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AN ISOGEOMETRIC BEM FOR EXTERIOR POTENTIAL-FLOW PROBLEMS AROUND LIFTING BODIES

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Abstract. In this paper, the Isogeometric Analysis (IGA) concept is combined with the Boundary Element Method (BEM) for solving the exterior Neumann problem associated with the steady lifting flow around a hydrofoil. The formulation of the problem is based on a Boundary Integral Equation for the associated velocity potential combined with the null-pressure jump Kutta condition at the trailing edge. The developed Isogeometric-BEM is based on a parametric NURBS representation of the hydrofoil and employs the very same basis for representing the velocity potential. The Boundary Integral Equation is numerically solved by collocating at the Greville abscissas of the knot vector of the hydrofoil's parametric representation. Numerical error analysis of the Isogeometric-BEM using h-refinement is performed and compared with classical low-order panel methods.

1 Introduction

Performance of many ship-hull types is dependent on the hydrodynamic properties of their keels, rudders, hydrofoils and similar appendages operating as lifting bodies. Interaction of free-surface flows with lifting surfaces constitutes therefore an interesting problem, finding applications in the design of yachts and sailing boats and the performance of stabilizers, hydrofoils and similar devices; see, e.g. Belibassakis (2011) [1]. Furthermore, biomimetic flapping-wing systems are currently investigated as unsteady propulsors for augmenting ship propulsion in rough seas; see Belibassakis & Politis (2013) [2], Filippas & Belibassakis (2014) [3].

Numerical methods to treat such complex systems face major difficulties due to the simultaneous presence of water free-surface and the free vortex system generated by the lifting components, which in some cases could strongly interact, leading to extremely non-linear phenomena, as, e.g., wave breaking (Duncan 1983) [4]. Moreover, in case of rapid flows and low submergence of hydrofoils cavitation could be initiated and further evolved (see e.g., Bal et al 2001 [5]). Restricting ourselves in the case where the free vortex sheets are relatively far from the free surface, the problem could be treated in the framework of Boundary Integral Equations applications, as e.g., in Bal (1999) [6], Xie & Vassalos (2007) [7], Chen (2012) [8] and other works.

Boundary element methods (BEM) in aero/hydrodynamics are established as main tools for the solution of flow problems around isolated or systems of bodies, with or without lift; see, e.g., Katz & Plotkin (1991) [9], Paris & Canas (1997) [10], Dragos (2003) [11]. Starting from the pioneering work by Hess & Smith (1962) [12] where the 3D panel method based on source-sink distribution is presented for analysing the flow around arbitrary non-lifting bodies, this approach has been further extended by Hess (1972) [13] for lifting flows and applied by Hess and Valarezo (1985) [14] for the simulation of a steadily translating and rotating propeller. Concentrating on marine propulsors, in the years to follow, various alternative formulations of the boundary element method have been applied for the solution of propeller related steady or unsteady problems. Most boundary element methods for lifting and propeller flows solve directly for the unknown velocity potential (see, e.g., Kinnas 1996) [15], although several alternatives exist, as for example the methods based on surface vorticity distributions developed by Belibassakis & Politis (1995, 1998) [16], [17].

In the last period, higher-order BEM, based on B-splines or NURBS representations for both the geometry and the solution, have been presented for the analysis of flow around marine propellers; see, e.g., Kim et al (2007) [18], Gao & Zou (2008) [19]. In the same direction, more recently, high order BEMs based on Isogeometric Analysis have been developed, with application to potential flows on the plane (Politis et al 2009) [20] and applied to the wave resistance problem (Belibassakis et al 2013) [21].

In this work, the Isogeometric Analysis (IGA) concept introduced by Hughes et al (2005) [22], in the context of Finite Element Method, is applied to Boundary Element Method (BEM) for solving the exterior Neumann problem associated with the steady lifting flow around a hydrofoil. This work extends our previous analysis (Politis et al 2009) [20] concerning similar problems without circulation. The formulation of the problem is based on a Boundary Integral Equation for the associated velocity potential combined with the null-pressure jump Kutta condition at the trailing edge (Morino 1993 [23], Gennaretti et al 1998 [24]). The developed Isogeometric-BEM is based on a parametric NURBS representation of the hydrofoil and employs the very same basis for representing the velocity potential.

