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NUMERICAL SIMULATION OF CASING CENTRALIZATION

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Abstract. One of the main requirements to casing that provides cementing of oil and gas wells during drilling and operation is their optimum centralization, which allows achieving a better homogeneity of the slurry flow in the annulus. Optimum standoff between the borehole wall and the casing is ensured with special devices, centralizers put on the casing and spaced along it in a certain pattern. The paper offers a numerical solution to the centralization of casing. The model considers 3D dynamic equations of the lateral and axial motion of a long pipe in the wellbore with constrained deflections in borehole during tripping operation considering all the major factors typical of casing exploration. A numerical method that enables to determine contact and friction forces as well as standoff between the borehole wall and the casing is proposed. Examples of standoff ratio calculations for casings of varying sizes and wells with different inclination and tortuosity are presented.

1 INTRODUCTION

One of the main requirements to casing that provides cementing of oil and gas wells during drilling and operation is their optimum centralization, which allows achieving a better homogeneity of the slurry flow in the annulus (Fig. 1). It appropriate to recall about the recent Macondo disaster [1], when BP responsible for well cementing miscalculated about the number of centralizers. This resulted in inadmissible reduction of standoff between the borehole wall and the casing in some sections preventing homogeneous flow of cement slurry in the annulus required for its filling and integrity. Finally, drilling mud was left in the standoff that could not prevent oil and gas travel up the well.

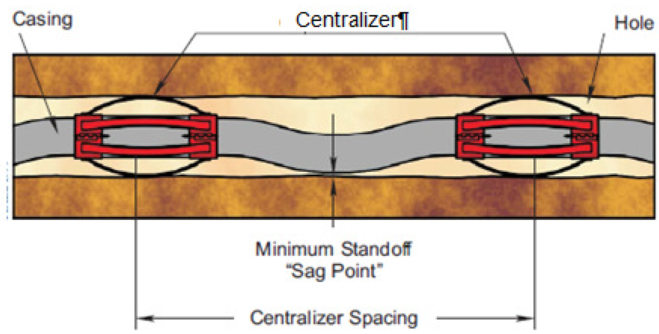


Figure 1: Scheme of casing centralization

There are two types of centralizers: rigid and flexible (bow-spring) ones. Rigid centralizers can be considered as bulges on the drill pipe body or BHA in the form of tooljoints or stabilizers. Contact and friction forces as well standoff between the borehole wall and the casing for such centralizers can be calculated based on the drillstring contact model using the FEM [2] or the finite difference method [3]. Methods of calculation the standoff between the borehole wall and the casing body for bow-spring centralizes are described in specification [4]. Unfortunately, the model used in the above paper does not account for elastic deflection of centralizers, which can result in the standoff underestimation for wells with substantial variation of hole size and high tortuosity. In paper [5] a serious step was made towards improvement a conventional model [4], however, it also requires further refinement of the model of contact forces acting on the casing with the varying diameter as well as improvement of the numerical method for solution of a contact problem

The present paper offers a numerical solution to the centralization of casing. The model considers 3D dynamic equations of the lateral and axial motion of a long string in the well with constrained deflections in wellbore considering all the major factors typical of casing exploration. This model represents a further development of multi-functional DYNTUB model, previously designed for the dynamic simulation of tubular in 3D wellbore during drilling with rotation and without rotation, tripping operation, buckling and whirling of drillstring, etc. [6]. Examples of standoff ratio calculations for casings of varying sizes and wells with different inclination and tortuosity are presented.

2 MATHEMATICAL MODEL OF CASING

The detailed description of the mathematical model of drillstring dynamics in the 3D well was provided by Tikhonov et al [7]. It is a set of equations for lateral, axial and torsional motion of the rod with bending, extension and torsional stiffness subject to its constrained lateral deflection in the borehole. A similar model can be used for a casing with centralizers.

The equations of rod motion in the coordinate system connected with the wellbore axis line can be represented as follows:

$$m\ddot{x} = -Q' + f_x, \tag{1}$$

$$m^* \ddot{\mathbf{r}} = Q\mathbf{r}'' - EI(\mathbf{r}'' + j\boldsymbol{\kappa}_0)'' + j[M(\mathbf{r}'' + j\boldsymbol{\kappa}_0)]' + \mathbf{f_n}, \tag{2}$$

