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STRESS – STRAIN RELATIONSHIP FOR THE CONFINED CONCRETE

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Abstract. *The objective of this study is to present a numerical analysis of circular and rectangular sections reinforced with longitudinal and transversal bars. In this case, a stress-strain relation of confined concrete is proposed. For the purpose of investigating confinement effect, an effectively confined ratio was introduced. It takes into account the effects according to concrete compressive strength, the ratio and the diameter and the configuration of the transversal bars, the diameter and the ratio of the longitudinal bars and sections geometry. This study provides an empirical stress-strain equation to determine the concrete strength until the peak stress. After the peak stress, two empirical stress-strain equations are proposed for describe the confined concrete behaviour in this case. Comparison with experimental stress-strain curves illustrates the validity of the proposed model.*

1 INTRODUCTION

The flexural behavior of reinforced concrete beams section is a well-known problem. Several classical studies about this subject permit the simulation of the flexural behavior of the reinforced concrete beams section but, in these classical studies the concrete confinement is often neglected.

The spiral reinforcement or the rectilinear ties in reinforced concrete play an important role in enhancing the strength and ductility. Under axial loads, concrete pressure in the lateral direction of the sections acts on the lateral ties and the resistance of the ties may restrain the core of concrete to a degree.

The mechanical behaviour of the confined concrete is characterized by the increase in strength and ductility. The magnitude of the increase is established by various confinement parameters such as the compressive strength of the concrete, the volumetric, the diameter the configuration and the strength of the ties and the ration and the diameter of the longitudinal bars and the section geometry, etc.

There have been many attempts to describe the stress-strain relation of confined concrete. Sheikh and Uzumeri [10] and sheikh and Yeh [11] made analytical and experimental studies on the mechanism of confined. They introduced the concept of the effectively confined concrete area and presented the stress-strain relations of confined concrete. Yong and al [12] proposed an empirical stress-strain relation of confined high-strength concrete. Mander and al [6] proposed a stress-strain relation of confined concrete with according the confinement effects to the various configurations of lateral ties. Kent and Park [5] developed a stress-strain relation of confined concrete from the stress-strain relation of unconfined concrete. Park and al [7] modified the stress-strain relation proposed by Kent and Park [5]. Heo-Soo and al [2] proposed a stress-strain curve of laterally confined concrete with according the confinement effects to various parameters.

The objective of this study is to present an empirical stress strain relation according the confinement effects to various parameters variables (compressive strength of the concrete, the volumetric, the diameter the configuration and the strength of the ties and the ration and the diameter of the longitudinal bars and the section geometry, etc.). This paper present a model made up of three empirical relations for describe the behaviour of confined concrete. The simplicity of the model permit it uses easily on various calculation methods. The comparisons with other stress-strain curves permit to establish the validity of the proposed model.

Notations

f_{c0} : Compressive strength of unconfined concrete.
f'_l : Effective lateral stress of the confinement
f'_{lx} : Effective lateral stress of the confinement on x direction
f'_{ly} : Effective lateral stress of the confinement on y direction
f_l : Lateral confinement stress.
f_{yh} : Yield strength of lateral ties.
ke : Effective confinement coefficient.
ρ_{cc} : Longitudinal reinforcement ratio.
A_{sp} : Transverse reinforcement area
s : Distance between longitudinal steels bounded by perimeter ties
ρ_s : Volumetric ration of circular ties.
ρ_x : Volumetric ration of ties on x direction.
ρ_y : Volumetric ration of ties on y direction.
w'_i : Distance between a successive longitudinal bares.
E_s : The slope of the descending curve and can be defined as:
bc : Distance between extremely vertical ties
dc : Distance between extremely horizontal ties
ds : Circular ties diameter
s' : Internal distance between a successive longitudinal bares
ε_{c0} : Unconfined concrete strain corresponding to the peak stress
ε_{cc0} : Confined concrete strain compounding to the peak stress
ε_{ccu} : Confined concrete ultimate strain

ε_c : Confined concrete strain
ε_{65} : The strain corresponding to the stress equal $0.65f_{cc}$.
E_{bc} : Initial confined concrete Young modulus
σ_{cc} : Confined concrete stress

2 GENERAL SPECIFICATIONS

The compressive strength is developed by Mander and al [6] and it can be defined by:

$$f_{cc} = f_{c0}\left(-1{,}254 + 2{,}254\sqrt{1+\frac{7{,}94 f'_l}{f_{c0}}} - \frac{2 f'_l}{f_{c0}}\right) \qquad (1)$$

The effective lateral stress f'_l of the confinement is defined separately for the circular and rectangular section.