The Boundary Integral Equation is numerically solved by collocating at the Greville abscissas of the knot vector of the hydrofoil's parametric representation. Numerical error analysis of the Isogeometric-BEM, using h-refinement technique (knot insertion), is performed and compared with classical low-order panel methods. It is the intention of the authors to proceed by developing a parametric geometric model for the hydrofoil which will be used for optimum shape design by embedding the hydrodynamic solver to an optimization process.

2 Formulation of the problem

We consider a two-dimensional body whose boundary is $\partial\Omega_B$, as shown in Figure 1, moving with a constant speed $\vec{U}_B$ in an ideal fluid of infinite extent. In the body-fixed coordinate system Oxy this problem is equivalent to a uniform stream with velocity $\nabla\Phi_\infty = \vec{U}_\infty = -\vec{U}_B$, where $\Phi_\infty(x,y) = u_\infty x + v_\infty y$ is the far-field asymptotic form of the velocity potential $\Phi(x,y)$ of the resulting flow. The potential $\Phi(x,y)$ is the solution of the following boundary-value problem(BVP):

$$\nabla^2\Phi = 0, \quad \vec{x} = (x,y) \in \Omega, \tag{1}$$

$$\frac{\partial\Phi}{\partial n} = 0, \quad \vec{x} \in \partial\Omega_B \tag{2}$$

$$\Phi - u_\infty x - v_\infty y \to 0, \quad as \ \|\vec{x}\| \to \infty \tag{3}$$

where, Ω is the fluid domain outside $\partial\Omega_B$ and $\vec{n}$ denotes the unit normal vector on $\partial\Omega_B$ directed inwards to the body. The above BVP has a unique solution up to an additive constant and, in order to fix a unique solution, we normally consider, for smooth bodies, zero circulation around the body. The difference between potential flows around a smooth body and a hydrofoil is that, in order for the flow around the hydrofoil to have a physical meaning, the circulation must be different from zero. To obtain non-zero circulation a wake has to be considered behind the trailing edge of the hydrofoil, which, in the case of the steady-state potential flow it can be chosen arbitrarily starting from the trailing edge and extending to infinity; see Figure 1. In this case, the vortex wake is a material force-free surface $\partial\Omega_S$ where the pressure p and the normal fluid velocity $\partial\Phi/\partial n$ have no jump. Mathematically, these conditions are stated as follows:

1. The kinematic condition of continuity of the normal velocity on $\partial\Omega_S$

$$\frac{\partial\Phi^+}{\partial n} = \frac{\partial\Phi^-}{\partial n} \tag{4}$$

2. The dynamic condition of continuity of the pressure

$$p^+ = p^- \tag{5}$$

where, $\Phi^+(p^+)$ and $\Phi^-(p^-)$ denote the velocity potential (pressure) on the upper $\partial\Omega_{S^+}$ and lower $\partial\Omega_{S^-}$ wake surface respectively.