$$\rho I_p \ddot{\psi} = M' + \mu, \tag{3}$$

where $\mathbf{r} = y + jz$; x, y and z are projections of the rod center line on wellbore axes OX, OY and OZ, respectively; ψ = torsion angle; $Q = EA(x' - 0.5|\mathbf{r}'|^2)$ = axial force; $M = GI_p \psi'$ = torque; EI, EA and GI_p are bending, axial and torsional stiffness of the rod; m = rod mass per unit length; ρI_p = moment of inertia relative to the rod center line; $\boldsymbol{\kappa_0} = \kappa_{y0} + j\kappa_{z0}$, κ_{y0} and $j\kappa_{z0}$ are projections of the wellbore curvature vector $\boldsymbol{\kappa_0}$ on axes OY and OZ; $\mathbf{f_n} = f_y + jf_z$; f_x, f_y and f_z are projections of the external force $\mathbf{f_n}$ per unit length on axes OX, OY and OZ; μ = external torque per unit length; $j = (-1)^{1/2}$; the prime means a derivative with respect to string arc length s, and the point is derivative with respect to time t.

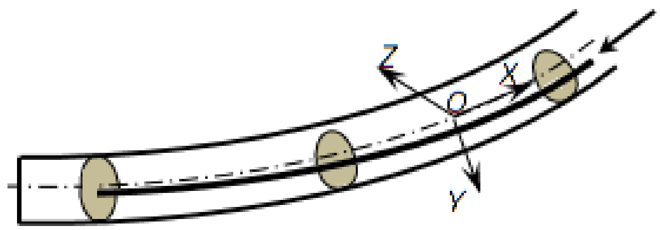


Figure 2: Coordinate system connected with axes of wellbore

The Eq. (1) describes the dynamic equilibrium of axial forces on the rod, the Eq. (2) describes the equilibrium of lateral forces as projections on axes OY and OZ, and the Eq. (3) describes torque equilibrium.

Major components of external force $\mathbf{f_n}$ include weight of rod per unit length w, and vectors of normal contact force $\mathbf{p}$ and friction force $\mathbf{f_t}$ per unit length.

The boundary conditions for the Eqs. (1)-(3) are defined by the specified lateral deflections of the top string end, casing running speed, conditions on its ends (free/hinge/cantilever) as well as loading conditions on the bottom end.

Contact and friction forces will be acting at centralizer installation points, models of such forces will be considered separately.

3 MATHEMATICAL MODEL OF CENTRALIZERS

The diameter of a bow-spring centralizer in unload state D_c should be larger than the nominal borehole diameter D_{h0}. Contact and friction force calculations for this type of centralizers have some peculiarities.

According to the Specification [4], diameter of a bow-spring centralizer is selected as 1/3 of the nominal standoff between the borehole wall and the casing:

$$D_c = D_{h0} + \frac{1}{3}(D_{h0} - D), \tag{4}$$

where D = borehole diameter.

Depending on the ratio of the actual well diameter D_h and the centralizer diameter, four contact types are possible:

1) Contact along the entire perimeter of the borehole when the centralizer diameter is more

than borehole diameter (Fig. 3a);
2) Linear contact when the centralizer diameter is more than borehole diameter (Fig. 3b);
3) No contact with the borehole (Fig. 3c);
4) Point contact when the borehole diameter is more than the centralizer diameter (Fig. 3d);