2.1. Circular section

The circular section shown in figure 1 can be idealized in to the same of effectivelly confined concrete, steel spiral and unconfined concrete.

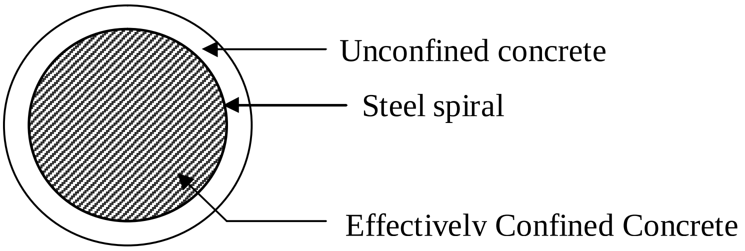

Figure 1: Circular section representation.

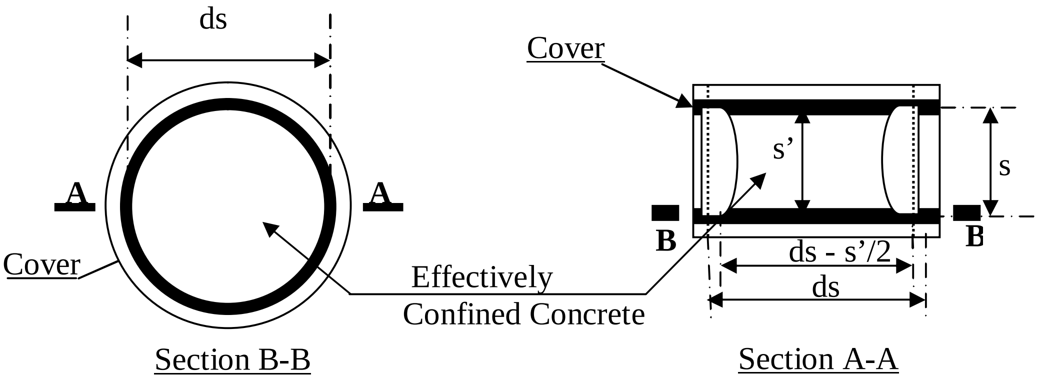

Figure 2: Effectively confined concrete on circular section

The confinement coefficient is defined bay Mander and al [6] as:

$$k_e = \frac{\left(1-\frac{s'}{2d_s}\right)}{1-\rho_{CC}} \qquad (2)$$

The loads equilibrium in the section can be written as:

$$2 f_{yh} \times A_{sp} = f_l \times s \times d_s \qquad (3)$$

And the confinement stress is given by:

$$f'_l = \frac{1}{2} \rho_S \times k_e \times f_{yh} \qquad (4)$$

2.2. Rectangular section

In the case of the rectangular section, as the axial load increase from the initial stages of loading, the concrete is longitudinally contracted and laterally expanded with internal micro cracks. The transversal reinforcement resist the high expanded pressure, and the effective confinement of the by lateral ties leads to the enhancement of the axial load-carrying capacity. For this section, effective confined area is shown in figure (3) and defined by Mander and al [6] as:

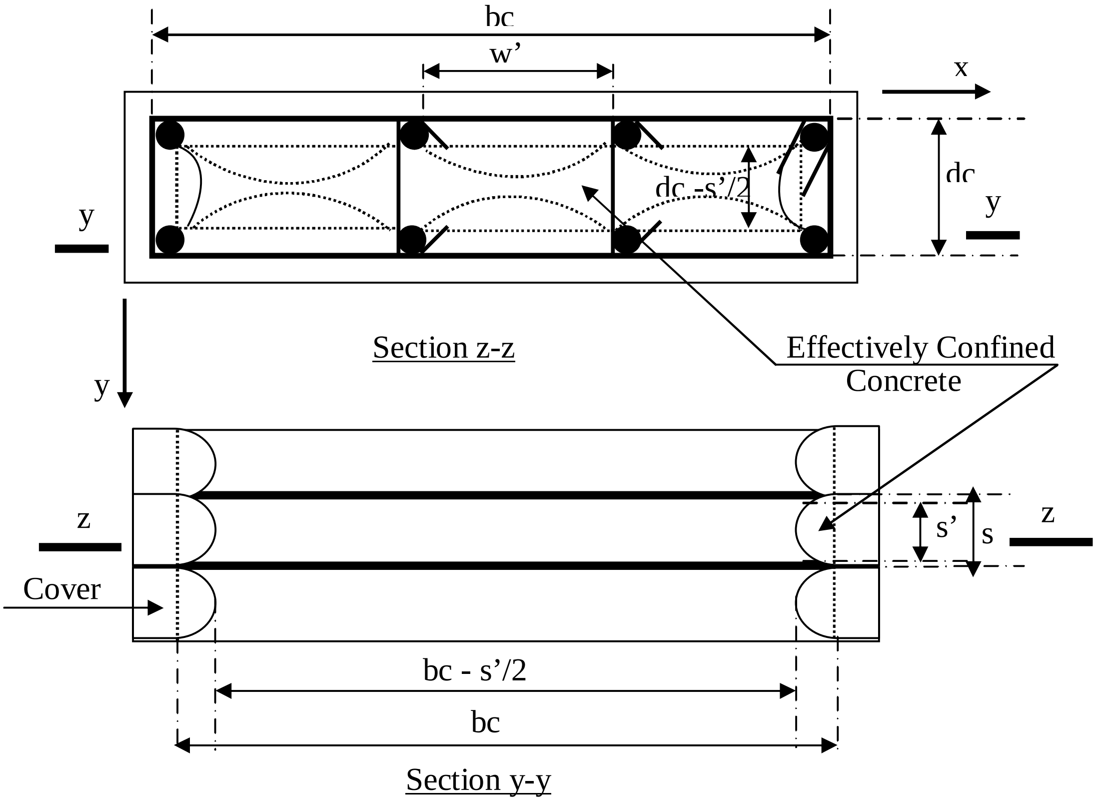

Figure 3: Effectively confined concrete on rectangular section.

For one section with n bares, the unconfined area can be written as:

$$A_i = \sum_{i=1}^{n} \frac{(w'_i)^2}{6} \qquad (5)$$

And the confined concrete area is given by:

$$A_e = \left(bc \times dc - \sum_{i=1}^{n} \frac{(\omega_i)^2}{6} \right) \times \left(1 - \frac{s'}{2bc} \right) \times \left(1 - \frac{s'}{2dc} \right) \qquad (6)$$

The confinement coefficient is defined as:

$$ke = \frac{\left(1 - \sum_{i=1}^{n} \frac{\left(\omega_i^{'}\right)^2}{6 \times bc \times dc} \right) \times \left(1 - \frac{s'}{2bc} \right) \times \left(1 - \frac{s'}{2dc} \right)}{(1 - \rho_{cc})} \qquad (7)$$

And the lateral pressures from x and y direction are defined by Mander and al [6] and given by:

$$f'_{lx} = ke \frac{A_{sx}}{s \times dc} f_{yn} = ke \,.\, \rho_x \,.\, f_{yh} \qquad (8)$$

$$f'_{ly} = ke \frac{A_{sy}}{s \times bc} f_{yn} = ke \,.\, \rho_y \,.\, f_{yh} \qquad (9)$$

Distribution of the confinement stresses is done according to an angle 45°, we define the mean value of lateral confining stress $f_l^{'}$, it can be evaluate considering the two lateral pressure value as:

$$f_l^{'} = \frac{f'_{lx} + f'_{ly}}{2} \qquad (10)$$

3. STRESS – STRAIN CURVE OF CONFINED CONCRETE

The stress-strain curve of confined concrete can be predicted by three coordinates. The coordinate (ε_{cc0} – f_{cc}) corresponding to the peak stress-strain, (ε_{65} – $0.65f_{cc}$) corresponding to the representative point of the stress – strain curve at $0.65f_{cc}$ after the peak and (ε_{ccu} – $0.65f_{cc}$) corresponding to the ultimate strain as shown by the figure (4). In the ascending region between the zero and the first coordinate can be derived bay an equation based on the Sargin equation for unconfined concrete [9] as:

$$\sigma_{cc} = f_{cc} \times \frac{k_c \times \bar{\varepsilon}_c + \left(k_c^{'} - 1\right) \times \bar{\varepsilon}_c^{\,2}}{1 + \left(k_c - 2\right) \times \bar{\varepsilon}_c + k_c^{'} \times \bar{\varepsilon}_c^{\,2}} \qquad \text{Pour: } 0 \leq \varepsilon_c \leq \varepsilon_{cc0} \qquad (11)$$