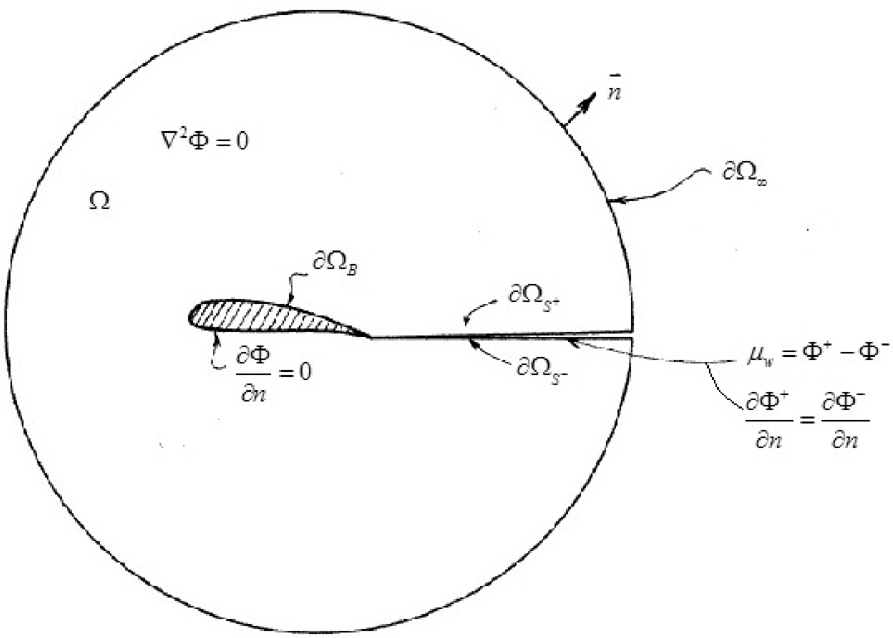

Figure 1: Geometric configuration of the hydrofoil

Applying Green's second identity for the potential $\Phi(P)$, $\quad P = (x, y)$, and the fundamental solution for the 2D Laplace equation $G(P,Q) = (1/2\pi)lnr, \quad r = |P - Q|$, we can reformulate the above boundary-value problem (1)-(3) as an integral equation of the second kind on the body boundary $\partial\Omega_B$, taking into account the conditions (4), (5) on the wake surface $\partial\Omega_S$ [23]:

$$\frac{\Phi(P)}{2} + \int_{\partial\Omega_B} \Phi(Q)\frac{\partial G(P,Q)}{\partial n_Q} ds_Q - \mu_w \int_{\partial\Omega_S} \frac{\partial G(P,Q)}{\partial n_Q} ds_Q = \Phi_\infty(P), \quad P \in \partial\Omega_B \tag{6}$$

where,

$$\mu_w = \Phi^+ - \Phi^- \tag{7}$$

is the jump of the velocity potential on the wake, which is constant along it and expresses the circulation around the hydrofoil. Equation (7) is a form of the Kutta condition, the so called Morino-Kutta condition, stating that the fluid velocity at the trailing edge of the hydrofoil must be bounded, ensuring, in this way, the unique solution of the BVP (1)-(3).

Without loss of generality, the wake surface can be considered as a straight line starting from the trailing edge, considered to be at the point (x_e, y_e), and extending to infinity; see Figure 1. In this case, the integral term on the wake $\partial\Omega_S$, appearing in equation (6), can be easily calculated resulting in:

$$\int_{\partial\Omega_S} \frac{\partial G(P,Q)}{\partial n_Q} ds_Q = \int_{x_e}^{\infty} \frac{\partial G(P,(x_Q,y_Q))}{\partial n_Q} dx_Q = \frac{1}{2\pi} \arctan\left(\frac{y_P - y_e}{x_P - x_e}\right), \tag{8}$$

which shows that the influence of the wake to the fluid flow is equivalent to a point vortex located at the trailing edge.

After solving the integral equation (6) we can calculate the pressure coefficient c_p on the hydrofoil using Bernoulli's equation:

$$c_p := \frac{p - p_\infty}{\frac{1}{2}\rho|\vec{U}_\infty|^2} = 1 - \frac{v_t^2}{|\vec{U}_\infty|^2} \tag{9}$$

where, v_t is the tangential fluid velocity on the hydrofoil given by $v_t = \partial\Phi/\partial s$.