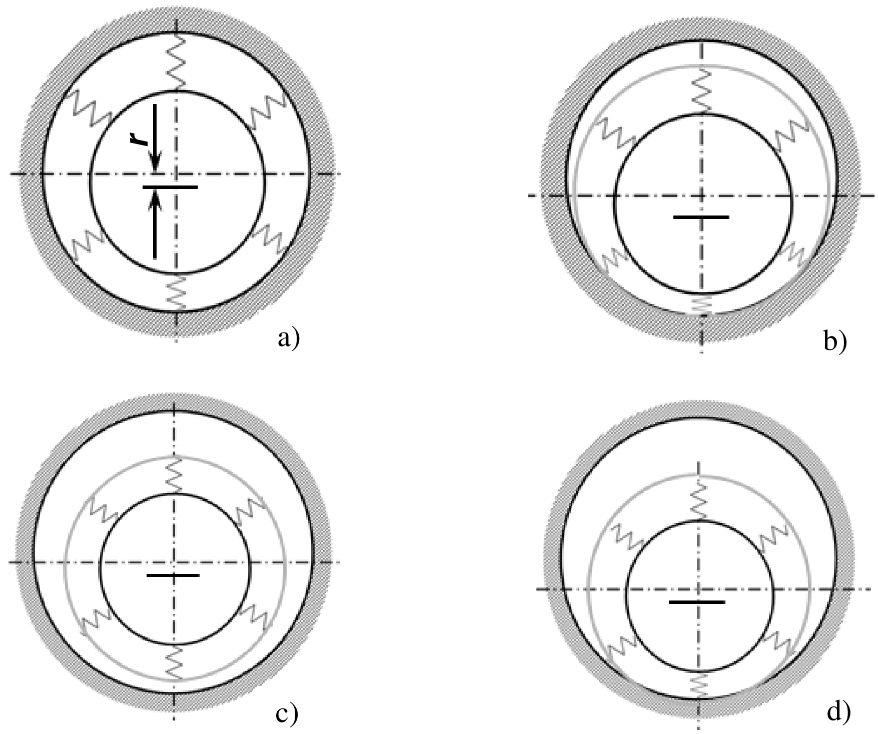


Figure 3: Types of centralizer contact

The centralizer stiffness k_c depends on the degree of its compression in the well and is determined by special tests. Such test includes measuring of the so-called restoring force F_r under compression of the centralizer put on the pipe (casing model) with the diameter D by 1/3 of the minimal standoff in a steel pipe (well model) with the diameter D_{h0} [4]. In this case the centralizer elastic ratio is:

$$k_c = \frac{6F_r}{(D_{h0} - D)}. \quad (5)$$

The contact force is determined based on the following considerations:

- If the diameter of the centralizer D_c exceeds the actual well diameter D_h (contact types 1 and 2, Figs. 3a, 3b), the contact force is proportional to the ratio $(D_c - D_h)/(D_c - D_{h0})$ [5];
- For the same case with small deflection $|\mathbf{r}|$ of the casing axial line, corresponding to contact type 1 (Fig. 3a), the contact force will be proportional to deflection;
- In case of linear or point contact of type 2 and 4 (Figs. 3b, 3d), the contact stiffness is halved as compared to the case with contact type 1 (Fig. 3a). At that the dependence of

contact force on deflection $|\mathbf{r}|$ while transfer from contact type 1 to contact type 2 and from contact type 3 to contact type 4, $|\mathbf{r}| = 0.5|D_c - D_h|$ shall remain continuous.

Taking into account the above conditions, the contact force $|\mathbf{f}_{nc}|$ per unit of casing length, is determined by the formula:

$$|\mathbf{f}_{nc}| = \begin{cases} 0, \ D_c \le D_h \cup |\mathbf{r}| \le \frac{1}{2}(D_h - D_c), \\ \frac{k_c}{2\Delta l}\left[|\mathbf{r}| - \frac{1}{2}(D_h - D_c)\right], \ D_c \le D_h \cup |\mathbf{r}| > \frac{1}{2}(D_h - D_c), \\ \frac{k_c}{\Delta l}\frac{D_c - D_h}{D_c - D_{h0}}|\mathbf{r}|, \ D_c > D_h \cup |\mathbf{r}| \le \frac{1}{2}(D_c - D_h), \\ \frac{k_c}{2\Delta l}\left[|\mathbf{r}| + (D_c - D_h)\left(\frac{D_c - D_h}{D_c - D_{h0}} - \frac{1}{2}\right)\right], \ D_c > D_h \cup |\mathbf{r}| > \frac{1}{2}(D_c - D_h), \end{cases} \tag{6}$$

where Δl = centralizer base.

The friction force in the centralizer while tripping is also determined by testing. For this purpose, the pipe of the diameter D with a centralizer is placed in the steel pipe of the diameter D_{h0} and its starting force or running force F_t is measured. In this case the friction force f_{xc} per unit of casing length can be determined by the formula:

$$f_{xc} = \begin{cases} 0, \ D_c \le D_h \cup |\mathbf{r}| \le \frac{1}{2}(D_h - D_c), \\ -\frac{k_s}{2k_{ss}\Delta l}F_t\frac{1}{D_c - D_{h0}}\left[|\mathbf{r}| - \frac{1}{2}(D_h - D_c)\right], \ D_c \le D_h \cup |\mathbf{r}| > \frac{1}{2}(D_h - D_c), \\ -\frac{k_s}{k_{ss}\Delta l}\frac{D_c - D_h}{D_c - D_{h0}}F_t, \ D_c > D_h \cup |\mathbf{r}| \le \frac{1}{2}(D_c - D_h), \\ -\frac{k_s}{2k_{ss}\Delta l}\frac{1}{D_c - D_{h0}}F_t\left[|\mathbf{r}| + \frac{3}{2}(D_c - D_h)\right], \ D_c > D_h \cup |\mathbf{r}| > \frac{1}{2}(D_c - D_h), \end{cases} \tag{7}$$

where k_s = steel-rock friction factor; k_{ss} = steel-steel friction factor [5].