Where: $\bar{\varepsilon}_c = \frac{\varepsilon_c}{\varepsilon_{cc0}}$ and $\varepsilon_{cc0} = \varepsilon_{c0} \times \left[1 + 5\left(\frac{f_{cc}}{f_{c0}} - 1 \right) \right]$

$$k_c = \frac{E_{bc0} \times \varepsilon_{cc0}}{f_{cc}} \quad , \; E_{bc0} = 11000\sqrt[3]{f_{cc}} \quad \text{and} \quad k_c^{'} = k_c - 1$$

The stress-strain relation of the descending part between the first and the second coordinates can be determined by:

$$\sigma_{cc} = f_{cc} - E_s\left(\varepsilon_c - \varepsilon_{cc0}\right) \text{ for: } \varepsilon_{cc0} < \varepsilon_c \leq \varepsilon_{65} \qquad (12)$$

Where: E_s is the slope of the descending curve and can be defined as:

$$E_s = \frac{6 \times f_{c0}^2}{ke \times \rho_s \times f_{yh}} \qquad (13)$$

And ε_{65} is the strain corresponding to the stress equal $0.65 f_{cc}$. It can be defined by:

$$\varepsilon_{65} = \frac{0.35 \times f_{cc}}{E_s} + \varepsilon_{cc0} \qquad (14)$$

After reaching the stress $0.65 f_{cc}$, the stress of the confined concrete is a constant value $0.65 f_{cc}$ regardless of the increasing strain until the ultimate strain (ε_{ccu}).

The ultimate strain of confined concrete adopted in this study can be defined taking account the transverse reinforcement ultimate strain as:

$$\varepsilon_{ccu} = 0.4x\frac{f_l}{f_{c0}} + \varepsilon_{cu} = 0.0035 + 0.4x\frac{f_l}{f_{c0}} \qquad (15)$$

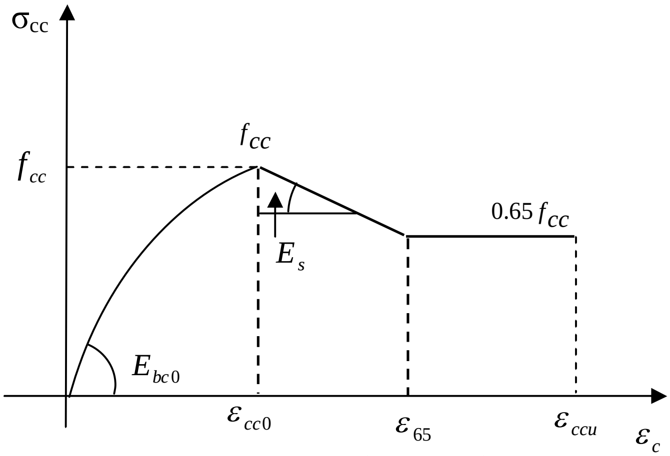

Figure 4: Stress-strain curve for the confined concrete

4. APPLICATION OF THE MODEL

The proposed strain-stress curve is used to simulate the behaviour of some reinforced concrete rectangular and circular sections tested by J .B. Mander and al [6].

4.1. Circular sections

The geometrical characteristics of the circular sections and the materials characteristics are shown in the table 1.

Table 1: Geometrical and materials characteristics of the circular sections.

N° test	F_{c0} (MPa)	Ds (Cm)	S (Cm)	A_{sl} (Cm^2)	A_{sp} (Cm^2)	ρ_S (%)
1	29	43,80	04,10	24,12	1,13	2,50
2	29	43 ,80	06,90	24,12	1,13	1,50
3	29	43,80	10,30	24,12	1,13	1,00
4	29	44,00	11,90	24,12	0,79	0,60
5	29	44,00	03,60	24,12	0,79	2,00
6	29	43,40	09,30	24,12	2,01	2,00
7	32	43,80	05,20	49,24	1,13	1,98
8	30	43,80	05,20	49,74	1,13	1,98
9	32	43,80	05,20	50,24	1,13	1,98
10	30	43,80	05,20	48,23	1,13	1,98

The figure 5 to the figure 14 show the comparison of the theoretical and the experimental stress-strain curves of the circular reinforced concrete sections tested by Mander and al [6].

