Towards superresolution delay-Doppler maps

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October 22, 2010

GNSS-R 2010 – Barcelona, 21-22 October 2010

Motivations and framework

- Delay-Doppler maps generated in a GNSS–R setup are useful for exploring surface properties. Delay and Doppler (as well as spatial) resolution is limited by the ambiguity function of the GNSS waveform. Improving resolution can help to retrieve more detailed information on the sensed surface.

- We will explore new processing schemes to move further the limits for the achievable delay and Doppler resolution.

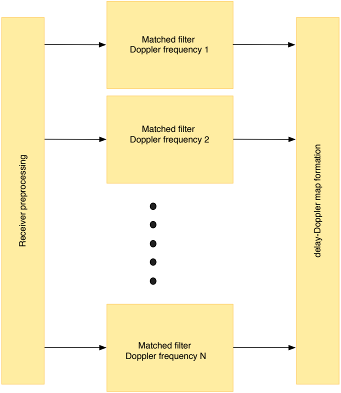
Ambiguity function of GPS signal

- For the GPS signal, the ambiguity function can be approximated (for small values of τ and f) as

$$\chi(\tau, f) = \mathrm{sinc}(fT)\Lambda\left(\frac{\tau}{\tau_c}\right)$$

- The main lobe of the ambiguity function regulates the delay-Doppler resolution of the matched filter: returns that lie less than half main lobe apart are seen as a single return.

Usual GPS-R processing



Concept

- The received waveform is the sum of many overlapping contributions from effective scatterers over the ocean surface. Each contribution has a different delay, Doppler and amplitude.
- Matched Filtering (MF) is able to separate contribution if they are sufficiently spaced in delay and Doppler. The output of the MF is the sum of cuts of the ambiguity function.
- In a subspace approach we find a basis for the ensemble of delayed waveforms. Interestingly, they form a set of orthogonal functions.
- The projection of the waveforms (cuts of the ambiguity function) on the noise subspace generates sharp nulls that can be detected to estimate delay and Doppler parameters.

GNSS-R superresolution

- *Scattered signal*: superposition of contributions from N scattering points, having attenuation a_k, delay τ_k and doppler frequency shift f_{dk}, plus noise,

$$y(t) = \sum_{k=1}^{N} a_k s(t - \tau_k) e^{j2\pi f_{dk} t} + n(t)$$

- *delay-Doppler profile*: the received signal correlated with a frequency–shifted replica of the transmitted signal

$$z(\zeta, \varphi) = \sum_{k=1}^{N} a_k \chi(\zeta - \tau_k, \varphi - f_{dk}) + r_{sn}(\zeta, \varphi)$$

GNSS-R superresolution

- *delay profile*: the delay-Doppler profile evaluated at a fixed frequency φ_0

$$z(\zeta) = \sum_{k=1}^{N} a_k r_c(\zeta - \tau_k) + r_{sn}(\zeta)$$

The delay profile is the superposition of an unknown number of replicas of cuts of the ambiguity function at fixed frequencies plus the mutual correlation of the PN sequence and the noise process.
We want to estimate

- the number of replicas
- the delay and Doppler location of replicas
- the amplitude of replicas.

GNSS-R superresolution

The number of returns

In a signal space approach the number of returns N is equal to the dimension of the signal subspace. To find the principal components of the signal subspace

- the delay profile $z(\zeta)$ is sampled at M equispaced lags, with M greater than the expected number of signals N; we obtain the samples $z(\zeta_i) \quad i = 1 \cdots M$.
- from a vector representation of the delay profile: $\mathbf{z} = \mathbf{\Gamma g} + \boldsymbol{\nu}$, we find the correlation matrix of the delay profile

$$\mathbf{R} = E[\mathbf{z}\mathbf{z}^\dagger] = \mathbf{\Gamma G \Gamma}^\dagger + \sigma_n^2 \mathbf{R}_n$$

- a generalized eigenvalues decomposition of $\mathbf{R}$ is derived and the eigenvalues are sorted in descending order: the first N eigenvalues belongs to the signal subspace, the remaining $M - N$ to the noise subspace.

Delay super resolution

Super resolution delay profile

The super resolution delay profile (SDP) is based on projection of a cut of the ambiguity function on the eigenvectors of the noise subspace. By defining the *steering vectors*

$$\mathbf{r}_c(\tau) = [\chi(\zeta_1 - \tau; \varphi_0), \chi(\zeta_2 - \tau; \varphi_0) \ldots \chi(\zeta_M - \tau; \varphi_0)]$$

it can be demonstrated that $\mathbf{r}_c(\tau)$ lies entirely in the signal subspace when $\zeta = \tau_i$. Thus the SDP

$$\mathsf{SDP}(\tau; \varphi_0) = \frac{\mathbf{r}_c(\tau)\mathbf{R}_n^{-1}\mathbf{r}_c^{\dagger}(\tau)}{\left|\sum_{i=D+1}^{M} \mathbf{r}_c^{\dagger}(\tau)\mathbf{e}_i\right|}$$

diverges at the true delay points.

