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MODELING OF DENSE EXPANSIVE SOILS SUBJECTED TO WETTING AND DRYING CYCLES BASED ON SHAKEDOWN THEORY

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Abstract. Expansive soils generally contain smectite clayey particles that join the other soil components to form aggregates. The microstructure, corresponding to the aggregates and the macrostructure, corresponding to arrangement of the aggregates were observed in previous studies. The volume change of these materials exposed to several wetting and drying cycles, show a reversible equilibrium state at the end of several cycles. For the dense materials, the hysteresis phenomenon is negligible, because macropores are absent in this material. Additionally, the existing models developed for expansive soils are based on step-by-step calculation method, leading to unrealistic calculation time, especially for large number of cycles. In this context, an analytical model based on shakedown theory for dense expansive soils with a combined hardening is developed. The required parameters of the proposed model are calibrated by test results obtained for dense expansive soils in oedometric tests. The comparisons of the test results with the model estimations show that the proposed model is able to simulate the hydromechanical behaviour of dense expansive soils subjected to suction cycles.

1 INTRODUCTION

Unsaturated expansive soil is susceptible to be affected by environmental conditions, because an increase in water content causes the soil to swell, while water removal causes the shrinkage of soil. Many engineered structures built on this material are significantly damaged and cracked, such as shallow foundations, drainage channels and buffers of radioactive waste disposal. Determination of possible swell and shrinkage of expansive soils due to suction changes are generally considered as a necessary part of the construction design. Furthermore, the hydromechanical response of expansive soils affected by wetting and drying cycles is the essential information to understand its constitutive behaviour.

A review of literature suggested that many studies have been reported on cyclic swell-shrinkage behaviour of compacted expansive soil [1, 2, 3, 4, 5, 6, 7]. In their works, swelling and shrinkage deformations of expansive soils with an increasing number of wetting and drying cycles have been shown to reach an equilibrium state where the soil nearly behaves in a reversible manner in terms of deformations. Besides, Nowamooz and Masrouri [8] observed that the hydraulic hysteresis is disappeared for dense expansive soils due to the absence of marcopores in these materials, where the final equilibrium state becomes linearly elastic.

Barcelona Expansive Model (BExM) developed by Alonso et al. [9] is the most accepted framework to simulate the hydromechanical behaviour of expansive soils. However, it requires numerous model parameters to be determined. In addition, it will lead to unrealistic calculation time for numerical modeling with large number of suction cycles. Shakedown theory developed by Zarka [10] can replace the step-by-step method to estimate the stabilized limit state, but this simplified method aims at modeling kinematic hardening materials such as metallic structures. Previous modeling of the fatigue behaviour of unbound granular materials have been carried out based on Zarka method [11, 12].

In this context, it is necessary to complete the shakedown modeling with a combined hardening mechanism and this proposed model is validated by the experimental results of dense expansive soils to demonstrate its capacity.

2 SHAKEDOWN METHOD WITH A COMBINED HARDENING PLASTICITY

The purpose of this section is to present a shakedown-based model with isotropic and kinematic hardening to improve means of representing hardening behaviour in materials under cyclic loadings.

2.1 General mechanical problem

In the case of 1-dimensional elastoplastic problem, the elastic behaviour of the material can be supposed linear, independent of time and temperature. Thus, the mechanical problem can be solved as follows:

$$\epsilon(t) = \frac{1}{E} \cdot \sigma(t) + \epsilon^p(t) + \epsilon^I(t) \tag{1}$$

where $\sigma(t)$ is the stress, $\epsilon(t)$ the strain, $\epsilon^p(t)$ the plastic strain, E the elastic modulus, and $\epsilon^I(t)$ the initial strain (taken equal to 0 in this study).

The yield condition with the combined hardening can be defined by the form:

$$f = |\sigma - y_\alpha| - [\sigma_\alpha + K \cdot \alpha] \tag{2}$$

where, K is plastic modulus and α is plastic strain controlling yield surface variation.

Additionally, the evolution law of back stress (y_α) is related to the strain ϵ^p in a linear relation:

$$y_\alpha = h \cdot \epsilon^p \tag{3}$$

where, h is kinematic hardening.