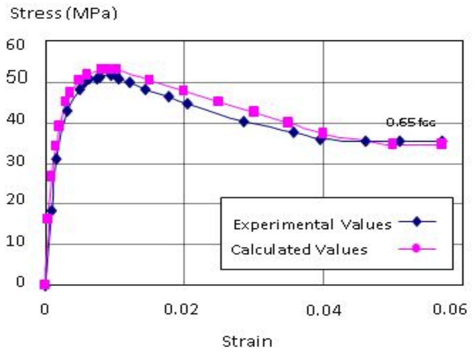

Figure 5: Comparison of experimental and theoretical Stress-Strain curve (test N° 1)

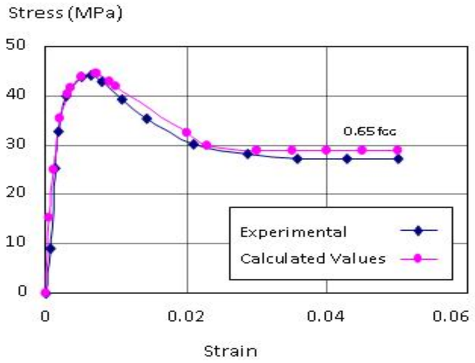

Figure 6: Comparison of experimental and theoretical Stress-Strain curve (test N° 2)

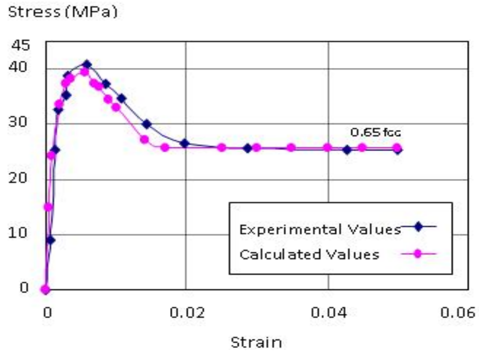

Figure 7: Comparison of experimental and theoretical Stress-Strain curve (test N° 3)

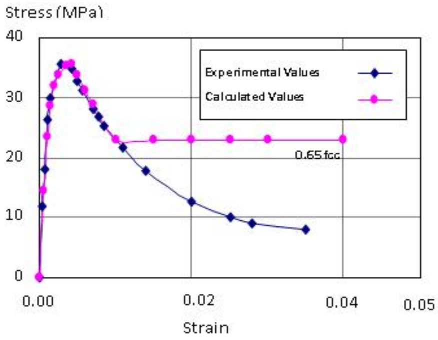

Figure 8: Comparison of experimental and theoretical Stress-Strain curve (test N° 4)

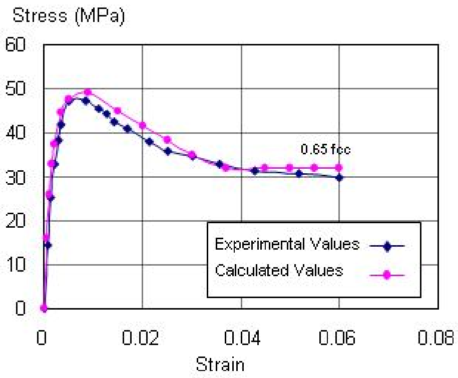

Figure 9: Comparison of experimental and theoretical Stress-Strain curve (test N° 5)

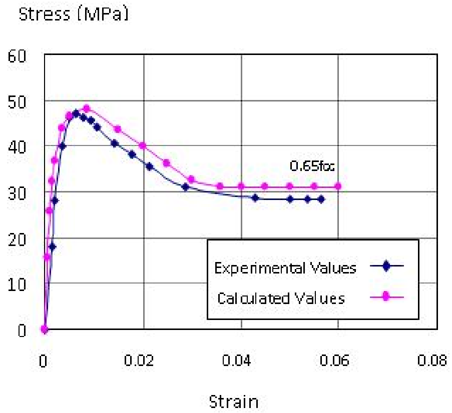

Figure 10: Comparison of experimental and theoretical Stress-Strain curve (test N° 6)

