Design and optimization of grid shell structures using Christoffel duality

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Abstract

Nowadays, free-form grid shell structures are a big part of contemporary architecture. Nevertheless, their geometry and construction properties can be rather unfitting when overlooking other aspects of the production and static performance, and therefore must be optimized. There are a lot of techniques for making the optimal performance of spatial grid elements, depending on the different construction and design goals. The major aim is to obtain equilibrium between the rod elements, as well as the desired shape. The overviewed parameters for optimization are direction, size and number of the elements.

In the paper, the authors deal with enabling static equilibrium within the grid meshes, by using geometry optimization methods, which were not used enough for purposes of shell structures designs. They include designing, discretization and modification of the meshes (size and edge's direction) with the help of the circle geometry patterns [1]. The edges of used meshes in the surface division are known for defining the direction of principal curvatures and therefore represent fundamental shape characteristics.

After getting the meshed surface, with the help of Christoffel duality of diagonal meshes, the authors will determine the static equilibrium of the grid shell. This technique is defined and mentioned within the Koebe mesh system (Fig. 1), that helps to define the regular edge offset meshes, and cannot be applied to arbitrary mesh system [2]. The Koebe mesh is used by different plane intersections on the sphere, making a variety of scaled inscribed circles of faces. That being said, the modified mesh has to satisfy certain criteria. All the meshes emerging from one vertex, of the designed surface, must construct the corresponding Christoffel dual mesh (Fig. 2). This means that transformed meshes must be connected at the same point, in order to obtain static equilibrium. Moreover, that will enable the higher stability of corresponding rods.

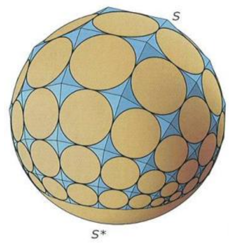

Fig. 1 Koabe meshes on the sphere [2]

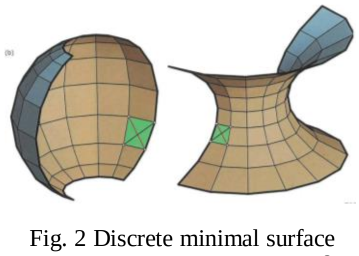

Fig. 2 Discrete minimal surface M obtained from Koabe mesh [2]

Bearing in mind that this cannot be the case for every free-form grid structure and design, the authors explore more techniques for shell design. Christoffel duality is not affected by different projective and affine transformations, so by using them, a lot of designs can emerge. As a result, a combination of these techniques will help to optimize the shape and position the grid elements in free-form like shells structure. In addition, it will show various interesting designs, ensuring their static performance in construction and stability, which remains the challenge for grid shells.

References

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[2] H. Pottmann, D. Bentley (eds.), *Architectural Geometry*, Exton : Bentley Institute Press, 2012.