

# From Polyhedral to Minimal Surface Funicular Spatial Structures

Mostafa AKBARI\*, Mohammad BOLHASSANI, Masoud AKBARZADEH

\* Polyhedral Structures Laboratory, School of Design, University of Pennsylvania, Philadelphia, USA  
Pennovation Center, 3401 Grays Ferry ave. Philadelphia, PA, 19146  
akbariae@design.upenn.edu

## Abstract

Exploring an unlimited variety of funicular spatial structural typologies is now feasible by using 3D (polyhedral) graphic statics [1]. These structural forms have polyhedral configurations including vertices, edges, polygonal faces, and cells where the edges of the cells can be materialized as members of a structure with a cross section associated with the magnitude of the force (area of the reciprocal face) in the force diagram. The structural performance of such systems relies heavily on the buckling performance of the members [2]. Besides, if the loading scenario changes, the mechanical behavior of the system will immediately change. Moreover, the fabrication process of such systems is quite challenging, due to its complex spatial configuration. Unlike polyhedral systems, minimal surface structures consist of a continuous surface where the internal forces are distributed on a surface with consistent mean curvature rather than the cross-section of the members. Recent studies show that the high surface-to-volume ratio in minimal surface geometries enhances cell proliferation and cell-to-cell interactions, maximizing both porosity and mechanical performance [3].

This paper proposes a novel approach to translate a polyhedral geometry of a funicular form designed by 3D graphic statics with  $n$ -manifold volumetric meshes to a minimal surface structure with two-manifold meshes for materialization purposes. Using such minimal surface structure for materialization in place of polyhedral geometry might improve the structural performance of the system and facilitate its fabrication process. Moreover, spatial shell structures can expand the boundaries of the thrust lines on their surface and thus improve the load-bearing capacity of the system under asymmetric loading scenarios. The proposed approach introduces a new typology of funicular spatial structures consisting of a minimal surface for given boundary conditions. Followed by introducing the computational procedure to generate such geometries, the paper concludes by evaluating the mechanical performance of a cellular specimen and a spatial continuous surface structure based on the same polyhedral geometry with the same volume of construction material.

## References

- [1] M. Akbarzadeh. *3D Graphic Statics Using Polyhedral Reciprocal Diagrams*. PhD thesis, ETH Zürich, Zürich, Switzerland, 2016.
- [2] A. T. Ghomi, M. Bolhassani, Nejur, and M. Akbarzadeh. The effect of subdivision of force diagrams on the local buckling, load-path and material use of founded forms. In *Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2018*, MIT, Boston, USA, July 2018.
- [3] S. C. Han, J. M. Choi, G. Liu, and K. Kang. A Microscopic Shell Structure with Schwarzs D-surface. *Scientific Reports*, 7(1):13405, 2017.