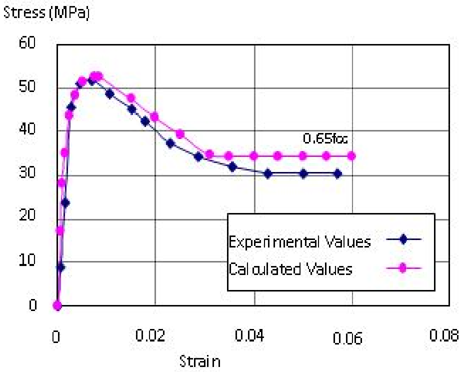

Figure 11: Comparison of experimental and theoretical Stress-Strain curve (test N° 7)

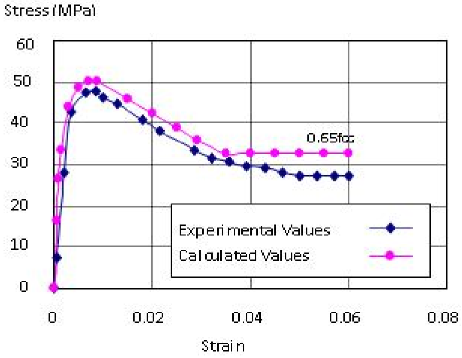

Figure 12: Comparison of experimental and theoretical Stress-Strain curve (test N° 8)

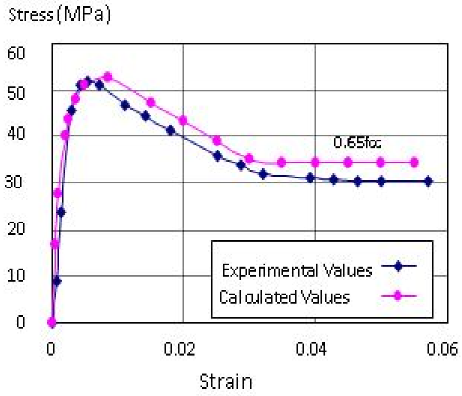

Figure 13: Comparison of experimental and theoretical Stress-Strain curve (test N° 9)

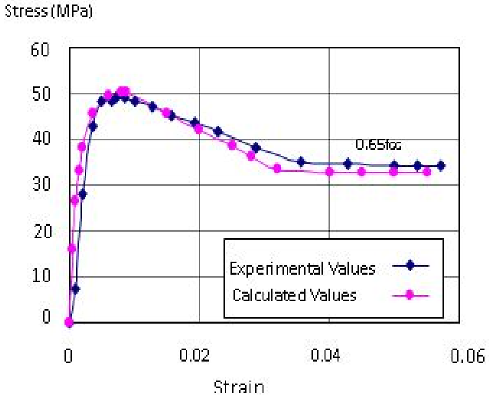

Figure 14: Comparison of experimental and theoretical Stress-Strain curve (test N° 10)

4.2. Rectangular sections

The geometrical characteristics of the circular sections and the materials characteristics are shown in the table 2.

Table 2: Geometrical and materials characteristics of the rectangular sections

N° test	F_{c0} (MPa)	A_{sl} (Cm^2)	A_{sp} (Cm^2)	Dc (Cm)	Bc (Cm)	S (Cm)	S' (Cm)	ρ_{cc}	ρ_{sx}	ρ_{sy}	ρ_s (%)
1	26,00	18,09	0,28	9,40	64,40	02,50	01,90	0,0299	0,0238	0,0139	03,77
2	29,00	18,09	0,28	9,40	64,40	02,50	01,90	0,0299	0,0238	0,0139	03,77
3	26,00	18,09	0,28	9,40	64,40	05,00	04,40	0,0299	0,0119	0,0122	02,41
4	26,00	18,09	0,28	9,40	64,40	02,50	01,90	0,0299	0,0238	0,0139	03,77
10	43,00	32,15	0,28	9,00	64,00	02,50	01,90	0,0558	0,0249	0,0140	03,89
11	43,00	32,15	0,28	9,40	64,40	05,00	04,40	0,0531	0,0119	0,0122	02,41
12	43,00	32,15	0,79	9,00	64,00	04,20	03,20	0,0558	0,0418	0,0235	06,53
13	43,00	31,30	0,79	9,00	64,00	03,00	02,00	0,0196	0,0585	0,0206	07,91

The figure 15 to the figure 22 show the comparison of the theoretical and the experimental stress-strain curves of the rectangular sections tested by Mander and al [6].

