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# Continuum form and force diagrams in NURBS-based shell form-finding

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## Abstract

This paper introduces force diagrams represented by a set of NURBS surfaces.

A NURBS-based computational form-finding method that utilizes Airy stress functions for continuum shell form-finding was presented at SIGGRAPH 2015 [1]. The Airy stress function also has been explored for form-finding and analysis of discrete 2D trusses, with the two graphics statics approaches being mathematically identical [2]. In 2D graphic statics, the connection between form and force diagrams and the Airy function is well-understood [3,4]. However, continuum form-finding methods, though it includes the Airy function, currently lacks force diagrams.

As multiple methods that manipulate force, or reciprocal, diagrams have already proven successful in the discrete case [5], force diagrams in the NURBS-based methods have a large potential to establish a user-friendly interactive interface to manipulate continuum shell form while maintaining equilibrium. Moving away from manipulation of the Airy function will improve the user interface because the shell surface is extremely sensitive to small changes in the Airy function, making it difficult to use directly in shell form-finding as in [1]. This is because shell stresses are related to the second derivatives of the Airy function, but only the first derivatives of the continuum force diagram. Manipulating the force diagram is therefore less sensitive than manipulating the Airy function.

This paper focuses on force diagrams in the NURBS-based form-finding methods and indicates that a pair of continuous form-force diagrams is a pair of surfaces whose deformation gradient defined between them is symmetric, or, rotation free, to each other. It will also be pointed out that a NURBS-force diagram is essentially an ellipse-packing of ellipsoids that represent stress tensors in the form-diagram. As an application of NURBS-represented force diagrams, the famous Xochimilco restaurant of Felix Candela is picked up and isogeometric form-finding analysis using NURBS force diagrams is explained. The discussion continues to numerical methods to obtain rotation-free pair of NURBS surfaces and, furthermore, interactive user-interface to allow the designers to manipulate force diagrams directly.

## References

- [1] M. Miki, T. Igarashi. and P. Block, Parametric Self-supporting Surfaces via Direct Computation of Airy Stress Functions, *ACM Transactions on Graphics (TOG)*, vol. 34, no. 4, 2015.
- [2] A. Mazurek, J. Carrion, A. Beghini, W.F. Baker, Minimum weight single span bridge obtained using graphics statics, Amsterdam: Future Visions IASS, 2015
- [3] T. Mitchell, W.F. Baker, A. McRobie, A. Mazurek, Mechanisms and states of self-stress of planar trusses using graphic statics, part I: the fundamental theorem of linear algebra and the Airy stress function, *International Journal of Space Structures*, 2016; 31, pp 85-101
- [4] C. Hartz, A. Mazurek, M. Miki, T. Zegard, T. Mitchell, W.F. Baker, The applications of 2D and 3D Graphic Statics in Design, *International Journal of Space Structures*, 2018, vol. 59, No. 4, pp 235-242
- [5] Rippmann M., Lachauer L. and Block P. RhinoVAULT - Interactive Vault Design, *International Journal of Space Structures*, 27(4): 219-230, 2012