

# IMPLEMENTATION OF A CONSTITUTIVE MODEL FOR THE FINITE ELEMENT ANALYSIS OF LANDSLIDE TRIGGERED BY RAINFALL

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**Abstract.** The paper deals with the implementation of an extended Cam-Clay model for unsaturated soils [14]. The model is defined using the generalized Bishop stress as formulated in [3] and [2] and an extended hardening law that accounts for the rate of the degree of saturation. The model is implemented in the research code COMES-GEO ([4], [5], [6], [7], [8]) and the rate equation is integrated via an implicit return mapping scheme. The elastic predictor stage is modified to account for soil collapse due to wetting. This mathematical feature allows to use a single tangent matrix as discussed in the present paper. The model is applied in the simulation of a rainfall induced landslide. The purpose of the analysis is to assess the suitability of the model to reproduce a shallow landslide. The analysis of the stress path into the flowing mass show that the condition of instability corresponds to a phase transition of water vapour into liquid.

## 1 INTRODUCTION

Shallow rainfall induced landslides are a serious problem for ages. The failure mechanism of this type of landslide is rather complex and dependent on many factors and thus difficult to describe in an accurate way. Rainwater infiltration into an initially partially saturated soil gives rise to increasing moisture content and reduces the matric suction. The soil strength is lowered, which may induce slope collapse. Recently, the use of soil

bioengineering methods to stabilize soil slopes has become very popular. Many bioengineering models for slope stability analysis have been developed in recent years. Most models are based on the Limit Equilibrium Methods (LEM). The aim of this study is to develop a Finite Element (FE) tool for the analysis of the performance of bioengineering methods for slope stabilization.

## 2 THERMODYNAMICS

Lewis and Schrefler [2] provided a remarkable contribution to the extension of the consolidation theory for saturated porous materials to the case of two fluid phases. Their model was proposed in the 1990s when the validity of the Bishop's equation was questioned. The Bishop's equation was considered unable to explain the collapse during wetting [15]. However, no theoretical argumentation was provided to substantiate the above criticism. The present paper provides an explanation based on the Prigogine's theorem and shows that the collapse was not correctly identified as a phase transition phenomena.

Recently, several authors have reintroduced the Bishop's equation in their constitutive models (e.g. [13]). However, they are still of the opinion that the Bishop's proposal remains questionable for the same reasons discussed by Jennings and Burland. The authors suggest however that the Bishop's expression can be adapted to model collapsible soils by introducing a second tangent operator in the linearisation of the equilibrium equation but they do not give any argumentation to demonstrate their assertion on the need of a second stress variable even with the use of the Bishop's formula. The present paper addresses the identification of the stress and kinematic variables adopting the theory of non-equilibrium thermodynamics. For details of the theory of non-equilibrium thermodynamics, the readers are referred to [10] and [16].

In the following, the intensive thermodynamic variables are assumed to be function of the space coordinate vector  $\mathbf{x}$  and time  $t$ . The intensive variables are the temperature  $T(\mathbf{x}, t)$ , the fluid pressure  $p_\alpha(\mathbf{x}, t)$  of the  $\alpha$  component of the mixture, the total applied stress tensor  $\sigma_{ij}(\mathbf{x}, t)$  and the chemical potential  $\mu(\mathbf{x}, t)$ . The extensive variables are defined with their specific values and they are: the entropy  $s(\mathbf{x}, t)$ , the internal energy  $u(\mathbf{x}, t)$  and  $n_k(\mathbf{x}, t)$ , the number of mole of the reactant  $k$ . Non-equilibrium thermodynamics assumes that the system can be in an overall non-equilibrium condition in which the variables may vary from a point to another but the local equilibrium is satisfied for each elementary volume of the continuum. The second law of thermodynamics can be written as:

$$dS = dS_i + dS_e \quad (1)$$

in which  $dS_e$  is the change of the entropy of the system due to the exchange of energy and matter and  $dS_i$  is the internal production of entropy. The local entropy production is defined as:

$$\frac{dS_i}{dt} = \int_V \Phi(x, t) dV \quad \text{with} \quad \Phi(x, t) = \frac{ds_i(x, t)}{dt} \geq 0 \quad (2)$$