The fluid flow around the hydrofoil can be also formulated using the perturbation potential $\phi(x,y)$ which is defined as:

$$\phi(x,y) = \Phi(x,y) - \Phi_\infty(x,y) \tag{10}$$

Using this perturbation potential $\phi(x,y)$the integral equation (6) is written as follows:

$$\frac{\phi(P)}{2} + \int_{\partial\Omega_B} \phi(Q) \frac{\partial G(P,Q)}{\partial n_Q} ds_Q - \mu_w \int_{\partial\Omega_S} \frac{\partial G(P,Q)}{\partial n_Q} ds_Q = -\int_{\partial\Omega_B} (\vec{U}_\infty \cdot n(\vec{Q})) G(P,Q) ds_Q \tag{11}$$

3 The discrete BEM-Isogeometric formulation

The purpose of this section is to present a method that combines BEM with IGA for solving numerically the boundary integral equations (6) and (11). IGA philosophy is equivalent to approximating the field quantities (dependent variables) of the boundary-value problem in question by the very same basis that is being used for representing (accurately) the geometry of the involved body-boundary. In our case the dependent variables are the total potential $\Phi(\vec{x}), \vec{x} \in \partial\Omega_B$ (equation 6) or the perturbation potential $\phi(\vec{x}), \vec{x} \in \partial\Omega_B$ (equation 11). For this purpose, we shall presume that the body boundary $\partial\Omega_B$ can be (accurately) represented as a regular parametric NURBS curve $\mathbf{r}(t), t \in I$, as follows:

$$\mathbf{r}(t) = (x(t), y(t)) := \sum_{i=0}^{n} \mathbf{d}_i M_{ik}(t), \quad t \in I = [t_{k-1}, t_{n+1}], \tag{12}$$

where $\{M_{ik}(t)\}_{i=0}^{n}$ is a rational $B-$spline basis of order k, defined over a knot sequence $\mathcal{J} = \{t_0, t_1, \ldots, t_{n+k}\}$ and possessing non-negative weights $w_i,\ i = 0, \ldots, n$, while $\mathbf{d}_i$ are the associated control points; see, e.g., Piegl & Tiller [25]. Equation (11) can then be written in the following form:

$$\frac{\phi(t)}{2} + \int_I \phi(\tau) K(t,\tau) d\tau - \frac{\mu_w}{2\pi} \arctan\left(\frac{y(t) - y_e}{x(t) - x_e}\right) = g(t) \tag{13}$$

where, for the sake of notational simplicity, we define $\phi(t) := \phi(\mathbf{r}(t))$, $G(t,\tau)) := G(\mathbf{r}(t), \mathbf{r}(\tau))$ and $K(t,\tau) = (\partial G(t,\tau)/\partial n_\tau)\|\dot{\mathbf{r}}(\tau)\|$, $g(t) = -\int_I (\vec{U}_\infty \cdot \vec{n(\tau)}) G(t,\tau) \|\dot{\mathbf{r}}(\tau)\| d\tau$.

We note that if the hydrofoil boundary $\partial\Omega_B$ is a C^2 curve (with the exception of the trailing edge where $\partial\Omega_B$ is simply C^0) the kernel $K(t,\tau)$ of the integral equation (13) is a regular function (except at the trailing edge) expressed as:

$$K(t,\tau) = \frac{-\dot{y}(\tau)\left[(x(t) - x(\tau)\right] + \dot{x}(\tau)\left[y(t) - y(\tau)\right]}{\left[x(t) - x(\tau)\right]^2 + \left[y(t) - y(\tau)\right]^2}, \quad \text{if} \quad t \neq \tau, \tag{14}$$

and, through a careful limiting process as $t \to \tau$,

$$K(t,t) = \frac{-\dot{y}(t)\ddot{x}(t) + \dot{x}(t)\ddot{y}(t)}{2\left[\dot{x}(t) + \dot{y}(t)^2\right]}, \quad \text{if} \quad t = \tau. \tag{15}$$