Delay super resolution - Amplitude estimation

The peaks of the SDP do not correspond to the signal amplitudes that should be estimated. The delay-Doppler profile, in the estimated points is

$$z(\widehat{\tau}_k;\widehat{\varphi}_k) = \sum_{i=1}^{N} a_i \chi(\widehat{\tau}_k - \tau_i;\widehat{\varphi}_k) + r_{sn}(\widehat{\tau}_k;\widehat{\varphi}_k)$$

We define the vectors

$$\mathbf{z} = \left[z\left(\widehat{\tau}_1;\widehat{\varphi}_1\right), z\left(\widehat{\tau}_2;\widehat{\varphi}_2\right), \ldots, z\left(\widehat{\tau}_N;\widehat{\varphi}_N\right)\right]^T$$
$$\mathbf{a} = \left[a_1, a_2, \ldots, a_N\right]^T$$
$$\boldsymbol{\nu} = \left[r_{sn}(\widehat{\tau}_1;\widehat{\varphi}_1) \ldots r_{sn}(\widehat{\tau}_N;\widehat{\varphi}_N)\right]^T$$

Delay super resolution ... Amplitude estimation

and the matrix

$$\mathbf{S} = \begin{bmatrix} \chi(\widehat{\tau}_1 - \tau_1; \widehat{\varphi}_1) & \ldots, & \chi(\widehat{\tau}_N - \tau_i; \widehat{\varphi}_N) \\ \vdots & \ddots & \vdots \\ \chi(\widehat{\tau}_1 - \tau_N; \widehat{\varphi}_1) & \ldots, & \chi(\widehat{\tau}_N - \tau_N; \widehat{\varphi}_N) \end{bmatrix}$$

Thus, we have $\mathbf{z} = \mathbf{aS} + \boldsymbol{\nu}$

Simulations

Simulated received signal (at the moment, not tied to any GNSS–R scattering model):

- the generated signal is the sum of GPS-like PN sequences with chosen delay and Doppler shift. Amplitudes have a Rayleigh distribution and are uncorrelated from sequence to sequence. Additive gaussian noise is considered.
- Delay and Doppler shifts are inside a single chip interval.

Simulations

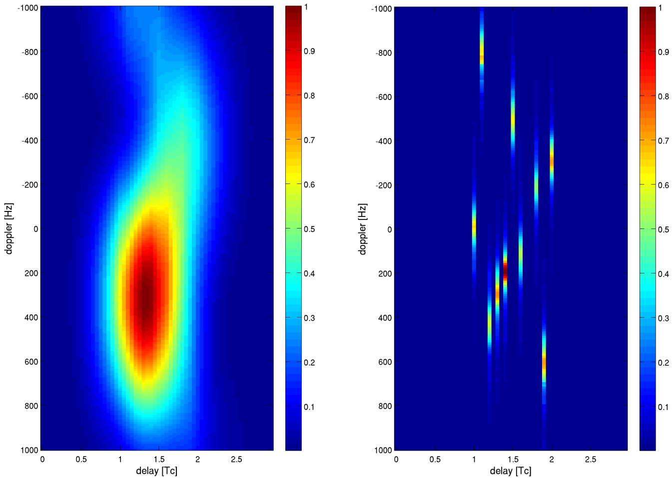
Parameters setup

- Number of delays: 10
- Number of samples per chip: 20
- SNR: 0, 10, 20 dB
- simulated delay-Doppler returns

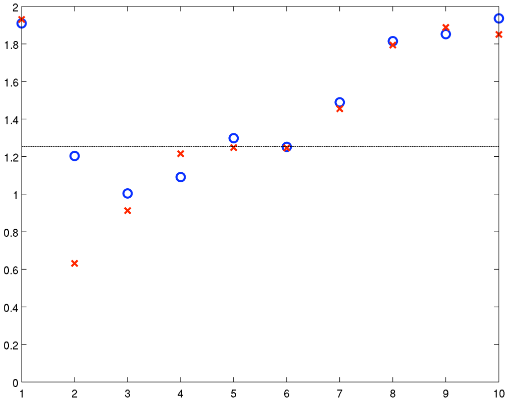
Delay (chip)	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.8	1.9	2.0
Doppler (Hz)	0	-800	400	300	200	-500	100	-200	600	-300

- Number of sequences 200 for superresolution; 1, 200 for amplitude estimation.