The change in the number of mole of the component  $k$  can be expressed as  $dn_k = dn_k^e + dn_k^i$  where the first term is the change of the number of mole due to transport and the second term is the internal production due to chemical reactions or phase transformation. The mass balance equation of the component  $k$  can be expressed by:

$$\frac{\partial n_k}{\partial t} = \frac{\partial n_k^e}{\partial t} + \frac{\partial n_k^i}{\partial t} = -\nabla \cdot (\mathbf{v}_k n_k) + \Theta [n_k] \quad (3)$$

in which  $\mathbf{v}_k$  is the velocity of the component  $k$  and  $\Theta$  is the amount of the component  $k$  produced inside the element volume. The velocity of the centre of mass of the mixture (generally associated with the motion of the solid skeleton) can be defined as:

$$\mathbf{v} = \frac{\sum_k M_k n_k \mathbf{v}_k}{\sum_k M_k n_k} \quad (4)$$

in which  $M_k$  is the molecular mass of component  $k$ . The diffusion flow  $\mathbf{J}_k$  of component  $k$  is defined as:

$$\mathbf{J}_k = n_k (\mathbf{v}_k - \mathbf{v}) \quad (5)$$

Introducing the equation (5) in equation (3) the following expression can be obtained:

$$\frac{\partial n_k}{\partial t} = \frac{\partial n_k^e}{\partial t} + \frac{\partial n_k^i}{\partial t} = -\nabla \cdot \mathbf{J}_k - \nabla \cdot (n_k \mathbf{v}) + \sum_j v_{jk} \nu_j \quad (6)$$

in which the last term is the internal production of the component  $k$  and it can be expressed by the product of the stoichiometric coefficient  $\nu_{jk}$  and the velocity of reaction. The equation (6) can be formulated in terms of the material time derivative  $d/dt$  with respect to the motion of the centre of mass of the mixture:

$$\frac{dn_k}{dt} = -\nabla \cdot \mathbf{J}_k - n_k \nabla \cdot \mathbf{v} + \sum_j v_{jk} \nu_j \quad (7)$$

The equation of motion for the system can be written:

$$\sum_k \rho_k \frac{d\mathbf{v}}{dt} = -\nabla \cdot \boldsymbol{\sigma} + \sum_k n_k \mathbf{F}_k \quad (8)$$

in which  $\mathbf{F}_k$  is the gradient of an external field  $\psi_k$ :

$$\mathbf{F}_k = \nabla \psi_k \quad (9)$$

The balance of the kinetic energy can be obtained by multiplying both terms of equation (8) with the velocity of the centre of mass of the mixture:

$$\rho \frac{d(0.5\mathbf{v}^2)}{dt} = -\nabla \cdot (\boldsymbol{\sigma} \cdot \mathbf{v}) + \boldsymbol{\sigma} : \nabla \mathbf{v} + \sum_k \rho_k \mathbf{F}_k \cdot \mathbf{v} \quad (10)$$

Considering the relation:

$$\rho \frac{da}{dt} = \frac{\partial \rho a}{\partial t} + \nabla \cdot (a \rho \mathbf{v}) \quad (11)$$

in which  $a$  is a general quantity, the equation (10) can be rewritten as:

$$\frac{\partial (0.5 \rho \mathbf{v}^2)}{\partial t} = -\nabla \cdot (0.5 \rho \mathbf{v}^2 \mathbf{v} + \boldsymbol{\sigma} \cdot \mathbf{v}) + \boldsymbol{\sigma} : \nabla \mathbf{v} + \sum_k \rho_k \mathbf{F}_k \mathbf{v} \quad (12)$$

The rate of the kinetic energy and potential energy can be defined as (see [16]):

$$\frac{\partial \rho (0.5 \mathbf{v}^2 + \psi)}{\partial t} = -\nabla \cdot \left[ \rho (0.5 \mathbf{v}^2 + \psi) \mathbf{v} + \sigma_{ij} \nu_j + \sum_k \psi_k \mathbf{J}_k \right] + \boldsymbol{\sigma} : \nabla \mathbf{v} - \sum_k \mathbf{J}_k \mathbf{F}_k \quad (13)$$

in an open system the conservation of total energy can be expressed by the equation:

$$\frac{\partial e}{\partial t} + \nabla \cdot \mathbf{J}_e = 0 \quad (14)$$

in which  $e$  is the total specific energy and  $\mathbf{J}_e$  is the energy flux. The total energy  $e$  is the sum of the kinetic energy, the internal energy (the energy that is not associated with the bulk motion) and the potential energy:

$$e = 0.5 \sum_k \rho_k \mathbf{v}_k^2 + \rho u + \rho \psi = 0.5 \rho \mathbf{v}^2 + 0.5 \sum_k \rho_k \mathbf{J}_k^2 + \rho u + \rho \psi \quad (15)$$

in the following the kinetic energy of diffusion associated with the vector  $\mathbf{J}_k$  will be disregarded. The energy flux can be written as:

$$\mathbf{J}_e = \mathbf{J}_q + \sum_k \rho_k \mathbf{v}_k + \sum_k \psi_k \mathbf{J}_k + \boldsymbol{\sigma} \cdot \mathbf{v} \quad (16)$$

The expression (14) for the rate of the specific internal energy is obtained by using equation (15) and (16) (see [16]):

$$\frac{\partial (\rho u)}{\partial t} = -\nabla \cdot (\mathbf{J}_q + \rho u \mathbf{v}) + \boldsymbol{\sigma} : \nabla \mathbf{v} - \sum_k \mathbf{F}_k \mathbf{J}_k \quad (17)$$

The equation (17) can be reformulated using the equation (11):

$$\rho \frac{du}{dt} = -\nabla \cdot \mathbf{J}_q + \boldsymbol{\sigma} : \nabla \mathbf{v} + \sum_k \mathbf{F}_k \mathbf{J}_k \quad (18)$$

The specific entropy production in the reference volume can be expressed as:

$$\frac{\partial \rho s}{\partial t} = -\nabla \cdot \mathbf{J}_{s,tot} + \Phi \quad (19)$$

in which  $\mathbf{J}_{s,tot}$  is the total entropy flux due to exchange of matter and heat flow. By the use of equation (11), the equation (19) results:

$$\rho \frac{ds}{dt} = -\nabla \cdot (\mathbf{J}_{s,tot} - \rho s \mathbf{v}) + \Phi \quad (20)$$

in which the entropy due to convection  $\rho s \mathbf{v}$  is subtracted from the flux. The expression of the time derivative of the specific entropy production can be obtained using the Gibbs equation in its eulerian form:

$$T ds = du - \nu p dv^e - \sum_k \nu \mu_k dn_k \quad (21)$$

in which  $\nu$  is the specific volume and  $p dv^e$  is the elastic reversible part of the volumetric strain work. Considering the relation  $\rho^{-1} = \nu$  and introducing equations (7) and (18) in (21), the following expression can be obtained:

$$\rho \frac{ds}{dt} = -\frac{1}{T} \nabla \cdot \mathbf{J}_q + \frac{1}{T} \boldsymbol{\sigma} : d\boldsymbol{\epsilon}^p - \frac{1}{T} \sum_k \mathbf{F}_k \cdot \mathbf{J}_k + \frac{1}{T} \sum_k \mu_k \nabla \cdot \mathbf{J}_k - \frac{1}{T} \sum_k \mu_k n_k \nabla \cdot \mathbf{v} - \frac{1}{T} \sum_j A_j J_j \quad (22)$$

in equation (22), the term  $A_j$  is the Affinity, the thermodynamic force that drives the chemical reactions and  $\boldsymbol{\epsilon}^p$  is the plastic strain tensor. The following relation between the Gibbs energy of the mixture and the chemical potential holds:

$$g = g_f + g_g = \sum_k \mu_k n_k \quad (23)$$

The second and the fifth term of equation (22) can be cast to define the effective stress of the mixture; the indices  $f$  and  $g$  indicate the fluid and gas phases.

$$\hat{\sigma}_{ij} = (p + g) \delta_{ij} + \Pi_{ij} \quad (24)$$

in which  $\boldsymbol{\Pi}$  is the deviator component of the stress. The equation (22) can be rewritten as:

$$\rho \frac{ds}{dt} = -\frac{1}{T} \nabla \cdot \mathbf{J}_q + \frac{1}{T} \hat{\boldsymbol{\sigma}} : d\boldsymbol{\epsilon}^p - \frac{1}{T} \sum_k \mathbf{F}_k \cdot \mathbf{J}_k + \frac{1}{T} \sum_k \mu_k \nabla \cdot \mathbf{J}_k - \frac{1}{T} \sum_j A_j J_j \quad (25)$$