This combined hardening is used when the yield surface not only expands (or contracts) but also translates in the stress space upon plastic loading and these two purely linear hardenings are widely accepted and applied to model strain-hardening materials.

2.2 Modified shakedown method

In this section, the isotropic hardening plasticity is added to modify classical Zarka shakedown method [10]. The convex is able to translate and expand at the same time in the transformed internal parameter (y_α) plane and the modified shakedown method is showed in Figure 1.

Figure 1 shows that the convex moves with the applied loading and linear elastic behaviour, elastic shakedown and plastic shakedown are defined in transformed internal parameter plane according to the amplitude of applied loading (see figure 1 a-e):

- *Linear elastic behaviour*

The material firstly presents linear elastic behaviour when the applied stress σ inside the convex (see figure 1a). In this case, there is no yield surface expansion because of $f < 0$ and the purely elastic strain is given by:

$$\Delta\epsilon^e = \frac{\Delta\sigma}{E} \tag{4}$$

where, $\Delta\sigma$ is stress increment.

- *Elastic shakedown theory*

Continuing loading, applied stress σ is beyond the yield stress σ_α where plastic strain is generated and the yield surface starts to expand presented Figure 1b. When plastic mechanism is activated ($f = 0$), the initial yield equation (Eq.2) can be written as the following form:

$$y_\alpha = \sigma - [\sigma_\alpha + K \cdot \alpha] \tag{5}$$

This equation represents that the convex, centered in applied stress σ with a convex radius of $(\sigma_\alpha + K \cdot \alpha)$, translates in the transformed internal parameter plane (y_α) plane.

Here, we define that elastic shakedown occurs when convex translation is going through two phases presented in figure 1c and 1d:

Case 1. If the two extreme positions of the convex, taking into account a combined hardening plasticity, have an intersection showed in figure 1c, elastic shakedown behaviour is obtained after several cycles. For this, elastic shakedown depends on whether there is an intersection between two extreme positions of the convex, the same as the classical Zarka method.

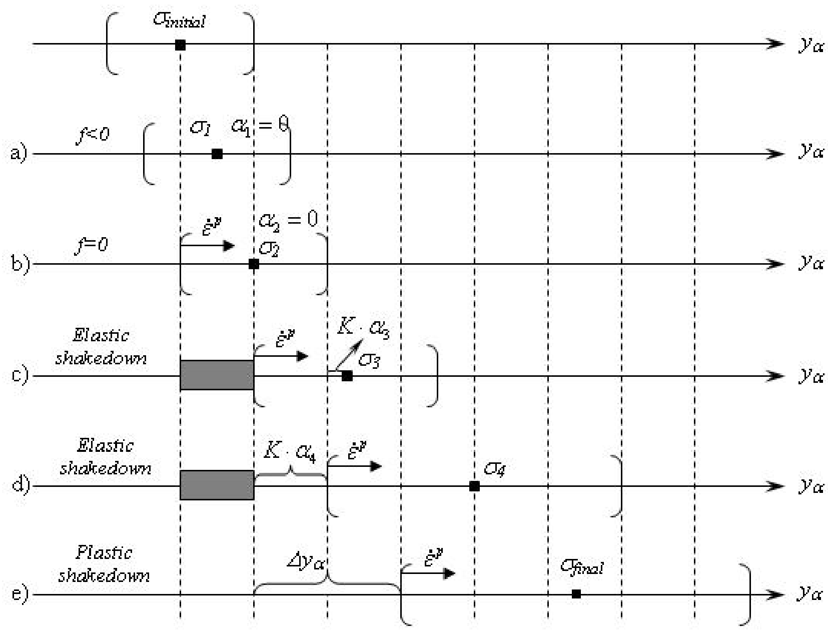

Figure 1: Transformed internal parameter plane with a combined hardening mechanism for: a) linear elastic behaviour; b) plastic mechanism; c and d) elastic shakedown; e) plastic shakedown

Case 2. If there is no a common part between the extreme positions of the yield surface, elastic shakedown behaviour can also be obtained when the applied stress is small, satisfied by the following equation:

$$\sigma \leq 2 \cdot (\sigma_\alpha + K \cdot \Delta\alpha) \tag{6}$$

In this case, the elastic region expands, because of isotropic hardening plasticity, to guarantee the response of materials becomes purely elastic after cyclic loadings, but the prerequisite is equation 6.