Aiming to employing IGA for handling equation (13), we project, in a suitably defined manner, the perturbation potential $\phi(t)$ on the spline space $\mathcal{S}^k(\mathcal{J}^{(\ell)})$, $\mathcal{S}^k(\mathcal{J}^{(0)}) := \mathcal{S}^k(\mathcal{J})$, expressed in the form:

$$\phi_s(t) := \mathcal{P}_s(\phi(t)) = \sum_{i=0}^{n+\ell} \phi_i M_{ik}^{(\ell)}(t), \quad t \in I, M_{ik}^{(0)} := M_{ik}, \tag{16}$$

where $\ell \in \mathbb{N}_0$ denotes the number of knots inserted in I. Recalling the fundamental property of knot insertion, we can say that $\{\mathcal{S}^k(\mathcal{J}^{(\ell)}), \ell \in \mathbb{N}_0\}$ constitutes a sequence of nested finite dimensional-spaces, i.e., $\mathcal{S}^k(\mathcal{J}^{(\ell)}) \subset \mathcal{S}^k(\mathcal{J}^{(\ell+1)})$. Equation (13) can then be written as

$$\frac{1}{2}\sum_{i=0}^{n+\ell} \phi_i M_{ik}^{(\ell)}(t) + \int_I \sum_{i=0}^{n+\ell} \phi_i M_{ik}^{(\ell)}(\tau) K(t,\tau) d\tau - \frac{\mu_w}{2\pi} \arctan\left(\frac{y(t) - y_e}{x(t) - x_e}\right) = g(t). \tag{17}$$

Several methods are available for defining the projection $\mathcal{P}_s$ (see eq.16) onto the finite-dimensional space $\mathcal{S}^k(\mathcal{J}^{(\ell)})$ and discretizing equation (13), like Galerkin and collocation. In the present work, a collocation scheme is adopted, which consists in projecting on $\mathcal{S}^k(\mathcal{J}^{(\ell)})$ through interpolation at a set of collocation points $t = t_j$, $j = 0, \ldots, n+\ell$, which are chosen to be the Greville abscissas associated with the knot vector $\mathcal{J}^{(\ell)}$. This leads to the following linear system for the unknown coefficients $\phi_i,\ i = 0, \ldots, n+\ell$:

$$\frac{1}{2}\sum_{i=0}^{n+\ell}\phi_i M_{ik}^{(\ell)}(t_j)+\sum_{i=0}^{n+\ell}\phi_i q_i(t_j)-\frac{(\phi_{n+\ell}-\phi_0)}{2\pi}\arctan\left(\frac{y(t_j)-y_e}{x(t_j)-x_e}\right)=g(t_j) \tag{18}$$

where, $q_i(t_j)=\int_I M_{ik}^{(\ell)}(\tau)K(t_j,\tau)d\tau$ and, taking into account the Kutta-Morino condition (7), the unknown μ_w is expressed through the difference $\phi_{n+\ell}-\phi_0$.

Having calculated $\phi_i,\ i=0,\dots,n+\ell$, the tangential velocity on the hydrofoil can be easily calculated using the derivatives of the rational B-spline basis functions $M_{ik}^{(\ell)}(t)$, without resorting to the finite difference scheme, as it is the the practice in low-order panel methods:

$$v_t=\vec{U}_\infty\cdot\vec{t}+\frac{\partial\phi}{\partial s}=\vec{U}_\infty\cdot\vec{t}+\sum_{i=0}^{n+\ell}\phi_i\frac{dM_{ik}^{(\ell)}(t)}{dt}\ \frac{1}{\|\dot{\mathbf{r}}(t)\|} \tag{19}$$

where, $\vec{t}$ denotes the unit tangential vector on the hydrofoil.

4 Numerical Results

As a first attemt to test the accuracy and effectiveness of the IGA-BEM method presented in Section 3 we present numerical results for the flow around the standard NACA-4412 profile. This hydrofoil is represented as a cubic B-spline curve with 47 knots and 43 control points.

In Figure 2 the total potential Φ on the hydrofoil is presented for an incident flow at an angle of attack of 5 deg. Numerical results of the present IGA method are compared with corresponding results of low-order panel method (Moran 1984, Sec.3) [26], considered as reference solution, using a large number of elements (N=500). The agreement of IGA solution, using considerably less degrees of freedom (n=120), is very satisfactory. We recall that in IGA method degrees of freedom (dof) is the number of the control coefficients used in the NURBS approximation of the potential Φ; see eq. (16).