Note that the rate of the term in equation (23) for isothermal condition can be written as:

$$dg = \sum_k n_k d\mu_k + \sum_k \mu_k dn_k = -\nu_g dp_g - \nu_w dp_w + \sum_k \mu_k dn_k$$

$$\Rightarrow -\nu_g dp_g - \nu_w dp_w - \sum_k n_k d\mu_k = 0 \quad (26)$$

in which the Gibbs-Duhem equation has been used. Introducing the porosity  $\phi$ , the degree of saturation  $S_r$  and the gas saturation degree  $S_g$  as:

$$\phi = \frac{V_v}{V} \quad S_r = \frac{V_w}{V_v} \quad S_g = \frac{V_g}{V_v} = 1 - S_r \quad (27)$$

The rate of equation (24) can be formulated as:

$$d\hat{\sigma}_{ij} = d\sigma_{ij} - \phi dp_g \delta_{ij} + \phi S_r (dp_g - dp_f) \delta_{ij} + \sum_k \mu_k dn_k \delta_{ij} \quad (28)$$

The first three terms of equation (28) are similar to the expression proposed by Bishop [1]. Note that the third term vanishes in phase transition in stationary condition. In the following the equation by Lewis and Schrefler [2] in which  $\phi S_r$  is replaced by  $S_r$  will be adopted. Rearranging the equation (25) for isothermal conditions, the following equation can be obtained:

$$\Phi = \hat{\boldsymbol{\sigma}} : d\boldsymbol{\epsilon}^p - \sum_k (\nabla \mu_k - \mathbf{F}_k) \mathbf{J}_k - \sum_j A_j J_j \geq 0 \quad (29)$$

The first term is the dissipation due to the work done by the effective stress, the second term is the dissipation due to the diffusion of the components  $k$  and the last term is the dissipation due to chemical reactions. It is interesting to investigate when the equation (29) remains valid imposing an arbitrary velocity field  $\mathbf{v}^\alpha$  to each component of the mixture. An important theorem of non-equilibrium thermodynamics to deal with this problem is due to Prigogine [11]. This theorem applies in the hypothesis of a negligible value of the gradient of velocity, in the following this hypothesis is removed to analyse the applicability of the theorem in slope stability analyses. This theorem states that: *if the local mechanical equilibrium in equation (8) is satisfied (the inertial term is disregarded) the entropy production  $\Phi$  in equation (29) is invariant under the transformation [10]:*

$$\mathbf{J}_k \rightarrow \mathbf{J}'_k = \mathbf{J}_k + n_k \mathbf{v}^\alpha \quad (30)$$

in which  $\mathbf{v}^\alpha$  is an arbitrary "drift" velocity that is applied to all components  $k$  of the mixture. The theorem can be demonstrated using the Gibbs-Duhem equation. The Gibbs energy is equal to:

$$dg = -sdT + \delta_{ij} d\sigma_{ij} + \sum_k \mu_k dn_k = \sum_k \mu_k dn_k + \sum_k n_k d\mu_k \quad (31)$$

Considering isothermal conditions:

$$\nabla \sigma_{ij} = \sum_k n_k \nabla \mu_k \quad (32)$$

Introducing the hypothesis in equation (30) and equation (32) in equation (29):

$$\Phi = \sum_k (\nabla \mu_k - \mathbf{F}_k) \mathbf{J}_k + \sum_k (\nabla \sigma_{ij} - n_k \mathbf{F}_k) \mathbf{v}^a - \sum_j A_j J_j \geq 0 \quad (33)$$

If equation (8) holds (with negligible inertia) the third term multiplying  $\mathbf{v}^a$  vanishes and this demonstrates the theorem. Now consider the condition of non-negligible gradient of the velocity field. The equation (7) reads:

$$\frac{dn_k}{dt} = -\nabla \mathbf{J}'_k - n_k \nabla \cdot \mathbf{v} - \nabla (n_k \mathbf{v}^a) + \sum_j v_{jk} v_j \quad (34)$$

