

# Algebraic Formulation for Controlling the Face Areas of 3D Graphic Statics

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## Abstract

Geometric construction of the reciprocal polyhedrons as the form and force diagrams are at the heart of 3D/polyhedral graphic statics [1]. Moreover, the face areas of the force diagram are associated with the magnitude of the applied loads and internal forces in the form diagram. Therefore, controlling the face areas in the design process is quite crucial in structural form finding and optimization. In a previous paper, authors presented an algebraic formulation for the construction of reciprocal polyhedral diagrams by computing the edge lengths of closed polygons around each edge of the primal diagram [2, 3]. These polygons thus represent the faces of the dual and reciprocal diagram.

Although this formulation is a novel approach in constructing the reciprocal diagrams, it does not address how the areas of the faces of the resulting polyhedron could be controlled for form finding and optimization purposes. In this paper, we will expand on the algebraic relationship between the area of a face and its edges in the context of reciprocal polyhedral diagrams. In addition, the paper explains the solution space of the equation system comprising of the equilibrium equations and the equations describing the areas of the faces specified by the user.

The approach of this research allows controlling the areas of convex as well as self-intersecting (complex) faces with a positive and a negative regions per face. With the approach of this paper, a designer can even assign zero area for certain faces of the force diagram in the form finding process and remove their corresponding force component/member in the form diagram.

## References

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