When a rotating casing is run in hole, centralizers on the pipe body should be free to rotate about its axis. In this case the friction torque acting on the pipe is determined only by friction in the “centralizer – pipe” pair, i.e. steel-steel friction:

$$\mu_c = -0.5Dk_{ss}p. \tag{8}$$

A running of casing with rotation at fixed centralizers is never carried out.

The forces f_{xc}, f_{yc}, f_{zc} and the torque μ_c are components of the forces f_x, f_y, f_z and the torque μ in the set of Eqs. (1)-(3) acting at centralizer installation points.

4 METHOD OF SOLUTION

Considering possible casing contact with the borehole wall between the neighboring centralizers, the borehole wall was artificially stiffened, which was expressed as a quadratic-elastic dependence of the contact force on the casing penetration depth [7]. At that the value of such elastic component should be high “enough”, at least considerably exceed the centralizer stiffness k_c.

The solution for the set of Eqs. (1)-(3) in combination with boundary conditions and the centralizer contact Eqs. (6)-(8) is determined numerically by the method of lines detailed in paper [6].

First, the derivatives with respect to variable s are approximated by finite differences of second order. At that a special scheme offered by Bakhvalov [8] is used to approximate the derivatives with respect to length s included in boundary conditions. The obtained set of ordinary second order differential equations can be formally converted into the set of equations of the first order by introduction of velocities: $v_x = \dot{x}, v_y = \dot{y}, v_z = \dot{z}, \omega = \dot{\psi}$, with subsequent integration by one of the standard methods, e.g. by the Runge-Kutta method.

Which variant of the method should be used to integrate obtained set of equations?

The implicit schemes are of little use for integration, since they suggest that each time step would necessitate solving of a set of substantially nonlinear algebraic equations by iteration. Through computational experiment it has been found that there is no guarantee that the iterative solution will be convergent.

The main problem for the explicit schemes is selection of the minimum step of time integration Δt, that will ensure numerical convergence of the solution. According to the analysis results, a sufficient condition of the studied system numerical stability was obtained:

$$\Delta t < \min\left[\Delta s^2\sqrt{\frac{m}{EA}},\ \Delta s^2\sqrt{\frac{m^*}{EI}},\ \sqrt{\frac{m^*}{\lambda}},\ \sqrt{\frac{\Delta_q m^*}{\lambda\delta}}\right], \tag{9}$$

where Δs = minimum step of mesh on the rod length; λ, δ and Δ_q are elastic parameters of the borehole wall [6].

The first restriction set by condition (9) and determined by the propagation velocity of longitudinal waves in the casing can be omitted as tripping operations are slow enough, and the axial dynamics can be ignored.

Static stress-strain state of casing with centralizers is determined by means of establishing a dynamic process, the initial condition for which is determined in a rather arbitrary way, e.g. on the assumption that the casing lies on the low side of the wellbore.

The authors used the explicit Runge-Kutta method modified by Gill to integrate the developed set of equations.

Note that the offered model accounts for rotation of the casing center line in relation to the borehole wall at the point of centralizer contact. All known works assume that these axes are always collinear.

5 EXAMPLES OF CALCULATION

The developed model of centering was adapted in DYNTUB software [6].

To illustrate how the software works, some examples of casing running in a well with varyung hole diameter were considered. The input data for calculations are given in Table 1.

Table 1: Input data

Parameter	Example 1	Example 2
Casing O.D. (mm)	178	168
Weight per unit length in air (N/m)	380	234
Nominal hole diameter (mm)	251	240
Restoring force (N)	4626	4270
Running force (N)	4626	4270
Density of external mud (kg/m^3)	1800	
Density of internal mud (kg/m^3)	1000	
Friction factor steel-rock	0.4	

Standoff ratio was determined by the following formula:

$$SR = 100\frac{D_h - (D + 2r)}{D_h - D}, \%. \tag{10}$$

Figure 4 represents dependence of casing standoff ratio on measured depth for Example 1 while running in a inclined 60-deg well with varying diameter of hole for different centralizer spacing *h*, 12 and 14 meters. It is evident that when the centralizer spacing doubles, the standoff ratio falls by more than two thirds. Note that this value should not be less than 67% for optimal casing centralization [5].

Figure 5 illustrates distribution of the casing standoff ratio in the same well for Examples 1 and 2, and Fig. 6 illustrates distribution of the standoff ratio for Example 1 in a vertical well with the tortuosity of 1.7 deg.