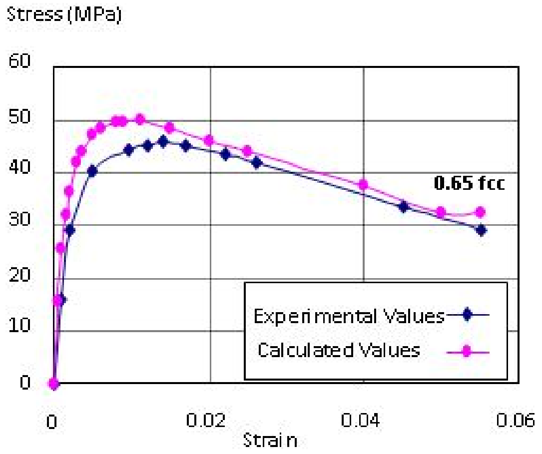

Figure 15: Comparison of experimental and theoretical Stress-Strain curve (test N° 1)

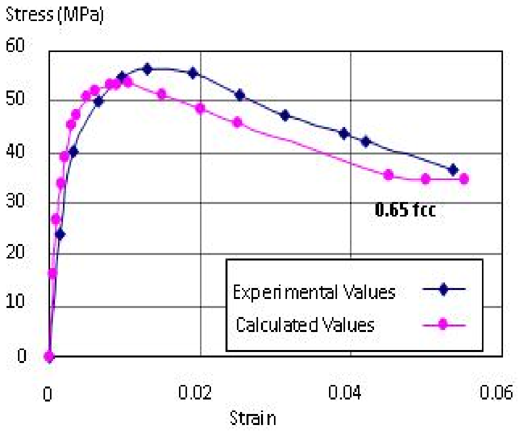

Figure 16: Comparison of experimental and theoretical Stress-Strain curve (test N° 2)

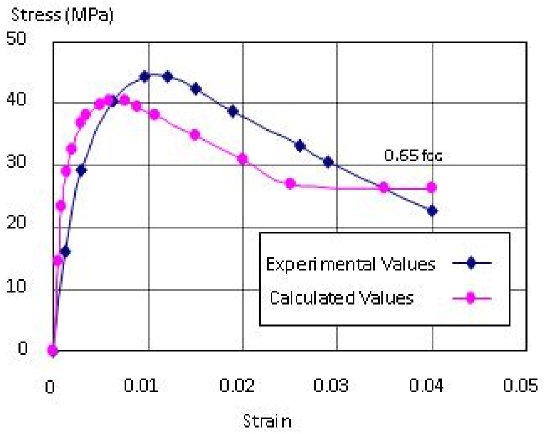

Figure 17: Comparison of experimental and theoretical Stress-Strain curve (test N° 3)

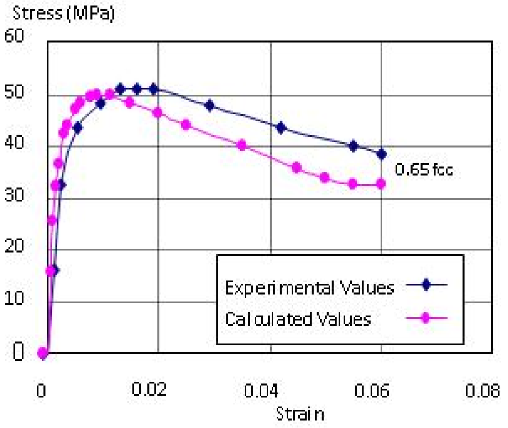

Figure 18: Comparison of experimental and theoretical Stress-Strain curve (test N° 4)

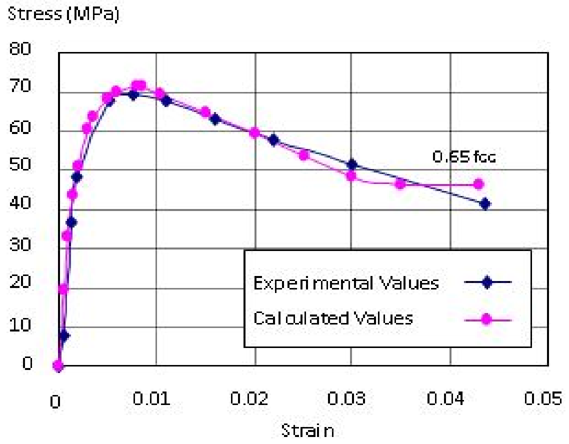

Figure 19: Comparison of experimental and theoretical Stress-Strain curve (test N° 10)

