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Study of dynamic behavior in cylindrical shell coupled with fluid

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Abstract

In this work, are free vibration of an empty or filled with an inviscid and incompressible fluid cylindrical tank is presented. The tank is modeled by a cylindrical shell and the fluid represented by an acoustic cavity, in which the impulsive and convective modes (sloshing modes) were investigated. The approach is based on two methods: an energetic formulation implemented in MAPLE software to obtain impulsive modes and the finite element method through ANSYS software to evaluate impulsive and convective modes. The fluid-structure interaction (FSI) is done in terms of fluid continuity conditions and boundary conditions at the fluid-structure interface. To validate the analyzes, the values found with the analytical procedure and numerical discretization were compared with the experimental results found in the literature. The natural frequencies, impulsive mode shapes and convective (sloshing) mode shapes were studied. The comparative analytical, numerical and experimental results of the literature were satisfactory.

Keywords: Cylindrical Tank, Fluid-Structure Interaction, Impulsive Modes, Sloshing Modes.

1. Introduction

Cylindrical shells are characterized by an object with a curved surface, small thickness compared to its other dimensions, and usually made of solid material. Structural dynamic analysis of shells is complex because they have flexural and in-plane vibrations that act together. Many simultaneous factors influence the dynamic response of a cylindrical shell, like the type of material inside it (fluid or solid), the boundary conditions of the shell, the presence or lack of a ring support, the influence of the connection between the shell wall and the bottom and the lid, among others.

The study of free vibration in cylindrical shells has been receiving a lot of attention by many researchers for many years. Based on the classic works of Donnell [1], Flügge [2], Vlasov [3], Timoshenko and Woinowsky [4], Sanders [5], many other theories were developed and improved upon to simulate the dynamic behavior of a shell. Other analytical techniques differ from each other by the strains and curvatures assumed for the shell, as well as by the solution methods used in the problem. Leissa [6] presents the different classic theories mentioned and many simulation results.

Based on the classic theories, there was a big advance in the analytical studies towards the calculation of cylindrical shells vibration characteristics containing fluid, it is worth mentioning the works of Lakis e Paidoussis [7], where they present a theory to study partially or completely filled with liquid cylindrical shells free vibration using cylindrical finite elements in the solution of the Sanders equations. Haroun [8] presents theoretical and experimental investigations of the dynamic behavior of cylindrical tanks with elastic supports.

Gonçalves and Batista [9] applied the Rayleigh-Ritz method using the energy formulation and created a theoretical analysis method to determine the vibration characteristics of partially filled or submerged cylindrical shells. Fernholz and Robinson [10], evaluated the free vibration in coupled cylindrical shells through the Donnell-Mushtari theory and the software NASTRAN.

Amabili and Dalpiaz [11], studied the free vibration of fully and partially filled circular cylindric tank theoretically and experimentally. Analytically, the shell was simulated by the Berry and Reissner theory and the fluid was modeled by the velocity potential function in the Laplace equation. França Júnior [12] evaluated the influence of the classic boundary conditions on the free vibration of coupled and uncoupled cylindric shells through analytical methods and numerical procedures. Ji et al. [13] developed an analytical method to calculate the free vibration characteristics of thick cylindrical shells filled with fluid. The fluid is modeled by the velocity potential incorporated in the shell wall by the impermeability condition.

In this article, the impulsive and convulsive modes (sloshing) in the vibration of a tank are investigated. The approach is based on an energetic formulation implemented in MAPLE software to evaluate the impulsive modes and the finite element method, through the ANSYS software, to study the impulsive and convective modes. The fluid-structure interaction is done in terms of fluid continuity conditions and boundary conditions at the fluid-structure interface. To validate the analyzes, the values found analytically and through the numerical discretization for the impulsive modes were compared with the experimental results obtained by Amabilli & Dalpiaz [11]. With the validated numerical modeling, the analyzes extended to sloshing modes of the tank. Therefore, the natural frequencies and mode shapes, with and without sloshing, of the tank are presented.

2. Analytical Formulation

The cylindrical shell is made of thin walls with length *L*, average radius *R*, radial angle ϕ and thickness *h*. The shell material is elastic with Young modulus *E*, Poisson coefficient μ and specific mass ρ. The displacement vector coordinates on the surface of the shell in the axial, circumferential and radial directions are *u*, *v* and *w*, respectively. Based on Flügge [2] linear theory, the shell strains and curvatures are given by:

$$\varepsilon_z{}^{(0)} = \frac{\partial u}{\partial z}; \quad \varepsilon_\emptyset{}^{(0)} = \frac{1}{R}\left(\frac{\partial v}{\partial \phi} + w\right);$$
$$\gamma_{z\phi}{}^{(0)} = \frac{\partial v}{\partial z} + \frac{1}{R}\frac{\partial u}{\partial \phi}\ ; \quad \chi_z = \frac{\partial^2 w}{\partial z^2}\ ; \tag{1}$$
$$\chi_\phi = -\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial \phi^2} - \frac{\partial v}{\partial \phi}\right)\ ; \quad \chi_{z\phi} = -\frac{1}{R}\left(2\frac{\partial^2 w}{\partial z \partial \phi} - \frac{\partial v}{\partial z}\right)$$

where (0) is the superscript related to the average surface, ε_z and $\varepsilon_\emptyset$ represent the membrane and transverse strains, respectively, $\gamma_{z\phi}$ represent the distortions, χ_z e χ_ϕ represent the bending curvatures and $\chi_{z\phi}$ represent the torsional curvatures. The total strains are assumed to be the sum of the membrane and flexural strains. Thus, the strain in any point of the shell is given by:

$$\varepsilon_z = \varepsilon_z{}^0 + x\ \chi_z$$
$$\varepsilon_\emptyset = \varepsilon_\emptyset{}^0 + x\ \chi_\phi \tag{2}$$
$$\gamma_{z\phi} = \gamma_{z\phi}{}^0 + x\ \chi_{z\phi}$$

where *x* is the distance from a point in the shell to the average surface. The strains and stress are based on Hooke's Law, and can be expressed as:

$$\sigma_z = \frac{E}{1-\mu^2}\left(\varepsilon_z + \mu\varepsilon_\phi\right)$$

$$\sigma_\phi = \frac{E}{1-\mu^2}\left(\varepsilon_\phi + \mu\varepsilon_z\right) \qquad (3)$$

$$\tau_{z\phi} = \frac{E}{2(1-\mu^2)}\gamma_{z\phi}$$

The equations of motion can be obtained by the Rayleigh-Ritz procedure using the Lagrange function (Γ), given by:

$$\Gamma = \mathrm{T} - \mathrm{U} \qquad (4)$$

where T is the maximum kinetic energy and U is the maximum strain energy potential of the cylindrical shell. The strain energy potential and the kinetic energy, according to Brush e Almoroth [14], are given by:

$$U = \frac{1}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_0^{2\pi}\int_0^L \left(\sigma_z\varepsilon_z + \sigma_\phi\varepsilon_\phi + \tau_{z\phi}\gamma_{z\phi}\right)dv \qquad (5)$$

$$T = \frac{\rho R h}{2}\int_0^{2\pi}\int_0^L \left[\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right] dz d\phi \qquad (6)$$

where *u*, *v* and *w* are, respectively, the longitudinal, circumferential and radial displacements and *t* is time. The equations for *u*, *v* and *w* displacements are given by:

$$u(z,\phi,t) = A\, u(z,\phi).\, e^{-j\omega t} \qquad (7)$$

$$v(z,\phi,t) = B\, v(z,\phi).\, e^{-j\omega t} \qquad (8)$$

$$w(z,\phi,t) = C\, w(z,\phi).\, e^{-j\omega t} \qquad (9)$$

where *e* is the Euler's number, *u*(z, ϕ), *v*(z, ϕ) and *w*(z, ϕ) are the modal shapes (modes of vibration), $\mathrm{j} = \sqrt{-1}$ is the imaginary unit, *A*, *B*, and *C* are constants that represent the amplitude of the axial (*u*), circumferential (*v*) and radial (*w*) respectively and *ω* is the natural frequency of vibration. In the work, the beam mode and its respective transcendental equation coefficients for a simply supported beam presented by Blevins [15] are used. The Rayleigh-Ritz method applied to the Lagrange function is given by:

$$\frac{\partial \Gamma}{\partial \mathrm{A}} = 0$$

$$\frac{\partial \Gamma}{\partial B} = 0 \qquad (10)$$

$$\frac{\partial \Gamma}{\partial \mathrm{C}} = 0$$

Thus, the system of three equations of motion is obtained. These equations can be written in a matrix format, with symmetrical terms, that leads to the following eigenvalues problem:

$$[\boldsymbol{K} - \omega^2 \boldsymbol{M}]\{X\} = \{0\} \tag{11}$$

Where $\{X\} = \{A \quad B \quad C\}^T$ is the eigenvector. The terms of the characteristic matrix $[\boldsymbol{C_{ij}}] = [\boldsymbol{K} - \omega^2 \boldsymbol{M}]$ consist in high complexity mathematical expressions, and the analytical computation of Det $[\boldsymbol{C_{ij}}] = 0$, which leads to the characteristic polynomial, and thus to the natural frequencies of the shell, becomes very hard and labor intensive. The problem has been solved using the MAPLE software.