Applying the velocity  $\mathbf{v}^a$ , the equation (29) reads:

$$\Phi = \hat{\sigma} : \nabla \mathbf{v} + \sum_k \mu_k \frac{\partial n'_k}{\partial t} - \sum_k (\nabla \mu_k - \mathbf{F}_k) \mathbf{J}'_k - \sum_j A_j J_j \geq 0 \quad (35)$$

in which  $\partial_t n'_k$  is the variation of the mole fractions of the components due to the application of the artificial *drift* velocity  $\mathbf{v}^a$  and the term  $\sum_k \mu_k \partial_t n'_k$  is the latent heat of transformation. For the theorem of minimum entropy production the latent heat vanishes in a stationary condition [11] This condition is verified when:

$$\mu_g = \mu_w \quad \text{and} \quad \frac{\partial n_w}{\partial t} = -\frac{\partial n_g}{\partial t} \quad (36)$$

The mole fraction of the two components can be expressed with a single equation including the interface between the two phases:

$$n_\alpha = n_w + I (n_g - n_w) \quad (37)$$

in which  $I$  is the Dirac delta function that has an integral equal to zero when the  $x$  belongs to the water phase and it is equal to one when  $x$  is in the gas phase. Then equation (34) can be rewritten:

$$\frac{\partial n_w}{\partial t} = -\frac{\partial n_g}{\partial t} - (n_g - n_w) \frac{\partial I}{\partial t} \quad (38)$$

Using equations (27) and (36) the following kinematic constraint is obtained:

$$\frac{\partial (\phi S_w + \phi - \phi S_w)}{\partial t} = \frac{\partial \phi}{\partial t} = (n_w - n_g) \frac{\partial I}{\partial t} = (n_w - n_g) \int_\Gamma \mathbf{v}^* \cdot n d\Gamma \quad (39)$$

in which  $\mathbf{v}^*$  is the velocity of the moving interface and  $n$  is the normal to the interface  $\Gamma$ . The equation states that a collapse of the porous network is the consequence of the

instability (undefined applied drift velocity) and this collapse is equal to the phase change of water vapour in liquid.

### 3 REMARKS ON NUMERICAL INTEGRATION

The adopted constitutive equation is the extension of the Modified Cam clay model to the partial saturation conditions [14]. The model is integrated with a modification [12] of a return mapping technique for fully saturated soils [9].

The modification of the original scheme consists in the possibility of modifying the plastic surface position in the elastic predictor stage.

$$p_{ctr} = p_{cn} \exp(-b\Delta S_r) \quad (40)$$

Many authors have proposed integration schemes for unsaturated soils in which the original Cam clay hardening law is modified introducing the matric suction. These authors suggest that the 'extended' hardening implies a second consistent tangent operator that accounts for the dependency of the Bishop's stress on suction. If the hardening law is modified to account for the effect of the degree of saturation, the consistency can be obtained with the expression:

$$df = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial p_c} dp_c = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial p_c} \theta p_c d\lambda \frac{\partial f}{\partial p} - \frac{\partial f}{\partial p_c} b p_c dS_r = 0 \quad (41)$$

The plastic multiplier is than obtained at:

$$d\lambda = \frac{1}{H} \left[ \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} - \frac{\partial f}{\partial p_c} b p_c dS_r \right] \quad \text{with} \quad H = -\frac{\partial f}{\partial p_c} \theta p_c \frac{\partial f}{\partial p} \quad (42)$$

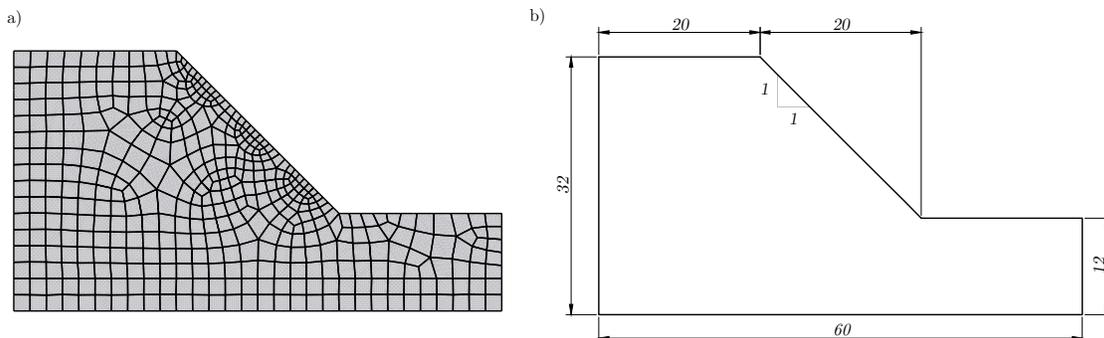
The original preconsolidation pressure in the plastic corrector stage is then:

$$p_{cn+1} = p_{cn} \exp(-b\Delta S_r) \exp \left[ \theta \frac{1}{-\frac{\partial f}{\partial p_c} \theta p_c \frac{\partial f}{\partial p}} \left( \frac{\partial f}{\partial p} \Delta p - \frac{\partial f}{\partial p_c} b p_c \Delta S_r \right) \frac{\partial f}{\partial p} \right] \quad (43)$$

where the trial value of the preconsolidation pressure has been introduced. Equation (43) shows that:

$$p_{cn+1} = p_{cn} \exp \left( -b\Delta S_r + \frac{\Delta p}{p_c} + b\Delta S_r \right) \Rightarrow \ln(p_{cn+1}) - \ln(p_{cn}) = \Delta \ln(p_c) = \frac{\Delta p}{p_c} \quad (44)$$

Equation (44) shows as the effect of  $S_r$  on the hardening law vanishes during the integration. This happens because the rate of the volumetric plastic strains and the degree of saturation are not independent variables as commonly thought by the mentioned authors.



**Figure 1:** Numerical model characteristics: a) Spatial discretization, b) Model dimensions.

#### 4 NUMERICAL ANALYSES

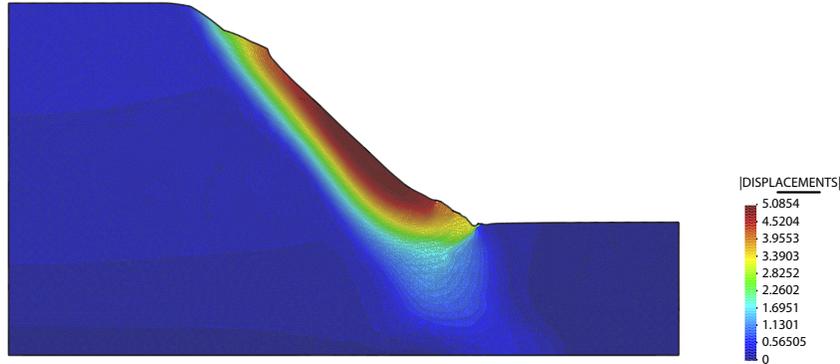
The implemented model is applied in the simulation of a rainfall induced landslide. A two-dimensional analysis is conducted using the FE code for non-isothermal multiphase porous media COMES-GEO ([4], [5], [6], [7], [8]). Initially unsaturated slope is subjected to the rainfall infiltration, until the failure occurs. The geometry of the slope, boundary conditions, soil and rainfall parameters as well as the obtained results are presented in this section.

A two-dimensional slope is spatially discretized using isoparametric quadrilateral 8-node elements. The finite element mesh and the geometry of the model are presented in Fig.1. The material parameters used in the calculations are listed in Tab.1.

**Table 1:** Soil parameters used in the calculations

Soil density	$\rho_s$	2670.0	kg/m <sup>3</sup>
Initial Young Modulus	$E_{\text{mod}}$	10000.0	kPa
Poisson's ratio	$\nu$	0.3	-
Initial void ratio	$v$	1.3	-
Intrinsic permeability of the soil	$k$	5.0 e-13	m <sup>2</sup>
Initial preconsolidation pressure	$p_c$	30.0	kPa
Plastic compressibility	$\lambda$	0.111	-
Elastic compressibility	$\kappa$	0.04	-
Parameter for partial saturation	$b$	10.0	-
Water density	$\rho_w$	1000.0	kg/m <sup>3</sup>

The initial condition of the analysis are: 200 kPa of matric suction, 293.15 K (constant temperature) and the gas pressure is imposed at the atmospheric value on the slope surface, the resulting initial degree of saturation is in the whole domain equal to 45%.