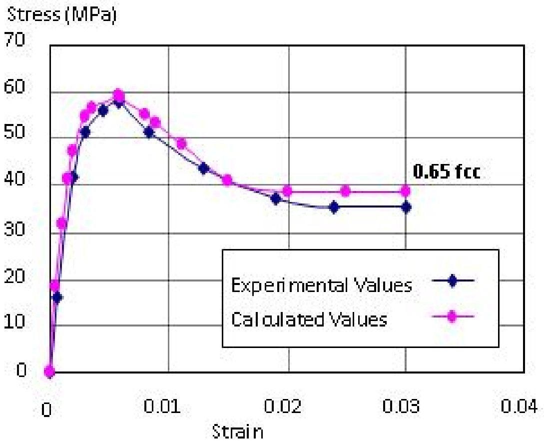

Figure 20: Comparison of experimental and theoretical Stress-Strain curve (test N° 11)

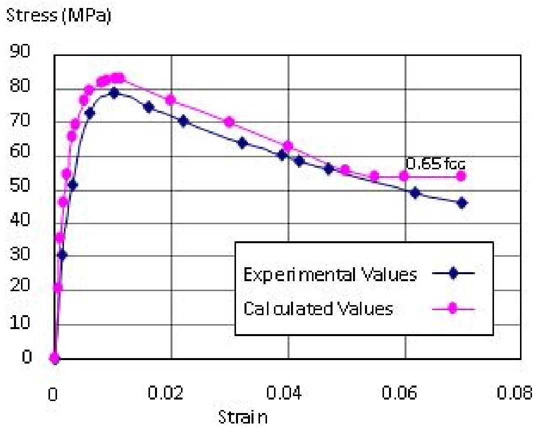

Figure 21: Comparison of experimental and theoretical Stress-Strain curve (test N° 12)

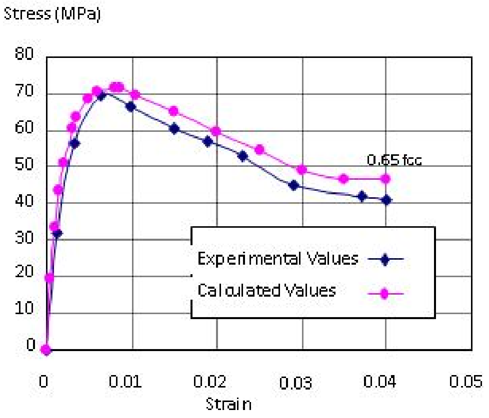

Figure 22: Comparison of experimental and theoretical Stress-Strain curve (test N° 13)

We can observe the theoretical stress-strain curve proposed for the confined concrete gives more reasonable values than the experience. The ascending branch and the descending of the behaviour of the confined concrete can be described, in the case of the circular sections and equally in the case of the rectangular sections, by the model with a good agreement.

CONCLUSION

In this study, a simple approach to define the confined concrete behaviour is proposed. The stress- strain curve proposed is characterised by three coordinates. In the ascending branch this relation is based on the Sargin model for unconfined concrete. The Compressive strength is this material is based on the relation proposed bay Mander and al.

The comparison of the obtained stress-strain curve and the experimental stress-train curve for circular sections and rectangular sections show that the proposed model show that the proposed model gives a reasonable values in the ascending branch and in the descending branch of the stress-strain curve of the confined concrete.

In the case of circular sections, the model gives good results for transverse steel percentages ranging from 1 to 2.5%. There is a difference between the calculated curve and the experimental curve in the post peak for a small percentage (0.6%) steel cross. By cons for rectangular sections, the results are quite satisfactory confrontation and to percentages ranging from 2.41 to 7.91%.

Looking ahead, we could estimate the contribution ductility structures railing posts - beams subjected to horizontal forces due to earthquake taking into account the real and effective confining of nodes and therefore better estimate the vulnerability of such structures. The model could also be extended to the case of circular sections of reinforced concrete confined by a metal tube.

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