In Figure 3 we compare the pressure coefficient distribution around a NACA-4412 profile at 7 deg. angle of attack, with corresponding results with a low-order panel method which adopts either the direct (Morino) formulation (see [26]), as in the present work, or the classical indirect formulation of Hess & Smith [13]. Again, considering the low-order panel solution with a large number of elements (N=500) as reference solution, we can see the good performance of IGA method, where it is needed considerably less dof (n=120) for the solution to converge.

In Figure 4 we compare the overall performance of IGA-BEM compared to low-order BEM. This is accomplished by plotting, for both methods, the error of the developed circulation of the flow μ_w around the hydrofoil (we recall that $\mu_w=\phi_{n+\ell}-\phi_0$; see eq. (7)) as a function of dof. Since the exact circulation around the hydrofoil is not known the error is calculated using a reference solution with a large number of dof=1000 for both

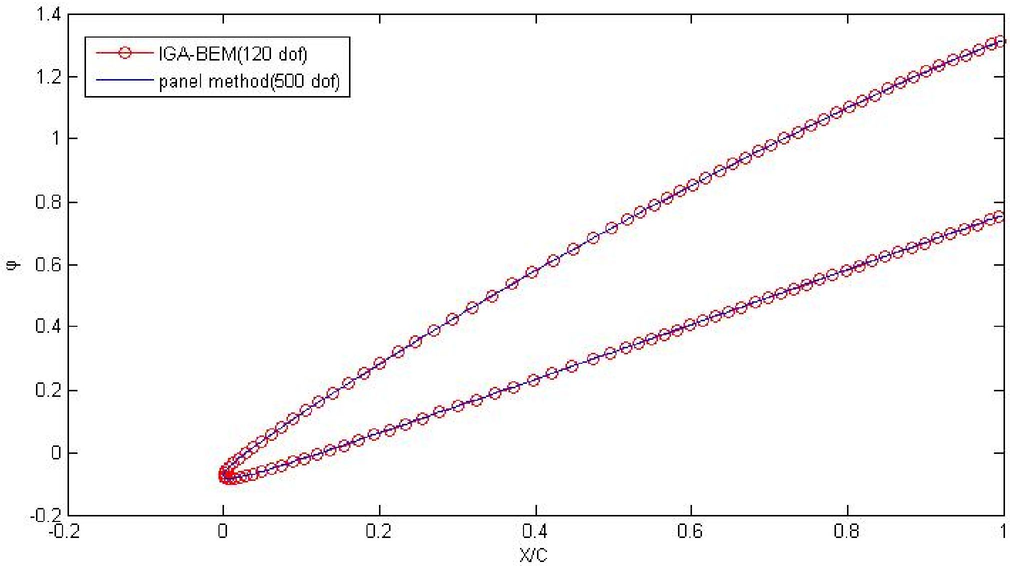

Figure 2: Potential distribution around a NACA-4412 profile at 5 deg. angle of attack and comparison with low-order panel method