Therefore, elastic shakedown occurs if equation 6 is met and the plastic strain during cyclic loading is defined:

$$\Delta\epsilon^p = \frac{\Delta\sigma - \Delta\sigma_\alpha}{h} \tag{7}$$

where, $\Delta\sigma_\alpha$ is the yield surface variation.

- *Plastic shakedown theory*

When the applied loading becomes large enough $(\sigma > 2 \cdot (\sigma_\alpha + K \cdot \Delta\alpha))$, Zarka [10] proves that a stationary limit state is reached. For this, we define that plastic shakedown occurs presented in Figure 1e and the plastic strain during cyclic loadings can be written as:

$$\Delta\epsilon^p = \frac{\Delta y_\alpha}{h} \tag{8}$$

and Δy_α is defined in Figure 1e.

3 SHAKEDOWN MODELING FOR HYDROMECHANICAL LOADINGS

In this part, a simplified shakedown-based model is presented to simulate the hydro-mechanical behaviour of unsaturated expansive soils during the successive suction cycles and the proposed shakedown method in section 2 is used to model dense expansive soils subjected to wetting and drying cycles.

3.1 A simplified constitutive model

It is generally accepted that a unique set of the net mean stress and the total suction are efficient to describe the hydromechanical behaviour of unsaturated soils. The equations of yield surfaces are given by Suction Increase (s_I) limit, Suction Decrease (s_D) limit, and Preconsolidation Stress(p_0),

$$s = s_I \tag{9}$$

$$s = s_D \tag{10}$$

$$p = p_0 \tag{11}$$

where, s_I, s_D and p_0 define the elastic region (see Figure 2).

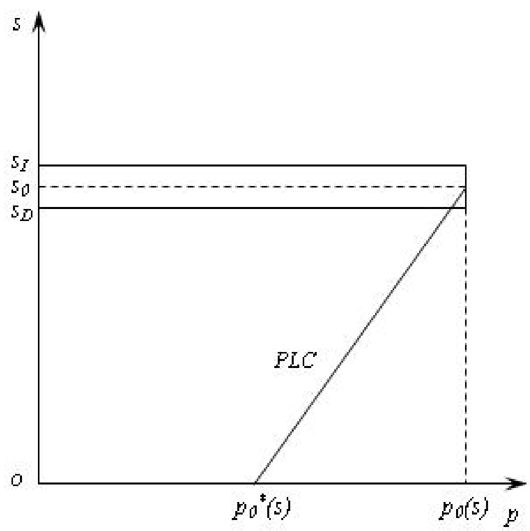

Figure 2: Yield surfaces for simplified model

In this work, we assume that the width of rectangular yield surface depends on suction cycles. In other words, the initial width of the elastic domain is very small and it expands with the wetting-drying suction cycles.

1) Elastoplastic formulation for suction variations

The suction variation within the yield surface will result in the elastic volumetric strain:

$$d\varepsilon_{vs}^{e} = \frac{\kappa_s}{v} \cdot \frac{ds_0}{s_0} \tag{12}$$

and if the boundaries ($s = s_I$ and $s = s_D$) are activated, the following total and plastic volumetric deformation will be generated,

$$d\varepsilon_{vs} = \frac{\lambda_s}{v} \cdot \frac{ds_0}{s_0} \tag{13}$$

$$d\varepsilon_{vs}^{p} = \frac{\lambda_s - \kappa_s}{v} \cdot \frac{ds_0}{s_0} \tag{14}$$

in which, κ_s and λ_s are elastic stiffness index and elastoplastic stiffness index for suction variation, respectively.

2) Elasoplastic strains for mechanical loadings

We define Pseudo Loading Collapse (PLC) with a linear relation between the preconsolidation stress and suction level. This relation can be given by:

$$p_0(s) = A \cdot s + B \tag{15}$$

where, A and B can be identified by the isotropic compression tests at different suctions.