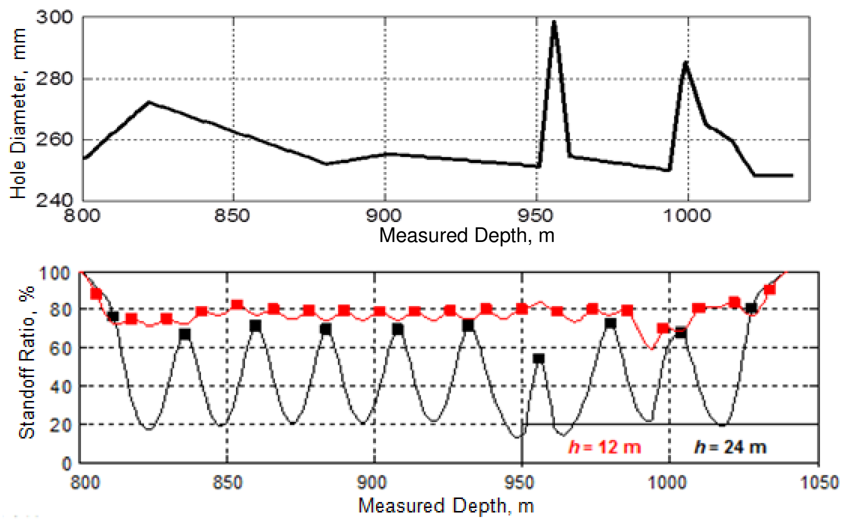
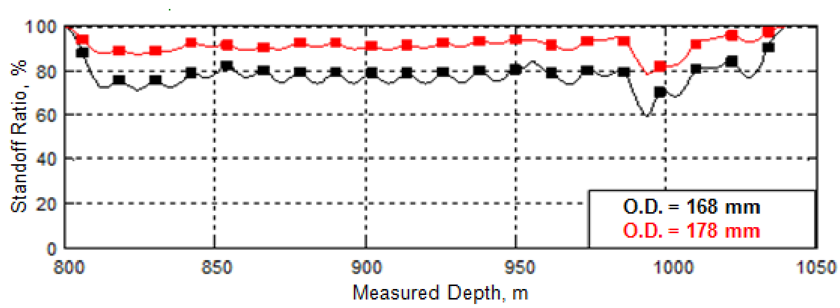


Figure 4: Dependence of hole diameter and SR on MD at various installation step of centralizers



Fugure 5: Dependence of SR on MD at various diameters of casing

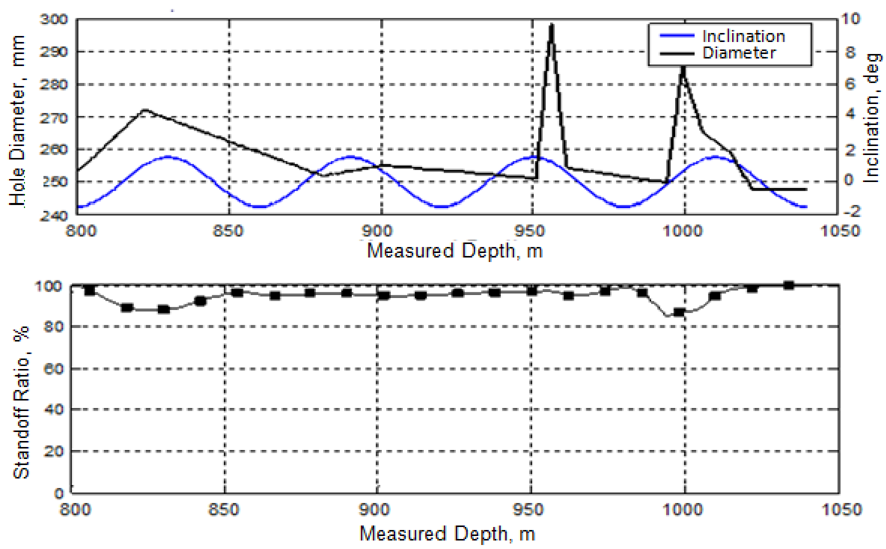


Figure 6: Dependence of hole diameter, well inclination and SR on MD for vertical well, Example 1

6 CONCLUSIONS

- More accurate model of contact and friction forces for bow-spring centralizers is developed.
- A complete model of casing centering on base of the DYNTUB code is developed.
- The module of the DYNTUB code for calculation of standoff ratio for wells with a high degree of well tortuosity and substantial change in hole diameter is developed and debugged.
- Comparative analysis of the results of field tests of casing with centralizers while running in hole or in a static state and the results obtained by software calculations is required.

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