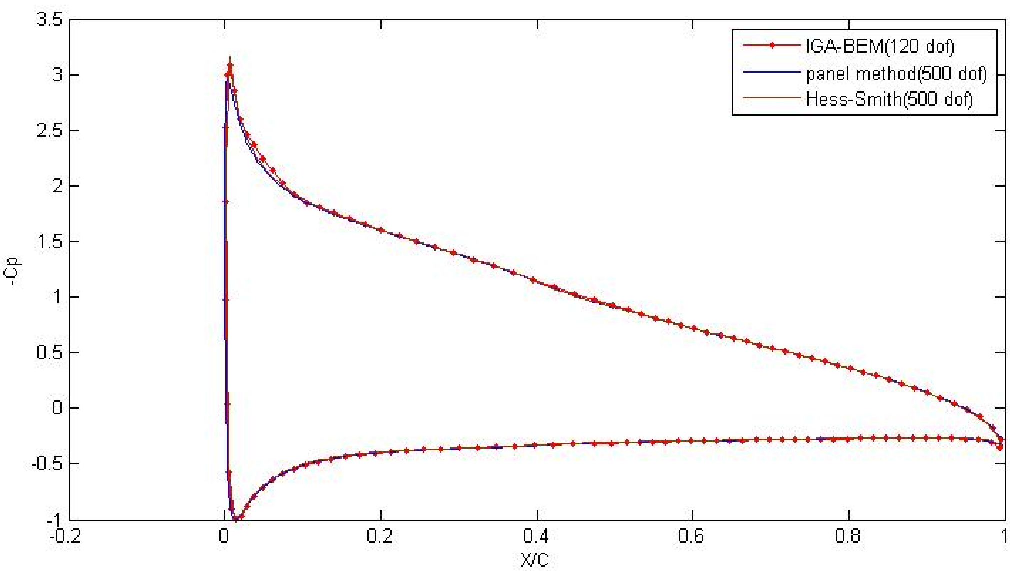

Figure 3: Pressure distribution around a NACA-4412 profile at 7 deg. angle of attack and comparison with low-order panel methods

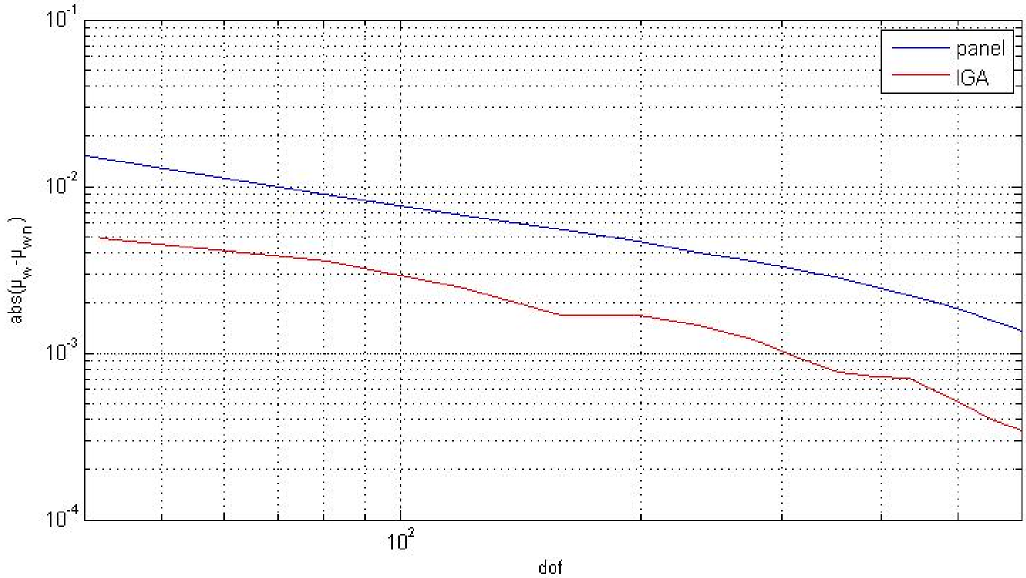

Figure 4: Error of the circulation μ_w around a NACA 4412 hydrofoil wrt dof

methods. In IGA-BEM refinement is performed by knot insertion (h-refinement). It can be observed that IGA-BEM performs better compared to low-order panel method (the error is smaller for all dof; see also Figures 2 and 3), although the rate of convergenge is not significantly increased, as expected from corresponding results concerning potential flows without circulation (see [20]) and application of the present IGA-BEM method in elasticity problems (see, e.g.,[27] and [28]). A possible source for this behaviour is the strongly singular character of the point vortex at the trailing edge, that could be remedied by using continuous vorticity distributions over the body surface, and this is left to be examined in future work.

5 Conclusions

An Isogeometric BEM for steady lifting flows around hydrofoils is developed and compared to existing classical low-order panel methods. The method is robust and effective since it needs much less degrees of freedom for the same accuracy. Future work will focus on the detailed investigation and enhancement of the rate of convergence, using $h-$ and $p-$ refinements, and application of the present method, in conjunction with a parametric geometric model for the hydrofoil, to optimum shape design problems.

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