Parameter A is a constant during suction cycles, controlling the slope of PLC yield surface.

Parameter B can be given by,

$$B = p_0^*(\gamma_d) \tag{16}$$

where, $p_0^*(\gamma_d)$ is the preconsolidation stress for saturated state, changing with the initial dry density(initial state).

Similarly, the increase of stress within the yield surface will generate the elastic volumetric strain:

$$d\varepsilon_{vp}^{e} = \frac{\kappa}{v} \cdot \frac{dp}{p} \tag{17}$$

and if the boundary $p = p_0(s)$ is reached, the following total and plastic volumetric deformation will be given by:

$$d\varepsilon_{vp} = \frac{\lambda(s)}{v} \cdot \frac{dp}{p} \tag{18}$$

$$d\varepsilon_{vp}^{p} = \frac{\lambda(s) - \kappa}{v} \cdot \frac{dp}{p} \tag{19}$$

in which, κ and $\lambda(s)$ are elastic stiffness index and elastoplastic stiffness index for loading variation, respectively.

3.2 Elastic shakedown behaviour of dense expansive soils

When highly dense expansive soils subjected to suction cycles, the hysteresis phenomenon is negligible because the macrostructure is absent in these materials [8]. In this way, we define the final equilibrium state is reached where elastic deformation varies linearly at the end of wetting and drying cycles.

In Figure 3, the convex translates along y_α axis from $s_{initial}$ to s_{final} and the material firstly shows purely elastic behaviour because the suction changes very small ($f < 0$) (see figure 3a). Continuing wetting, the plastic mechanism is activated and the yield surface starts to expand due to isotropic hardening mechanism (see figure 3b). When applied suction reaches the final value presented in figure 3c, elastic shakedown behaviour is eventually obtained because the final yield surface has an intersection with the initial yield surface.

In the suction-volumetric strain plane presented in figure 3d, expansive soils demonstrate elastic behaviour when suction variation is small and plastic flow is followed if applied suction outside the convex. After several suction cycles, the plastic strain is accumulated and final equilibrium state is achieved where only elastic behaviour can be observed because the yield surface expands to cover the whole range of suction variations. Moreover, the arrows in figure 3 represent convex movements where the arrow(ab) presents the first cycle that elastic and plastic deformation coexist while the arrow (cd) defines the linear elasticity at the equilibrium state after suction cycles.

The accumulated plastic strain $\Delta\epsilon^p$ in figure 3d is corresponding with the total plastic strain during suction cycles between $s_{initial}$ and s_{final} while plastic strain $\Delta\alpha$ is corresponding with isotropic hardening plasticity during cyclic loadings because point A represents the size variation of the yield surface during suction cycles.

Figure 3e shows the relation of the plastic strains ($\Delta\epsilon^p_{vs}$ and $\Delta\alpha$). This relation considers the simplest evolutionary equation for α to define the isotropic hardening plasticity, namely,

$$\dot{\alpha} = \dot{\epsilon}^p_{vs} \tag{20}$$

Finally, the total plastic strain ($\Delta\epsilon^p_{vs}$) during suction cycles is replaced by the plastic strain ($\Delta\alpha$) because $\Delta\alpha$ is easily determined from elastic shakedown theory if initial and final size of the yield surfaces are known.

Elastic shakedown modeling. For dense expansive soils, the initial size of the yield surface is supposed small and the final size of the yield surface is given due to no plastic strain at the equilibrium state (see figure 3d). Therefore, elastic shakedown is used to model dense expansive soil subjected to suction cycles and the accumulated plastic strain during suction cycles can be defined by the following equation:

$$\Delta\alpha = \frac{\Delta s_\alpha}{K} \tag{21}$$

where, Δs_α is the yield surface variation during suction cycles.

Elastic volumetric strain at the equilibrium state. The linear variation of the elastic volumetric strain with the suction can be found at the equilibrium state where demonstrates the linear elastic behaviour (see figure 3d). It can be written as:

$$\Delta\epsilon^e_{vs} = \frac{\kappa_s}{v} \cdot \frac{\Delta s}{s} \tag{22}$$

where, κ_s is the elastic stiffness index for suction variation.

