OBSERVATION PERFORMANCE
OF A PARIS ALTIMETER
IN-ORBIT DEMONSTRATOR

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Acknowledgment to: Nicolas Floury, Roberto Pietro Cerdeira

TEC-ETP, Electrical Engineering Department
European Space Agency
# PARIS IoD Mission Summary Table

<table>
<thead>
<tr>
<th>Instrument</th>
<th>PARIS Altimeter</th>
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<td>Mesoscale Ocean Altimetry</td>
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<tr>
<th>In-orbit Demonstrator</th>
<th>Operational Mission</th>
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<tr>
<td>Orbit</td>
<td>Polar Sun Synchronous</td>
</tr>
<tr>
<td>Orbital Height</td>
<td>800 km</td>
</tr>
<tr>
<td>Swath</td>
<td>900 km</td>
</tr>
<tr>
<td>Revisit Time</td>
<td>3 days</td>
</tr>
<tr>
<td>Spatial Resolution</td>
<td>100 km</td>
</tr>
<tr>
<td>Antenna Diameter / Gain</td>
<td>0.9 m / 30 dB</td>
</tr>
<tr>
<td>Number of Beams</td>
<td>4</td>
</tr>
<tr>
<td>Frequencies</td>
<td>GPS L1+L5, Galileo E1+E5</td>
</tr>
<tr>
<td>Total Altimetry Accuracy (1σ)</td>
<td>&lt; 17-20 cm</td>
</tr>
<tr>
<td>Platform</td>
<td>PROBA-like</td>
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<td>Launcher Configuration</td>
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## Factors affecting altimetry accuracy

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| Allowed incidence angle of specular point | Higher incidence angle reduces the height precision, but increases swath  
  Scanning reduce antenna gain |
| Instrument Characteristics:            |             |
| 1. Antenna Gain, Receiver NF           | 1. Improve SNR |
| 2. XC Coherent integration time        | 2. Fundamental for performance optimization |
| Waveform Retracking Performance        | Optimum retracker to be defined |
| **Accuracy**                           |             |
| Uncertainties on Propagation:          |             |
| 1. Ionosphere                          | 1. Use of 2 or 3 Frequencies |
| 2. Troposphere                         | 2. Estimated or measured |
| 3. EM Bias                             | 3. Frequency Dependent |
| 4. Skewness Bias                       | 4. Fraction of EM Bias |
| Uncertainties on Geometry/Orbit        |             |
| Residual instrument errors after      | On-board Delay/Amplitude Calibration needed |
| calibration                             |             |
PARIS IOD System Concept

- Direct Cross-correlation (DxR)
  - High gain beams for direct signals (D)
  - High gain beams for reflected signals (R)
  - Implicit use of full GNSS bandwidth (3x40 MHz)
- Dual or Trial Frequency for precise estimation of ionospheric delay
- Precise on-board delay calibration
- Precise on-board amplitude calibration
PARIS IoD L1 Power Waveform

- GPS L1 Composite, Orbit Height: 500Km, Down-Looking Antenna
- Gain: 19dBi, Nadir Looking Geometry

Normalized Cross-Correlation Power Waveform
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Max Incidence Angle (1/2)

The incidence angle plays an important role in the definition of critical parameters of the PARIS mission, such as:

(a) Precision

(b) Sampling and Coverage
Max Incidence Angle (2/2)

a) Altimetric performance degrades with incidence angle

b) Ionospheric delay and antenna losses increase with incidence angle

(a) \[ dh = -\frac{d\rho}{2\cos i} \]

The incidence angle i should be minimised.
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                                          • Scanning reduce antenna gain |
| Instrument Characteristics:           |             |
| 1. Antenna Gain, Receiver NF          | 1. Improve SNR  
                                          2. To be optimized for maximum precision |
| 2. XC Coherent integration time        |             |
| Waveform Retracking Performance       | Optimum retracker to be defined |
| **Accuracy**                           |             |
| Uncertainties on Propagation:         |             |
| 1. Ionosphere                         | 1. Use of 2 or 3 Frequencies  
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| 2. Troposphere                        |             |
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Cross-Correlation Integration Time

Interferometric Processing Schematic

For a given target along track spatial resolution, e.g. 100Km, the coherent integration time and the incoherent averaging are inversely proportional:

\[ T_{100\text{Km}} = T_c N_{inc} \]
Power Altimetry Waveform

\[
\langle |Z_S(\tau)|^2 \rangle = \int_{\theta,\phi} P_{R,\theta,\phi} ACF(\Delta \tau) \frac{\sin(\pi \Delta f T_c)}{\pi \Delta f T_c}^2
\]

1 chip delay
The role of $T_c$

\[ \langle |Z_S(\tau)|^2 \rangle = \int_{\theta,\phi} P_{R,\theta,\phi} \left| ACF(\Delta \tau) \right|^2 \frac{\sin(\pi \Delta f T_c)}{\pi \Delta f T_c} \]

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**High $T_c$**

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**Pulse Limited Footprint**

**3dB Antenna Footprint**

**Doppler Footprint For Low $T_c$**

---

**RX**

---

**PARIS Flight Dir**
$SNR_{SP} \propto \int_{\theta, \phi} P_{R, \theta, \phi} \left| ACF(\Delta \tau) \frac{\sin(\pi \Delta f T_c)}{\pi \Delta f T_c} \right|^2 T_c$
Cross-Correlation Integration Time ($T_c$)

- Analytical model for altimetry precision:

$$\sigma_h = \frac{c P_{Z,S}}{2 \cos \theta_{inc,SP} P_{Z,S}'} \frac{1}{\sqrt{N_{inc}}} \sqrt{\left(1 + \frac{1}{SNR}\right)^2 + \left(\frac{1}{SNR}\right)^2}$$

$$N_{inc} = \frac{T_{100\,Km}}{T_c}$$

$$SNR \propto T_c$$

$$\sigma_h = \text{const}_1 \sqrt{T_c} \sqrt{\left(1 + \frac{1}{\text{const}_2 T_c}\right)^2 + \left(\frac{1}{\text{const}_2 T_c}\right)^2}$$

- $T_c$ shall be chosen such to have a sufficient SNR (e.g. 6dB) but cannot be increased further otherwise the altimetric precision is degraded because speckle is not averaged enough.