Simulations

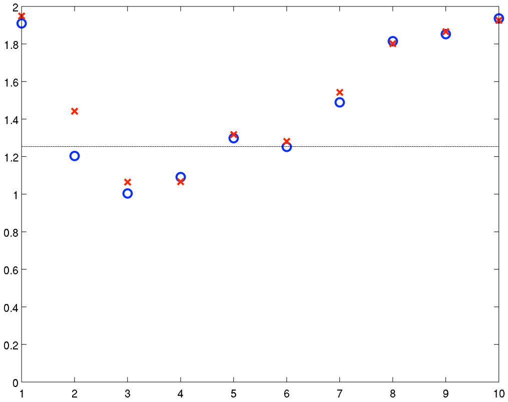


Simulations



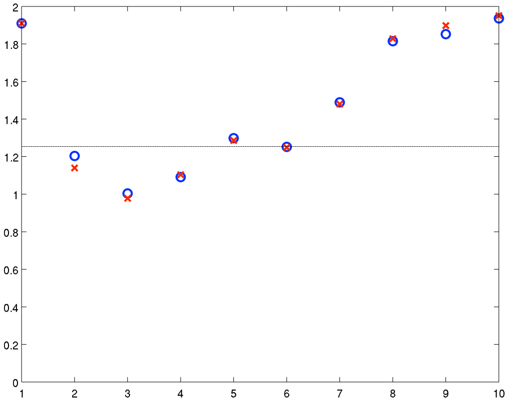
1 Snapshot, SNR = 0dB.

Simulations



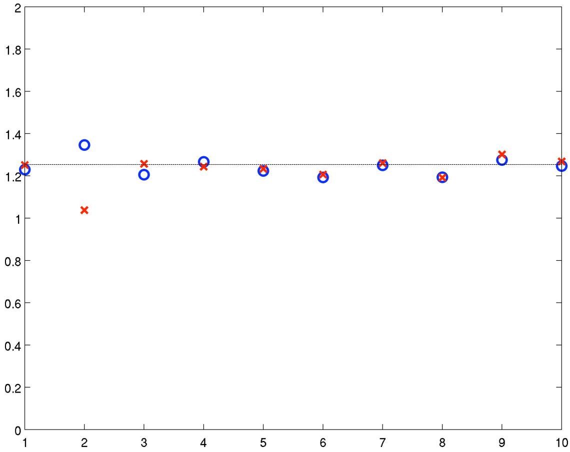
1 Snapshot, SNR = 10dB.

Simulations



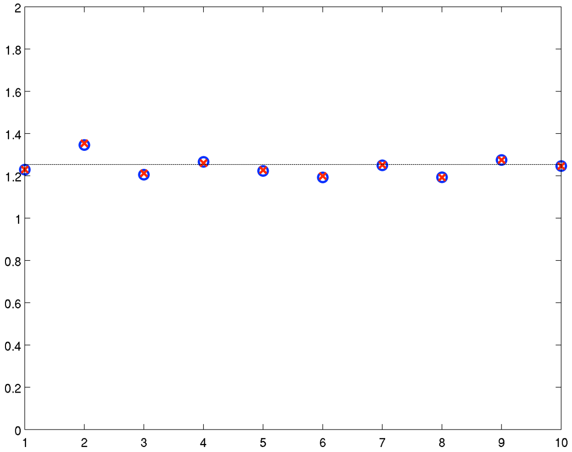
1 Snapshot, SNR = 20dB.

Simulations



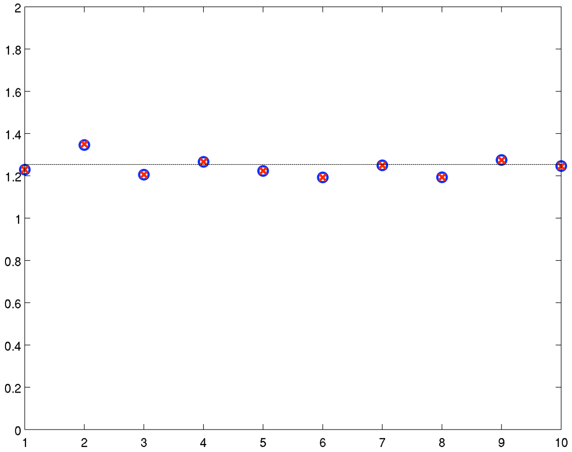
200 Snapshots, SNR = 0dB.

Simulations



200 Snapshots, SNR = 10dB.

Simulations



200 Snapshots, SNR = 20dB.

Conclusions

- Subspace approach seems to be effective for improving resolution in GNSS-R. It is supposed that the received amplitude returns are uncorrelated (this is acceptable after 1 ms) and that the wave shape remains substantially unchanged (this is also reasonable for hundreds of milliseconds).
- The setup is however not complete: we need realistic simulations of the received waveforms for computing delay-Doppler maps.
- Hardware should be modified to obtain oversampled waveforms.
- The introduced computational complexity is acceptable, especially using properties of quasi-circulant matrices.
- Doppler superresolution should be investigated.