2.1. Fluid-Structure Interaction

In the shell-cylindrical cavity fluid-structure interaction (coupled problem), two boundary conditions are present: the radial pressure at the shell surface is equal to the pressure on the cavity boundary, and the radial velocity at the shell surface is equal to the velocity on the acoustic cavity surface (impenetrability condition). Thus, as a simplification, the coupled shell modes of vibration are assumed to be equal to the structure modes of vibration in a vacuum.

Using the analytical method, the coupling of a simply supported cylindrical shell is done through an added fluid mass in the shell equation terms that correspond to the structure mass in the axial direction. The added mass equation for a partially or fully filled shell is presented by Gonçalves and Batista [9], given by:

$$\zeta = \left(\frac{\rho_f}{\rho}\right)\left(\frac{R}{h}\right)\left[\frac{H}{L} - \frac{sen\left(\frac{2m\pi H}{L}\right)}{2m\pi}\right]\left\{\frac{J_n(r_{\bar{\imath}})}{R\left[J'_n(r_{\bar{\imath}})\right]_{r=R}}\right\} \tag{12}$$

where ζ is the virtual added mass, ρ_f is the acoustic fluid density, $J'_n(r_{\bar{\imath}})$ is the Bessel function derivative. The added mass increases the inertial load in the shell radial direction. The shell displacements are bigger in the radial direction and including the added mass in this way yields more realistic results. The mass coupling is given by:

$$\left\{\begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} \\ k_{2,1} & k_{2,2} & k_{2,3} \\ k_{3,1} & k_{3,2} & k_{3,3} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,1} & m_{3,2} & m_{3,3} + \zeta \end{bmatrix}\right\}\begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = \{0\} \tag{13}$$

3. Numerical Formulation

The numerical analysis (Figure 1) was done using Finite Elements Method through the software ANSYS. The element SHELL63 was used to model the tridimensional cylindrical shell and the element FLUID30 was used to model the acoustic cavity. The modules Block Lanczos and Unsymmetric were used to solve the eigenvalues and eigenvectors problem. The mesh independence analysis was done and can be checked in França Jr. [16]. The numerical formulation used in the simulations is based on the structure displacement and fluid pressure (U-P), given by:

$$\begin{bmatrix} \boldsymbol{M_s} & \boldsymbol{0} \\ \boldsymbol{\rho FS} & \boldsymbol{M_f} \end{bmatrix}\begin{Bmatrix} \ddot{U} \\ \ddot{P} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{C_s} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{C_f} \end{bmatrix}\begin{Bmatrix} \dot{U} \\ \dot{P} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K_s} & -\boldsymbol{FS} \\ \boldsymbol{0} & \boldsymbol{K_f} \end{bmatrix}\begin{Bmatrix} U \\ P \end{Bmatrix} = \begin{Bmatrix} F_E \\ 0 \end{Bmatrix} \tag{14}$$

where {Fe} is a force vector, {U} is the structure displacement vector, {P} is the fluid pressure vector, **Ms** is the structure mass matrix, **Ks** is the structure stiffness matrix, **Cs** is the structure damping matrix, $\mathbf{M_f}$ is the fluid mass matrix, $\mathbf{K_f}$ is the fluid stiffness matrix, $\mathbf{C_f}$ is the fluid damping matrix, **FS** is the fluid-structure coupling matrix. In the system above, if $\{F_E\}$=0, **Cs**=0 and $\mathbf{C_f}$=0, the dynamic response obtained corresponds to the coupled undamped free vibration problem.

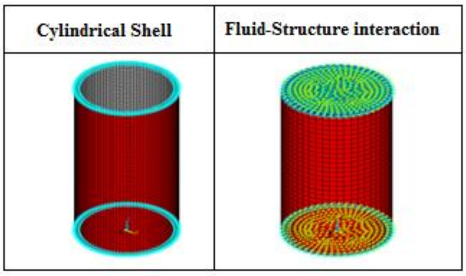


Figure 1: Numerical models.

4. Results

4.1. Impulsive modes

The first analysis is on the impulsive modes of tank in free vibration. The first six natural frequencies obtained through the analytical technique and the numerical method are presented and compared in Table 1. The index N represents the frequencies order, *m* is the number of longitudinal half waves, and *n* is the number of circumferential half waves. The lower natural frequencies are associated with the radial (w) modes of vibration.

Table 1: Comparisons of natural frequencies the cylindrical shell of impulsive modes the fluid.