- However, $T_c$ cannot be too low otherwise consecutive XC samples may be correlated and speckle would anyway not be averaged.
Interferometric Processing SNR

- Up / Down-looking Antenna Gain and Up/Down RX NF shall be such to have sufficient SNR
- In PARIS IoD with interferometric processing, the final power waveform SNR is given as:

\[
SNR_{\text{int}}(\tau) = \frac{SNR_{cr}(\tau)}{1 + \frac{1 + SNR_R}{SNR_D}}
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Waveform Retracking

- The retracking has the objective of estimating the sea state parameters and the sea mean surface height by fitting a theoretical model to measured waveforms.
- Maximum likelihood estimation (MLE) or on weighted least squares estimation has been extensively used in Conventional Radar Altimetry.
- The MLE method estimates the parameters by determining which values maximize the probability of obtaining the recorded waveform shape in the presence of noise of a given statistical distribution.

\[ W(\tau) = f(\tau_{sp}, \sigma_0, SWH) \]
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The impact of Ionosphere

- At L-band, the ionosphere is a major contributor to the propagation delay

\[ \rho = -2h \cos i + I \]

with

\[ I = \frac{40.3}{f^2} TEC \]

- Multi-frequency observations allow to remove the ionospheric effect

\[ \rho_1 = -2h \cos i + \frac{I'}{f_1^2} \]

\[ \rho_5 = -2h \cos i + \frac{I'}{f_5^2} \]

\[ h_{15} = 1.26 \frac{\rho_5}{2 \cos i} - 2.26 \frac{\rho_1}{2 \cos i} \]

- However this is at the expense of a severe error amplification factor

\[ \sigma_{h_{15}} = 2.59 \sigma_\rho \]
Possible Ionospheric Correction Method

• Starting from range equation: \[ y = -h + \frac{x}{f^2} \]

• Multi-frequency observations are taken:
  \[ \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -1 & f_1^{-2} \\ -1 & f_5^{-2} \end{bmatrix} \begin{bmatrix} h \\ x \end{bmatrix} \]

• The ionospheric delays are retrieved with the following uncertainties:
  \[ \frac{x}{f_1^2} = -1.26 \frac{\rho_1}{2 \cos i} + 2.26 \frac{\rho_5}{2 \cos i} \]

• A regression is performed over N samples of ionospheric delay

• The smoothed ionospheric delay is used in the range equations, resulting in an improved error amplification factor

\[ y = -h + \frac{\langle x \rangle_N}{2 \cos i f^2} \]

\[ h_{15} = -\frac{1}{2} \frac{\rho_1 + \rho_5}{2 \cos i} + \frac{1}{2} \left( f_1^{-2} + f_5^{-2} \right) \frac{\langle I' \rangle_N}{2 \cos i} \]

\[ \sigma_h = \sqrt{\frac{1}{2} + \frac{1.78^2}{4N} + \frac{3.2^2}{4N}} \sigma_y \]

\[ \sigma_{h_{15}} = 1.08 \sigma_y \text{ for } N = 5 \text{ (4th order regression)} \]

\[ \sigma_{h_{15}} = 1.27 \sigma_y \text{ for } N = 3 \text{ (linear regression)} \]
Ionospheric Correction: Example 1/2

- What matters in mesoscale ocean altimetry is the difference between the ionospheric delay at two specular points.

Worst case is between nadir (i=0°) and edge of swath (i=30°).
Ionospheric Correction: Example 2/2

Vertical Delay (m)

Mesoscale Delay (m)

Residual Delay (m)

200 km averaging of mesoscale delay applied

Derived from RA-2 real data
## PARIS IoD Error Budget

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<th>Parameter</th>
<th>IOD L1 Height Accuracy on 100Km, G=20dBi, H=800Km</th>
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<tr>
<td>Instrument Noise and Speckle</td>
<td>12.5 cm</td>
</tr>
<tr>
<td>Ionosphere Averaging Noise</td>
<td>9.5 cm (2 frequencies, N=3)</td>
</tr>
<tr>
<td>Ionosphere Residual</td>
<td>5 cm</td>
</tr>
<tr>
<td>Troposphere (Wet and Dry)</td>
<td>5 cm</td>
</tr>
<tr>
<td>EM Bias</td>
<td>2 cm</td>
</tr>
<tr>
<td>Skewness Bias</td>
<td>1 cm</td>
</tr>
<tr>
<td>Orbit / Geometry</td>
<td>5 cm</td>
</tr>
<tr>
<td>Instrument error residuals</td>
<td>2 cm</td>
</tr>
<tr>
<td><strong>Total RMS Height Accuracy</strong></td>
<td>18 cm at Edge of Swath</td>
</tr>
</tbody>
</table>