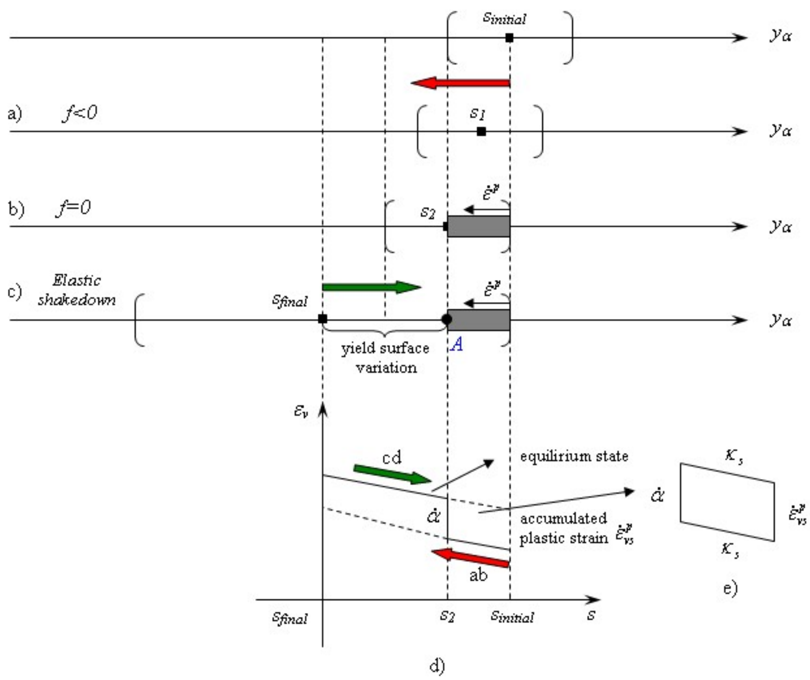

Figure 3: Elastic shakedown behaviour of unsaturated expansive soils during suction cycles

4 PARAMETER CALIBRATION FOR THE PROPOSED MODEL

In the following section, the studied material and oedometric test results on this studied material are presented. Comparing these experimental results, model calibration is carried out to show its modeling capacity.

4.1 Studied material and oedometeric test

The studied expansive soil comes from a depth between 5.20 and 5.70 m of a core sample near Le Deffend, which is located approximately 4 km southeast of Poitiers (France). These clayey soils had a liquid limit of 65%, a plastic index of 25%, a specific gravity of 2.62 and a clay content of 52%. The remolded samples were compacted with an initial water content of 10% to form dense state samples under the compaction pressure of 1500 kPa, and the initial dry densities is 1.75 Mg/m^3.

The influence of the wetting and drying cycles on above materials was studied by Nowamooz and Masrouri [13]. A series of tests with the same stress path were performed on dense samples meanwhile the successive wetting and drying cycles are applied between 8 and 0 MPa at three different vertical stresses (15, 30 and 60 kPa).

4.2 Model calibrations on compacted loose soils

Table 1 summarizes the required parameters of the proposed shakedown-based model for dense samples: plastic modulus (K), elastic stiffness index for suction variation (κ_s) and elastic region limit ($s_I - s_D$). For the modeling, we assume a very small elastic domain where ($s_I - s_D$) are equal to 0.1 MPa and κ_s is constant.

Table 1: Required parameters of shakedown-based model for dense sample studied by Nowamooz and Masrouri (2009) [13]

Parameters	15kPa	30kPa	60kPa
K(MPa)	-49.4	-48.3	-58.4
$s_I - s_D$(MPa)	0.1	0.1	0.1
κ_s	0.024	0.024	0.024

The linear fit of the inverse of plastic modulus ($1/K$) with net mean stress (p) for dense samples are illustrated in Figure 4. Based on this evolution law of plastic modulus, the accumulated plastic strain and total volumetric strain during suction cycles are presented in Figure 5.