N	***m***	***n***	**$\omega_{i,m,n}$ (Hz) Analytical**	**$\omega_{i,m,n}$ (Hz) FEM**	**Diff. (%)**	**$\omega_{i,m,n}$ (Hz) Experiment**	**Diff. (%)**
1	1	4	99,31	91,77	7,59	92,00	7,95
2	1	5	112,67	104,91	6,89	104,00	8,34
3	1	3	127,08	117,70	7,38	119,00	6,80
4	1	6	155,46	147,08	5,38	147,00	5,75
5	1	7	218,19	210,07	3,72	206,00	5,80

The coupled cylindrical shell analytical natural frequencies presented in Table 1 are satisfactory compared to the numerical results obtained with FEM using ANSYS and the experimental results. According to Lakis and Paidoussis (1971), the displacement amplitudes of a simply supported shell filled with liquid are smaller if compared to the empty shell due to the fluid influence. This behavior was observed herein, showing that the fluid presence reduces the natural frequencies and reproduce the same tendencies in the uncoupled shell. Figure 2 shows the impulsive modal shapes.

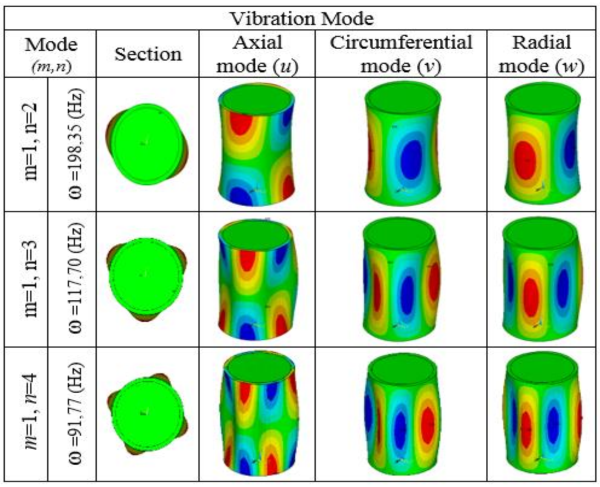


Figure 2: Natural frequencies and impulsive mode shapes of the tank.

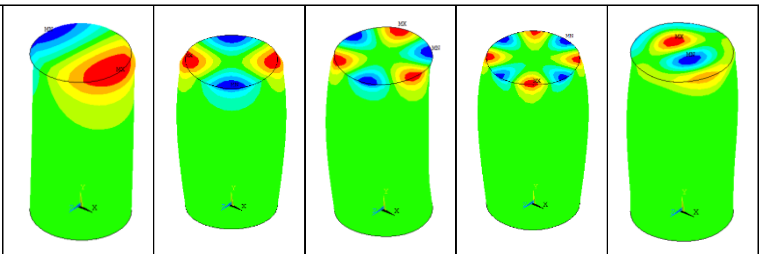
4.2. Convective modes

The second analysis is on the sloshing modes of tank in free vibration. With the analysis validated for impulsive modes, a free surface condition was applied to the numerical model. The first five natural frequencies obtained through the numerical method are presented in Table 2. The index N represents the frequencies order.

Table 2: Natural frequencies for the sloshing modes the tank.

N	**$\omega_{i,m,n}$ (Hz) FEM**
1	1,62
2	2,10
3	2,48
4	2,81
5	2,82

The impulsive pressure corresponds to high frequency oscillations, while the convective corresponds to low frequency oscillations. However, there is always a coupling between the free surface sloshing fluctuations of the liquid and the structural deformations. Thus, during a dynamic action the first modes to be excited are the sloshing modes, then the impulsive coupled modes, structure modes and acoustic cavity modes. Therefore, it is indispensable to impose the condition of free surface. Figure 3 shows the sloshing modes.



Mode Shapes ω (Hz)	1º 1,62 (Hz)	2º 2,10 (Hz)	3º 2,48 (Hz)	4º 2,81 (Hz)	5º 2,82 (Hz)

Figure 3: Natural frequencies and sloshing mode shapes of the tank.

When the excitation frequency is close to one of the natural frequencies of the sloshing liquid, large wave amplitudes can be generated. As a result, the hydrodynamic forces of sloshing waves affect the overall response of the tank and can cause damage to the top of the tank.

5. Conclusion

This work addressed the aspects of fluid-structure interaction in problems applied to the dynamic analysis of cylindrical tanks. The approach analytical, numerical and experiment was studied to investigate fully filled cylindrical shells free vibrations. In the problem study, the analytical technique based on the virtual added mass applied to the uncoupled shell formulation was satisfactory.

In the analysis of the impulsive modes, it was observed that despite having an influence in the system dynamic behavior, the fluid presence reproduced the characteristic structure dominant modes (same as the shell in a vacuum) in a significant range of frequencies analyzed. The convective modes (sloshing) corresponds to low-frequency oscillations, indispensable to dynamic analysis.

Thus, this work provides interesting results concerning the dynamic behavior of cylindrical shells containing fluid, which is a problem of big interest to Industrial Engineering.

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