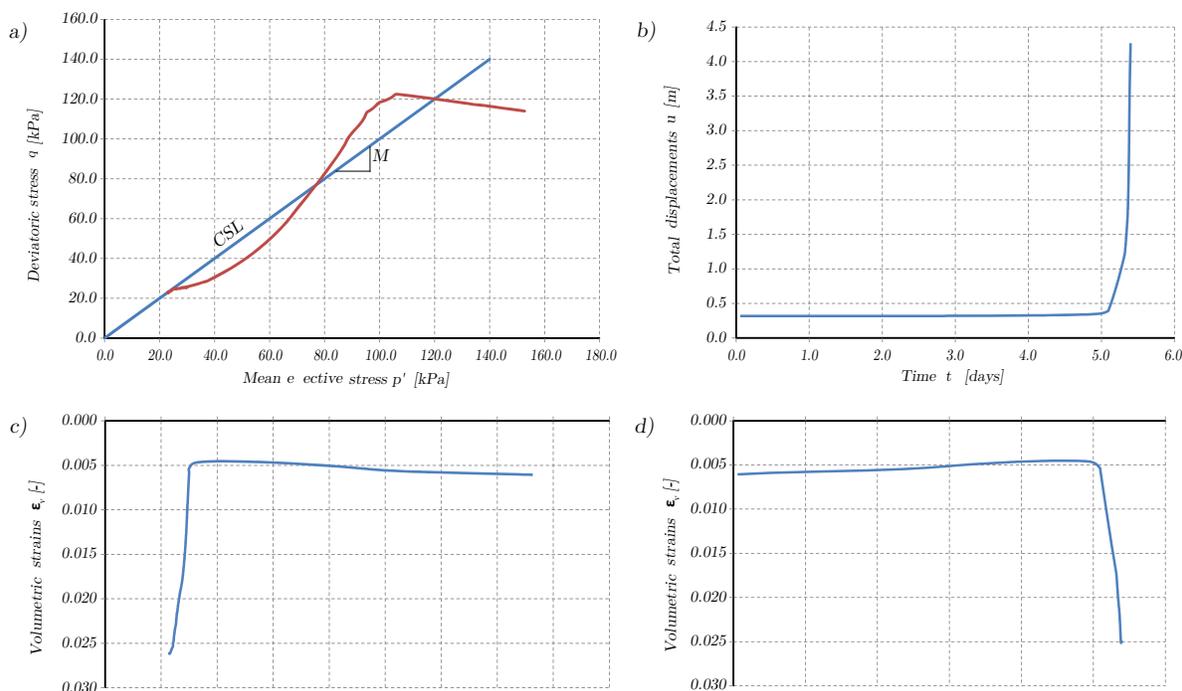
**Figure 2:** Contour of displacements norm obtained after 5 days and 8 hours of rainfall.

A rainfall of a constant intensity of 18 mm/h is imposed on a top surface. The extended Cam-clay constitutive model [14] is assigned for all elements. The slope becomes unstable after 5 days and 8 hours of rainfall. The wetting of the soil is a direct cause of a shallow landslide. The total displacements contour is shown in Fig.2.

The graphs presented in Fig.3 show the behaviour of the soil under constant infiltration rate. After the fifth day, a rapid increase in the total displacements and volumetric strains can be noticed. This is an indicator of the slope failure and it agrees with the thermodynamically based description of the previous section. Initially stiff soil starts to flow due to the wetting. This failure mechanism is characterized by a significant increase of the displacement at constant Bishop’s stress.

## 5 DISCUSSION AND CONCLUSIONS

The presented study shows that the wetting can induce instability of slopes. The numerical example shows that even if the soil is stiff and initially consistent, after an intense rainfall it can behave like a mud or other fluid-like material. The introduced extended Cam-clay model for unsaturated soil is able to describe the diffuse failure in an accurate way, because the Cam-clay ellipse can expand or shrink not only due to the increase of the plastic volumetric strains but also due to the changes in the degree of saturation. The soil collapse can be modelled using the Bishop’s stress and a single tangent matrix. The presented model is simple and requires few constitutive parameters. The model implementation in COMES-GEO allows capturing the failure and computing the development of strains, stresses and degree of saturation for further studies of bioengineered slopes. The FE analysis has an advantage with respect to the commonly used LEM, where an assumption concerning the shape and location of the failure surface has to be made. The model can also account for the real constitutive behaviour of the soil mass where the LEM assumes that it is a rigid body. The developed numerical tool will be applied in the study



**Figure 3:** Numerical results: a) stress path in a point in the middle of the slope mass, b) volumetric strains in the same point of (a), c) total displacement (Euclidean norm) of the point at the top of the slope, d) volumetric strains of the point of (b).

of the effects of evapotranspiration and roots mechanical effects on slope stability.

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