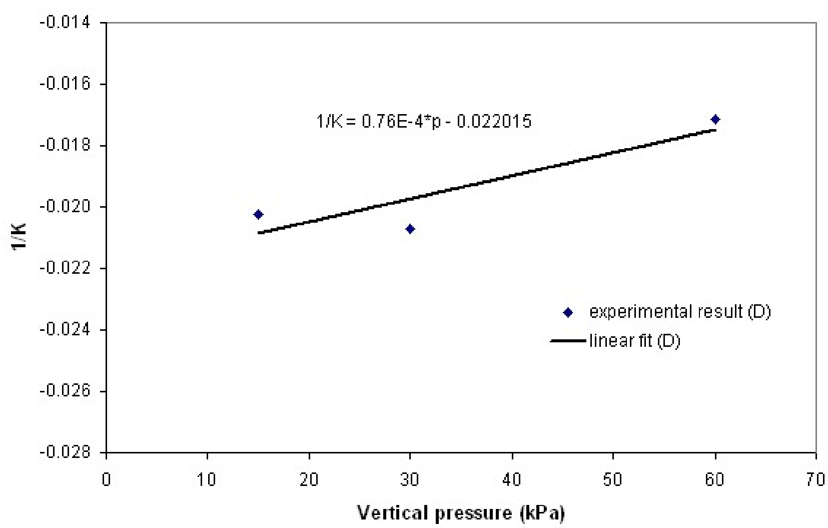

Figure 4: Evolution law of the inverse of plastic modulus ($1/K$) for studied materials

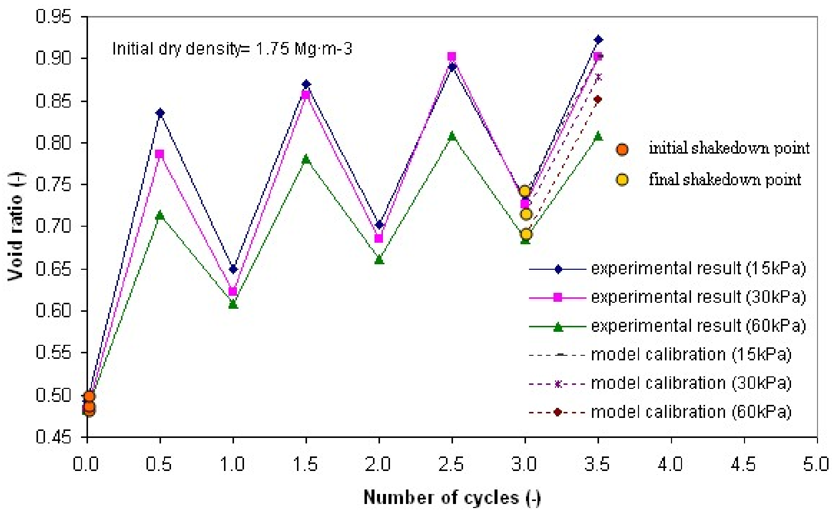

Figure 5: Comparisons of model estimations with experimental results in the plane of suction-void ratio for dense samples at the different vertical pressures[13]

Here, we emphasize that the parameters (K), (κ_s) and $(s_I - s_D)$ are able to describe the hydromechanical behaviour of dense expansive soils through the proposed shakedown-based model with the combined hardening mechanism. Eventually, the model calibration shows the ability of the model and produces a satisfactory simulation.

5 CONCLUSIONS

In this study, a shakedown-based model with two linear hardening mechanisms is developed to simulate the hydromechanical behaviour of dense expansive soils subjected to suction cycles:

- Elastic shakedown is investigated by the transformed internal parameter plane and it occurs for dense expansive soils at the end of suction cycles because of an intersection between two extreme positions of convex. Eventually, elastic shakedown theory explains the reversible elastic behaviour at the equilibrium state.

- Three parameters of the proposed shakedown-based model require to be determined by suction-controlled experiments: plastic modulus (K), elastic stiffness index for change in suction (κ_s) and elastic region limit $(s_I - s_D)$, and these three parameters are sufficient to describe the hydromechanical behaviour of unsaturated expansive soils.

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