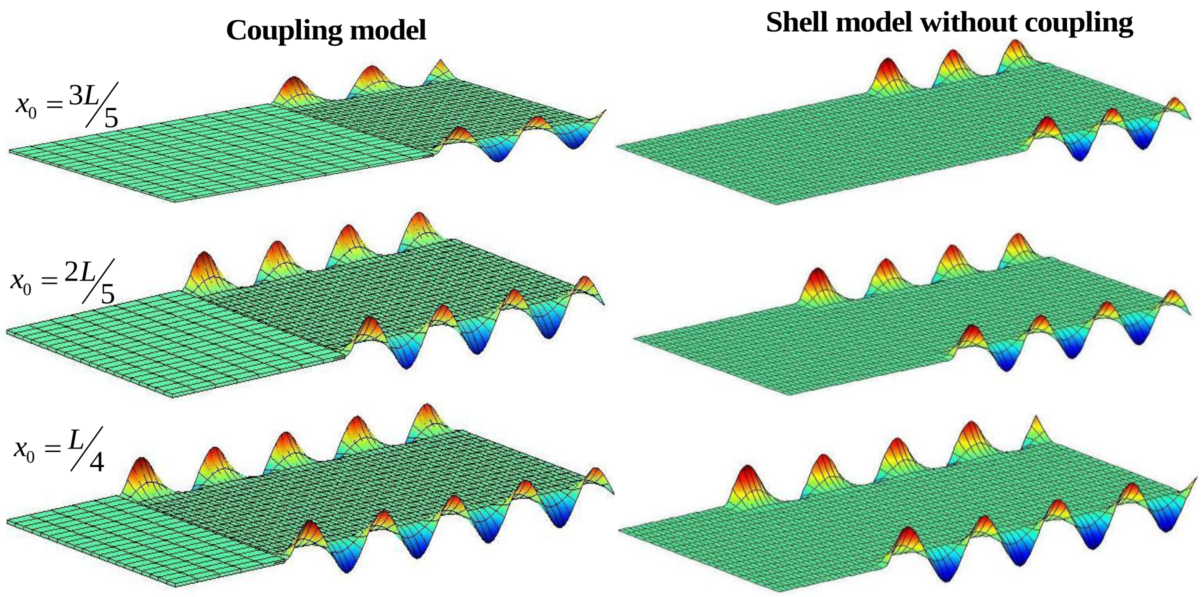

Fig. 5 Propagation of the “edge-waves” flatness defects

Furthermore, we validate the model by comparing load-displacement *Uz* curve at a maximum displacement point on the sheet buckled. Although the model without coupling is only an approximation of the model developed in this work, we notice that their results are in good agreement (see Fig. 6).

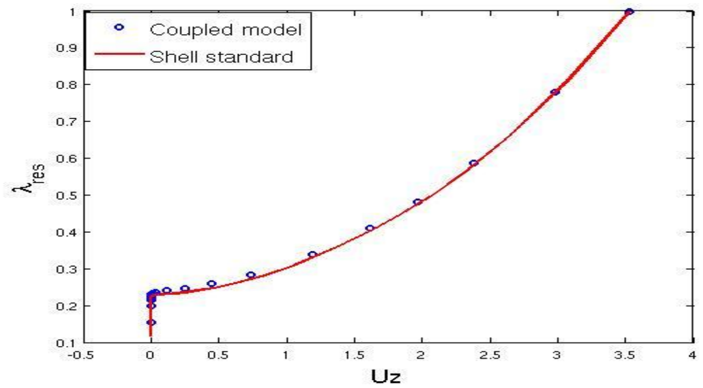

Fig. 6 Residual stress parameter λ^{res} versus the at a maximum vertical displacement point

Conclusions

We have developed a model that describes the buckling under residual stresses. The model is based on Arlequin and Asymptotic Numerical methods. To better approximate the rolling process, we have introduced a 3D FEM model that will describe the phenomena of the sheet at the upstream of the roll mill; and a shell model that will better describe the instability of the sheet – i.e. the flatness defects. The model we developed is a simplified model; then to be closer to a full rolling model, we have introduced in the 3D model a residual stress profile derived from an elastoplactic FEM. This residual stress will be transferred progressively to the shell model. Both models are coupled by Arlequin method. Meanwhile, to simulate the process, the residual stress field is translated along the sheet leading to a variation of the coupling area. Since the resulting problem is nonlinear, it is solved using Asymptotic Numerical Method which is a good tool for solving problems in the presence of instabilities. To validate the model we presented an application where residual stresses induce “edge-wave” buckling in the sheet. We also note that our procedure can be easily used to take into account the presence of several defects localized at the structure (“center-waves”, “quarter-waves”, “herringbone-waves”…). Work is in progress to introduce our procedure directly in a rolling code (LAM3) where residual stresses are computed by a thermo-mechanical procedure.

Acknowledgment

The authors wish to thank the French National Research Agency (ANR) for its financial support and the partners of the Platform Project for the authorization to publish this work.

REFERENCES

[1] H. Zahrouni, B. Cochelin and M. Potier-Ferry, Computing finite rotations of shells by an asymptotic-numerical method. *Computer Methods in Applied Mechanics and Engineering*, Vol. **175**, pp. 71-85, 1999.

[2] H. Ben Dhia and G. Rateau, The arlequin method as a flexible engineering design tool. *International Journal for Numerical Methods in Engineering*, Vol. **62**, pp. 1442-1462, 2005.

[3] B. Cochelin, N. Damil, M. Potier-Ferry, Méthode asymptotique numérique, *Hermès Science Publications*, 2007.

[4] N. Büchter, E. Ramm, D. Roehl, Three-dimensional extension of nonlinear shell formulation based on the enhanced assumed strain concept, *International Journal for Numerical Methods in Engineering*, Vol. **37**, pp. 2551-2568, 1994.

[5] S. Abdelkhalek, H. Zahrouni, M. Potier-Ferry P, N. Legrand, P. Monmitonnet, P. Buessler, Coupled and uncoupled approaches for thin cold strip buckling prediction, *International Journal of Material Forming*, Vol. **2** pp. 833-836, 2009.

[6] C. Counhaye, Modélisation et contrôle industriel de la géométrie des aciers laminés à froid (modelling and industrial control of the geometry of cold rolled steels). *PhD thesis, University of Liege*, 2000.

[7] A. Hacquin, P. Montmitonnet, P. Guillerault, A steady state thermo-elastoviscoplastic finite element model of rolling with coupled thermo-elastic roll deformation, *J Mater Proc Technol*, Vol. **60**, pp. 109-116, 1996.

[8]] F. D. Fischer, N. Friedl, A. Noe, F.G. Rammerstorfer, A study on the buckling behaviour of strips and plates with residual stresses, *Steel Res Int.* Vol. **76**, pp. 327-335, 2005.